

Fractal Characteristics of the Relationship between Industrial Water Use Efficiency and Economic Development at Multiple Temporal and Spatial Scales

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Abstract. The fractal theory is introduced to explore the stable fractal characteristics of the correlation between industrial water use efficiency (IWUE) and economic development level (EDL) at different spatial and temporal scales. The research was carried out based on statistical data including the water consumption per 10⁴ yuan of industrial added value (as IWUE) and per capita GDP (as EDL) of 336 cities in China from 1998 to 2017 and 142 countries across the world in 2017. The results show that during 1998-2017, the IWUE and the EDL of Chinese cities have improved gradually. Although the difference of IWUE and EDL among cities has gradually narrowed, the fractal dimension of the correlation between them has remained at around 1.45 in the past 20 years. The overall fractal characteristics is stable while the value of individual city is dynamic with time. Spatially, relationship between IWUE and EDL also maintains a stable fractal pattern for cities in different sub-regions and international countries. The stable fractal characteristics of EDL and IWUE at different spatial-temporal scales lays a theoretical foundation for further exploration of the structure of WUE regulation model and management target.

Keywords: IWUE, EDL, Fractal dimension, Correlation, Spatial-temporal scales.

1. Introduction

In 2012, the State Council issued the "Opinions of the State Council on Implementing the Strictest Water Resource Management System" (State Council Document No. 3 [2012]), which implemented the Strictest Water Resource Management Policy and also called the "Three Red Lines" policy [1]. By 2025, the Ministry of Water Resources and the National Development and Reform Commission plans that the national water consumption per 10k yuan of industrial added value (WC-IAV) is expected to decrease by 16% compared to 2020 [2]. Strict control of water use efficiency (WUE) is a key link in the implementation of the strictest water resource management policy [3], directly related to the control of total water use [4]. Exploring the key influencing factors of WUE, establishing the correlation pattern between driving factors and WUE, and investigating the collaborative rules of WUE regulation and prediction are important theoretical foundations for WUE management and a crucial starting point for water resource management in the new era [5].

WUE is influenced by many factors [6], among which the EDL is closely related to the efficient use of water resources [7, 8]. Many scholars have conducted research on the matching relationship between water resource use and economic development, and have confirmed that indicators such as matching degree [9, 10] and Gini coefficient [11, 12] can reflect the spatiotemporal matching status of economic development and water resource utilization to some extent. WUE improves with economic development, which is a universal development law proven by practice. However,

researches also show that the relationship between the two is not simply linear [13]. Various models have been used to simulate and predict changes in WUE [14, 15], all of which use economic indicators and can simulate and predict changes in WUE well [16, 17]. If the stable correlation pattern between WUE and EDL can be proven and established, the model structures used in existing research can be regarded as specific manifestations of this correlation pattern at specific spatial or temporal scales.

Fractal theory is a nonlinear scientific theory that can describe complex, disordered, and scale-invariant systems based on the self-similarity of a substance, exploring the fine structure within chaotic things [18]. Self-similarity theory has a wide range of applications in hydrology. Coastlines [19], water systems [20], and irrigation system canal structures [21] are manifestations of self-similarity at the spatial level. Fractal dimension is employed to describe similarity between the local and the entity, namely the fractal feature. Fractal theory can also be applied to the exploration of correlation between elements, such as soil particle size distribution [22] and rock surface fissure roughness [23].

The huge differences in the abundance of water resources and EDL at different temporal and spatial scales lead to complexity and irregularity in IWUE at the city, provincial, and international levels and at different temporal levels. Therefore, a stable self-similar pattern between EDL and IWUE may indicate a realistic correlation between the two, thus can serve as a theoretical foundation for IWUE regulation and prediction model. This paper takes the statistical data of 336 cities in China and 142 countries internationally as the research object, based on fractal theory and self-similarity, explores the evidence of an independently stable connection between EDL and IWUE from spatiotemporal scales. The conclusions can provide theoretical support for the subsequent establishment of a regional IWUE regulation model.

2. Research Methods and Data Sources

2.1. Data Sources

The basic data involved in this study include water use and socio-economic development data from 1998 to 2017. Domestic data are based on the urban administrative divisions announced by the Ministry of Civil Affairs, involving 336 prefecture-level and above administrative units in China, accounting for 99.7% of the national administrative units. Domestic water use data are mainly excerpted from the "China Water Resources Bulletin" and provincial "Water Resources Bulletins" from 1998 to 2017; socio-economic development data include industrial added value, GDP, total population, etc., mainly from the "Statistical Yearbook" and "China City Statistical Yearbook" from 1998 to 2017, and GDP, industrial added value, etc., are all converted to comparable prices in 2010 according to corresponding indices. International water use data mainly come from the FAO AQUASTAT database, and international economic data come from the World Bank WDI database; a total of 142 countries are involved, accounting for 72.8% of the total number of countries in the world that year.

