Mathematical Modeling of Multibeam Bathymetric Data Resolved Using Trigonometric Functions

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Abstract. Multibeam bathymetry is an important method for mapping seafloor topography in offshore waters. This method uses acoustic reflections to map underwater terrain accurately and is widely applied in marine resource development, environmental protection, and navigation safety. This study establishes two ocean models to solve multibeam bathymetry using trigonometric functions. To simplify, the seafloor is assumed to be a flat slope, and the transducer's opening angle is considered. The slope angle and depth at the center point are known, and the survey line direction varies. A new function model is constructed, similar to the objective function, and solved mathematically. In the simplified scenario, when the survey line is 600 meters from the center, the overlap rate is 3.21%, the most reasonable. In the optimized scenario, regardless of the distance, if the survey line angle is 90°, the coverage width remains 416.55 meters. This paper provides a new idea and method for ocean exploration.

Keywords: Multi-beam Measurement of Sea Depth, Trigonometric Model, Coverage Width, Single-test-line Direction.

1. Introduction

The multibeam bathymetric system is an important technical tool in marine cartography and seabed topographic surveys, which can provide high-precision underwater topographic data [1]. Multibeam bathymetric system is developed based on a single-beam bathymetric system, which can transmit dozens or even hundreds of beams at a time in the plane perpendicular to the track [2], and then receive the acoustic wave returned from the seafloor by the receiver transducer, and then determine the water depth by transmitting and receiving the acoustic wave reflections from the seafloor and map the seafloor topography. Due to its high efficiency and accuracy, this technology has a wide range of applications in marine resource development, marine environmental protection, and navigation safety [3].

In the actual measurement of the multibeam bathymetry system, the survey vessel usually adopts parallel routes to each other, which makes the overlap between neighboring strips [4]. Considering the change in seawater depth and other factors, the spacing of the parallel courses of the measuring vessel will affect the accuracy of the measurement data. Therefore, under the transducer opening angle and the water depth at the center point, we should investigate the relationship between the coverage widths of the bathymetric strips of different side lines, and make the coverage between the two side lines stay within a reasonable range.

In this paper, two specific seafloor situations are considered, and in a simplified situation, the intersection line between a plane perpendicular to the direction of the survey line and the seafloor slope is constructed to form a slanting line with an angle of the horizontal plane, which is called the slope. The opening angle of the multibeam transducer is 120 degrees, the slope is 1.5 degrees, and the depth of seawater at the center of the sea is 70 meters. From this data, the depth of the seawater, the coverage width, and the overlap with the previous line of measurement are calculated for different values of the distance of the line from the center. A mathematical model based on trigonometric relationships is constructed to solve the problem and the results are obtained using mathematics. In the optimization scenario, a rectangular sea area to be surveyed is constructed with the direction of the survey line and the projection of the normal direction of the seabed slope on the horizontal plane being β [5]. The angle between the direction of the survey line and the projection of the normal direction of the seabed slope on the horizontal plane is. In this process, the depth of deep water at the center of the sea area is 120 meters, and other data are consistent with the above data. At this point measure the measurement vessel from the center of the sea at different distances, and the measurement line direction angle is also different when the coverage width.

Also use the relationship between the trigonometric functions to create a mathematical model, according to the relationship between different degrees, three different models can be derived. Mathematics was used to find the results. This method provides fresh perspectives for ocean exploration [6].

2. Modeling Coverage Width and Overlap Rate in Multibeam Detection Systems

The data for this study were obtained from http://www.mcm.edu.cn/, according to http://www.mcm.edu.cn/Among the data, the data is first processed:

From 1 nautical mile is equal to 1852 meters, and converted the nautical miles in the data. The angles given in the data are also converted from the angular system to the radian system.

The simplified scenario requires the establishment of a mathematical model of the coverage width of the multi-beam sounding and the overlap rate between adjacent strips [7], through which the depth of the seawater measured at different distances from the center of the line, the coverage width and the overlap rate between adjacent strips can be solved by building a functional model. Since the depth of the central sea area and the direction of the survey line are determined, a trigonometric function can be used to construct a functional model of the depth of seawater measured at different distances from the center point. Based on this model, the sine theorem is used to construct a model of the coverage width. Finally, according to the spacing between two neighboring lateral lines, the overlap rate formula given in the question is used to solve the problem.

The optimization model is solved by building a function model [8]. Since the direction of the line is variable, the function is modeled by solving for the angle between the ray projected on the slope and the horizontal plane and is solved similarly to the simplified scenario.

3. Computational Approaches to Slope Line Angle Determination

To simplify the study, which is set here, θ Equal to 120 degrees, α Equal to 1.5 degrees.

Firstly, according to the trigonometric relationship, the seawater depth D can be modeled as a function of:

$$D = 70 - x \tan \alpha \tag{1}$$

Where X is the distance from the center of the sea.