IWUE is characterized by WC-IAV ($\text{m}^3/10\text{k}$ yuan domestically, $\text{m}^3/10\text{k}$ dollars internationally), and the indicator of EDL is per capita GDP (10k yuan/person domestically, 10k dollars/person internationally).

2.2. Analysis Methods

2.2.1. Calculation of Fractal Dimension

The fractal dimension in space can be calculated using the box-counting method[24]. Cover the study area with a square grid of side length l , and then count the number of grid cells containing the study object N (number of non-empty grids); changing the size by box length l , the number of grid cells $N(l)$ required to cover the study object at different l scales can be obtained.

If the length l and the number of non-empty grid cells $N(l)$ have a linear relationship on a double logarithmic coordinate axis, or if a straight line segment appears in the scatter plot, it can be judged

that the morphology of the study object satisfies the fractal rule [25]. The side line segment is called the scale-free region, whose slope D is the fractal dimension, which can be obtained by least squares regression:

$$D = -\lim_{l \rightarrow \infty} \frac{\log N(l)}{\log l} \quad (1)$$

The fractal dimension D is affected by both the degree of data irregularity and sample size. Under the condition that the sample size is similar, the more dispersed the point set, the larger the fractal dimension, and the weaker the regularity between elements; if the regularity between elements is similar, the larger the sample size, the larger the fractal dimension, and the higher the reliability of the regularity between elements.

2.2.2. Kernel Density Calculation

The spatial density of data points is quantitatively characterized using kernel density[25], and the density value calculation formula is as follows:

$$f(x, y, vX, vY, w_x, w_y) = \frac{1}{n} \sum_{i=1}^n \frac{1}{2\pi w_x w_y} \exp \left(-\frac{(x - vX_i)^2}{2w_x^2} - \frac{(y - vY_i)^2}{2w_y^2} \right) \quad (2)$$

Where xy is the defined grid, n is the number of points in vector vX or vY , vX_i is the i th element in vector vX , vY_i is the i th element in vector vY , and w_x and w_y are bandwidth values.

3. Results and Analysis

3.1. Temporal Stability of Fractal Character between IWUE and EDL in Cities

3.1.1. Overall Temporal Stability

From 1998 to 2017, the national average WC-IAV decreased from 196 m³/10k yuan to 47 m³/10k yuan, and per capita GDP increased from 1.3 10k yuan/person to 5.7 10k yuan/person. The national overall IWUE and EDL have steadily improved, and the differences in economic development and WUE among most cities are gradually narrowing. The fractal dimension values of the scatter plots of per capita GDP and WC-IAV for 336 cities in 1998, 2008, and 2017 are relatively close, and the overall shape of the highest part of the kernel density of the data points is highly similar (Fig 1), indicating that the correlation between IWUE and EDL has stable self-similarity over nearly 20 years.

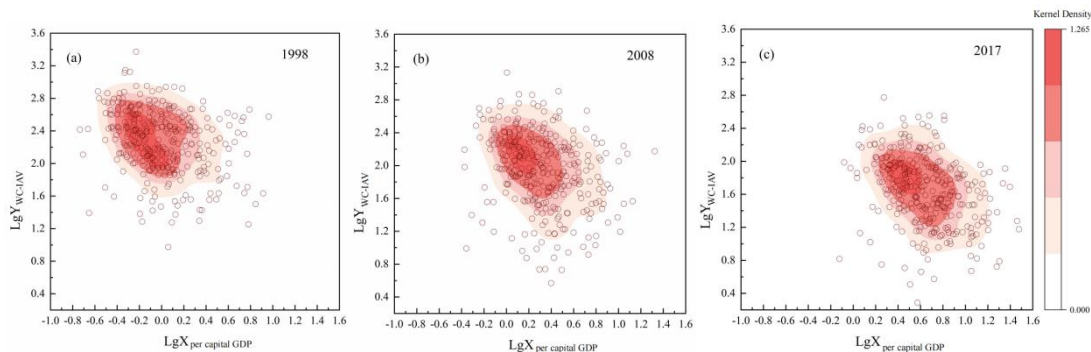


Figure 1. Point sets and Kernel density map of EDL and IWUE in China's cities in (a)1998 (b)2008 (c)2017

Calculating the fractal dimension of the scatter plots of WC-IAV and per capita GDP for Chinese cities from 1998 to 2017 (Table 1), the fractal dimension of each year has a significant linear relationship with the measure l and the number of non-empty grid cells $N(l)$, with a correlation coefficient above 0.99, showing an obvious fractal feature. The average fractal dimension of each year is 1.45, with a variance of 0.028. This indicates that the relationship between IWUE and EDL among cities involves stably, rarely affected by external factors.

Table 1. Fractal dimension of IWUE and EDL from 1998 to 2017

Year	D	Year	D	Year	D	Year	D
1998	1.401	2003	1.429	2008	1.477	2013	1.416
1999	1.405	2004	1.452	2009	1.448	2014	1.438
2000	1.447	2005	1.486	2010	1.459	2015	1.416
2001	1.469	2006	1.505	2011	1.444	2016	1.475
2002	1.470	2007	1.459	2012	1.475	2017	1.437

3.1.2. Changes in Typical Cities

To reveal whether the coordinated change rule of IWUE and economic development is static and fixed or dynamic and adaptive, some typical cities are selected and their time trajectories on the scatter plot is analyze. Beijing, Tianjin, Shanghai, and Chongqing, the four municipalities directly under the central government, are selected as typical cities, and all four cities are economically developed cities in their respective regions. In 2017, the WC-IAV for Beijing, Tianjin, Shanghai, and Chongqing was 12.4, 10.0, 43.9, and 85.4 m³/10k yuan, respectively, and per capita GDP was 11.65, 10.83, 12.81, and 4.71 10k yuan/person, respectively. The IWUE and EDL of Beijing, Tianjin, and Shanghai are higher than the national average, while Chongqing is both lower than the national average.

As depicted in the position changes of the four typical cities in the scatter plot from 1998, 2008, and 2017 (Figure 2), the relative position of Chongqing in the scatter plot remains stable, indicating that its economic development and the improvement of IWUE maintain the same speed as the overall national level. The relative positions of Beijing and Shanghai in the overall national city population move downward, indicating that the IWUE improves faster than the national average. The EDL of Tianjin is synchronized with that of Beijing. Beijing's IWUE was significantly lower than that of Tianjin in 1998, and catches up Tianjin by 2017, reflecting a significant improvement in Beijing's IWUE. The IWUE and EDL of Tianjin are both leading in the nation and remains stable in the plot with limited space for further improvement.

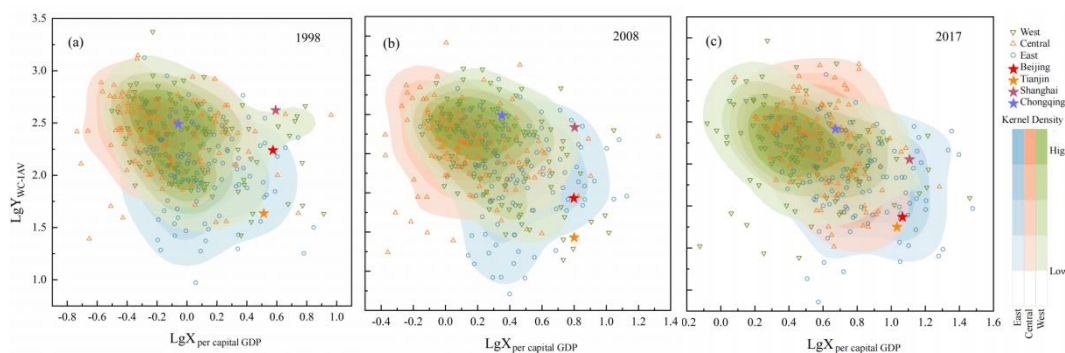


Figure 2. EDL and IWUE point sets and zoning kernel density in Chinese cities in (a)1998 (b)2008 (c)2017

In EDL and WUE point set, the position change of individual city points reflects its individual development path relative to the nation. The relative position change between cites reflects the different improving pace in EDL and WUE. In the temporal dimension, the development pattern of IWUE and EDL in Chinese cities is generally stable, but for specific city, the condition is dynamic rather than static. This "dynamic" reflects the local differences in water resource endowment and management policies.

3.2. Fractal Characteristics between IWUE and EDL at Different Spatial Scales

The dynamic interplay between IWUE and economic development raises questions about the stability of their fractal characteristics at different spatial scales. Given the spatial variations in water resource distribution, development, use patterns, economic conditions, and industrial structures

across regions and countries, it is crucial to examine whether the fractal patterns of this relationship persist across different spatial dimensions.

3.2.1. Fractal Characteristics in China's Regional IWUE and EDL

Chinese cities are categorized into three economic zones: East, Central, and West (Table 2). In 2017, the average WC-IAV was 32.85 m³/10k yuan in the East, 71.64 m³/10k yuan in the Central region, and 58.13 m³/10k yuan in the West, with corresponding per capita GDP of 8.16 10k yuan/person, 4.33 10k yuan/person, and 3.41 10k yuan/person, respectively. The East exhibits higher EDL and IWUE, as reflected in the kernel density plot (Figure 3), with cities in the West generally lagging in economic development and the Central region showing lower IWUE.