On this basis, a model of the depth of the seawater as a function of the distance from the center of the sea is established. As shown in Figure 1:

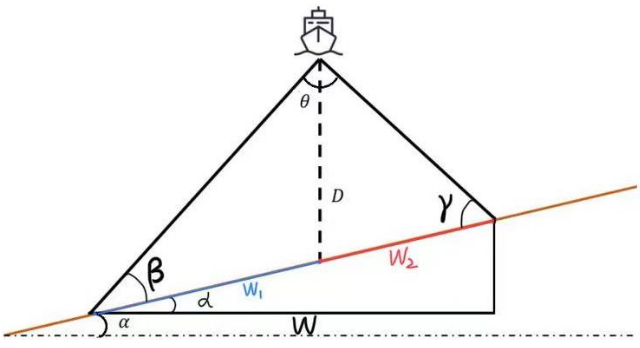

Figure 1. Schematic diagram of the simplified scenario

By the sine theorem:

$$\frac{W_1}{\sin\frac{\theta}{2}} = \frac{B}{\sin\beta}, \ \frac{W_2}{\sin\frac{\theta}{2}} = \frac{D}{\sin\gamma} \tag{2}$$

From the conversion of the angular relationship between the triangles, the following can be obtained:

$$\beta = \frac{\pi}{2} - \left(\frac{\theta}{2} + \alpha\right), \ \gamma = \frac{\pi}{2} + \left(\alpha - \frac{\theta}{z}\right) \tag{3}$$

Where β and γ represent only the angles labeled in Figure 1. Substituting (3) into (2) reduces to:

$$W_1 = \frac{\sin\frac{\theta}{2}}{\cos\left(\frac{\theta}{2}+\alpha\right)}, \ W_2 = \frac{\sin\frac{\theta}{2}}{\cos\left(\alpha-\frac{\theta}{2}\right)} \tag{4}$$

Combining Figures 1 and (4) and using trigonometric functions yields:

$$W = (W_1 + W_2)\cdot\cos\alpha = \left(\frac{1}{\cos\left(\frac{\theta}{2}+\alpha\right)} + \frac{1}{\cos\left(\alpha-\frac{\theta}{2}\right)}\right)\cdot\sin\frac{\theta}{2}\cdot\cos\alpha\cdot D \tag{5}$$

Substituting (1) into (5) yields a function of the coverage width:

$$W = \left(\frac{1}{\cos\left(\frac{\theta}{2}+\alpha\right)} + \frac{1}{\cos\left(\alpha-\frac{\theta}{2}\right)}\right)\cdot\sin\frac{\theta}{2}\cdot\cos\alpha\cdot(70 - x\tan\alpha) \tag{6}$$

Included among these θ Equal to 120 degrees, α Equal to 1.5 degrees.

According to the formula for the overlap rate [9]:

$$\eta = 1 - \frac{d}{W} \tag{7}$$

Using Mathematics, functions (1), (6), and (7) were used to input the variables, and the results were obtained as shown in Table 1:

Table 1. Solution results for the simplified case

The distance of the survey line from the center point/m	-800	-600	-400	-200	0	200	400	600	800
The depth of the sea/m	90.95	85.71	80.47	75.24	70.00	64.76	59.53	54.29	49.05
Coverage width/m	315.71	297.53	279.35	261.12	242.99	224.81	206.63	188.45	170.27
The overlap rate with the previous the distance of the survey line/%	——	36.65	32.78	28.40	23.42	17.69	11.04	3.21	-6.13

Now solve for the angle γ between the ray projected on the slope in the direction of the survey line and the horizontal plane.

As shown in Figure 2:

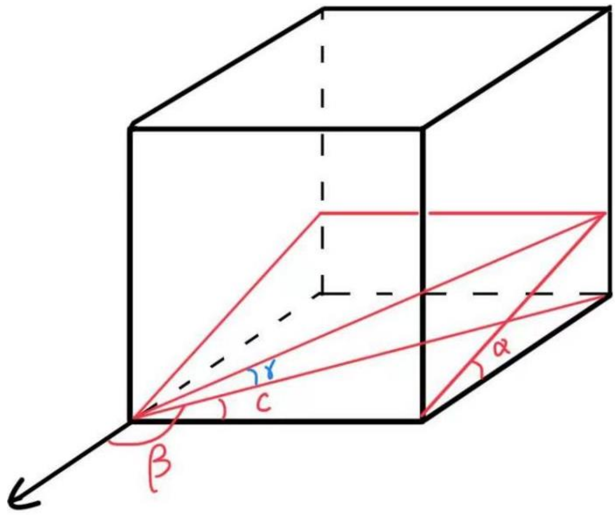

Figure 2. Schematic diagram of the optimization model

Assuming that the length of the side of the cube as shown in the figure is 1, the trigonometric relationship gives:

$$\tan\gamma = \frac{1\cdot\tan\alpha\cdot\tan c}{\frac{1}{\cos c}} = \tan\alpha\cdot\tan c\cdot\cos c = \tan\alpha\cdot\mathrm{sinc} \tag{8}$$