The fractal dimensions for each region reveals linear distributions in log-log plots with squared correlation coefficients above 0.99, indicating significant fractal characteristics. The fractal dimensions were 1.352, 1.337, and 1.373 in 1998; 1.351, 1.341, and 1.341 in 2008; and 1.299, 1.304, and 1.312 in 2017, with the maximum difference between regions being 0.053, suggesting stable fractal features. The highest kernel density regions in 2017 (Figure 2) show slight variations, with the Central region's axis tilting negatively, indicating that cities with higher WUE also have higher economic levels, while the West's axis is roughly horizontal, suggesting a lesser contribution of economic development to WUE. Despite these differences, the overall fractal characteristics of the relationship between economic development and WUE remain highly stable across regions.

Table 2. Division of three major regions

Region	Provinces	Cities
East	Beijing, Tianjin, Hebei, Liaoning, Shanghai, Jiangsu, Zhejiang, Fujian, Shandong, Guangdong, Hainan	110
Cental	Shanxi, Jilin, Heilongjiang, Anhui, Jiangxi, Henan, Hubei, Hunan	112
West	Niemen, Guangxi, Chongqing, Sichuan, Guizhou, Yunnan, Xizang, Shaanxi, Gansu, Qinghai, Ningxia, Xinxiang	114

3.2.2. Fractal Characteristics of the Relationship at the International Scale

Expanding the analysis to the international scale, we examine the fractal characteristics of IWUE and economic development across countries. In 2017, China's industrial water use was 1,335 billion m³, second only to the United States, and its IWUE ranked 94th out of 142 countries with available data. The fractal dimension from 142 countries in 2017 is 1.37 with a squared correlation coefficient of 0.99 (Fig 3), indicating fractal characteristics in the international scale.

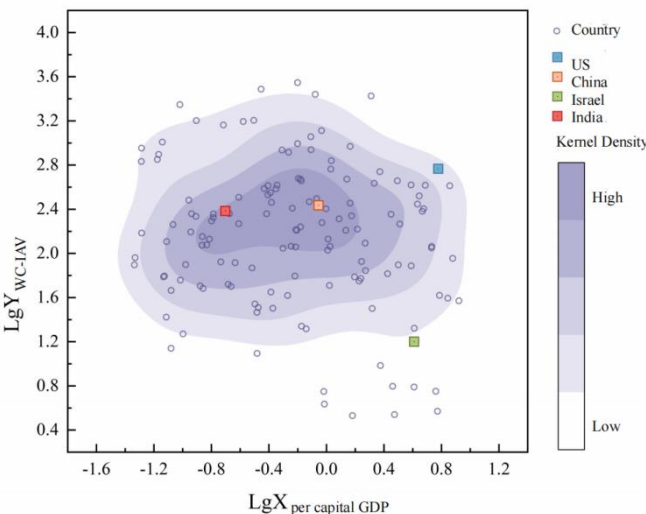


Figure 3. the international EDL and IWUE point sets, typical countries and kernel density in 2017

Examining specific countries such as the US, China, India, and Israel, we observe that Israel, despite its high economic development, has the highest IWUE. China, the United States, and India

are major industrial water users, with the United States showing low IWUE despite its economic prowess. Israel stands out with both high EDL and IWUE, while China and the US are within the international norm.

Our findings indicate that the fractal characteristics of the relationship between IWUE and economic development are consistent across time and space, and such stable synergistic development pattern may provide a methodological foundation for further exploration of regulatory models. However, economic development is a stable but not exclusive factor influencing IWUE. Future research should delve deeper into these potential influencing factors to compile a comprehensive set of determinants, enabling the development of more accurate IWUE regulatory scenario models. This will assist urban management in adjusting factors affecting IWUE and promoting its improvement.

4. Conclusions and Discussion

a. From 1998 to 2017, the national average WC-IAV decreased from 196 m³/10k yuan to 47 m³/10k yuan, with an average annual decline of 7.66% per five years. Per capita GDP increased from 1.3 10k yuan/person to 5.7 10k yuan/person, with an average annual growth rate of 8.88% per five years. The IWUE and EDL in China have steadily improved, with the disparities between most cities gradually narrowing.

b. The fractal patterns of the relationship in the temporal dimension show a dynamic system that is overall stable but subject to individual adjustments. Over the period 1998-2017, the average fractal dimension of the scatter plot of log-transformed per capita GDP and WC-IAV for 336 Chinese cities was 1.45, exhibiting stable fractal characteristics. Specific cities may deviate from the average development rate due to regional conditions and economic structures.

c. In the spatial dimension, the relationship between IWUE and EDL also shows stable fractal characteristics. In 2017, the fractal dimensions were 1.299, 1.304, and 1.312 for the East, Central, and West regions, respectively, and 1.37 for the 142 countries internationally. The relationship between IWUE and EDL maintains stable fractal patterns across China's cities and international countries.

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