Based on the solution process of Problem 1, a new functional relationship is established:

$$W = \left(\frac{1}{\cos\left(\frac{\theta}{2}+\gamma\right)} + \frac{1}{\cos\left(\gamma-\frac{\theta}{2}\right)}\right)\cdot\sin\frac{\theta}{2}\cdot\cos\gamma\cdot(120 - x\tan\gamma) \tag{9}$$

The following discusses the relationship between the direction angle of the required sideline and the constructor function. As shown in Fig. 3, the:

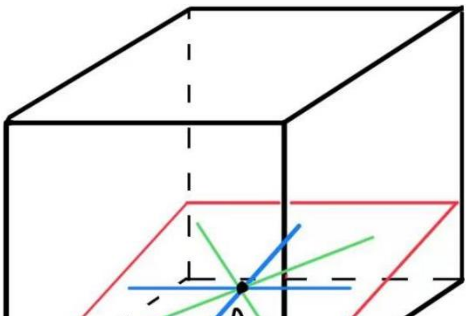

Figure 3. Schematic diagram of the optimization model

Where point A is the center point, the two green lines indicate the projection of the sideline on the slope when the angle of the sideline is 45 degrees 225 degrees, 135 degrees, and 315 degrees [10]. Observe the data of the direction angle of the sideline can be obtained: 0 degrees and 180 degrees, 45 degrees and 225 degrees, 90 degrees and 270 degrees, 135 degrees and 315 degrees of the sideline for the same line in both directions, so they are two and two of γ the same. According to Figure 3 can be seen, 45 degrees and 225 degrees, 135 degrees, and 315 degrees of the line on the center point of symmetry, so it has the same γ.

To summarize, only three cases can be discussed to solve the required case of different lateral line direction angles.

When the sideline direction angle is 0 degrees and 180 degrees, the objective function can be obtained:

$$W=\left(\frac{1}{\cos\left(\frac{\theta}{2}+\alpha\right)}+\frac{1}{\cos\left(\alpha-\frac{\theta}{2}\right)}\right)\cdot\sin\frac{\theta}{2}\cdot\cos\alpha\cdot(120-x\tan\alpha) \quad (10)$$

When the angle of the lateral line direction is 90 degrees and 270 degrees, the objective function can be obtained:

$$W=\left(\frac{1}{\cos\left(\frac{\theta}{2}+\alpha\right)}+\frac{1}{\cos\left(\alpha-\frac{\theta}{2}\right)}\right)\cdot\sin\frac{\theta}{2}\cdot\cos\alpha\cdot 120 \quad (11)$$

When the lateral line direction angle is 45 degrees versus 225 degrees, 135 degrees versus 315 degrees, the objective function can be obtained:

$$W=\left(\frac{1}{\cos\left(\frac{\theta}{2}+\gamma\right)}+\frac{1}{\cos\left(\gamma-\frac{\theta}{2}\right)}\right)\cdot\sin\frac{\theta}{2}\cdot\cos\gamma\cdot(120-x\tan\gamma) \quad (12)$$

Included among these $\tan\gamma=\frac{\sqrt{2}}{2}\tan\alpha$.

In functions (9), (10), (11), and (12), θ = 120 degrees α = 1.5 degrees, which denotes the distance to the center point (x can be negative due to constructing the two angles as a function).

Using Mathematics, functions (10), (11), and (12) were used, and the variable x was entered to find the results as shown in the Table 2:

Table 2. Solution results of the optimized model

Coverage width/m		Measure the distance of the ship from the center point of the sea/n mile							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
The angle between the direction of the measurement line/°	0	416.54	467.05	517.56	568.06	618.56	669.06	719.57	770.07
	45	416.55	451.79	487.47	523.14	558.82	594.49	630.17	665.84
	90	416.55	416.55	416.55	416.55	416.55	416.55	416.55	416.55
	135	416.55	380.45	344.77	309.10	273.42	237.75	202.08	116.40
	180	416.55	366.05	315.54	265.04	214.54	164.04	113.53	63.03
	225	416.55	380.45	344.77	309.10	273.42	237.75	202.08	116.40
	270	416.55	416.55	416.55	416.55	416.55	416.55	416.55	416.55
	315	416.55	467.05	517.56	568.06	618.56	669.06	719.57	770.07

4. Conclusion

This research focuses on seabed topography exploration using multibeam bathymetric technology, developing two mathematical models under different scenarios. This paper considered both the seabed slope and transducer's opening angle while analyzing variations in survey line directions, leading to the creation of a trigonometric-based mathematical model. These models encompass key parameters such as coverage width and line overlap rate, with a focus on optimizing multibeam measurements for different survey line directions and distances.

To validate the models, this study applied a range of mathematical tools and computational methods. The models were optimized and tested across various scenarios. Results show that, in the simplified scenario, the most efficient line overlap rate is 3.21%. In the optimized scenario, the coverage width remains consistent under different conditions, demonstrating strong practical potential.

The model is found to have good practical value and wide application prospects. Future research can further explore the application of the model in other ocean exploration fields and contribute to the development of China's marine industry.

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