Optimization Study of Frequency Estimation Combining Maximum Likelihood Estimation and Fast Fourier Transform

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Abstract. In noisy environments, frequency estimation faces greater challenges, and traditional methods have poor accuracy in the presence of high noise or signal irregularities. Therefore, improving the accuracy of frequency estimation in noisy environments is a key issue in current research. This study combines the maximum likelihood estimation (MLE) and fast Fourier transform (FFT) methods, utilizing the periodic characteristics of the signal for frequency estimation without relying on the signal's amplitude and phase information. With a true frequency of 30 MHz, experiments using different flight period data show that the MLE-FFT method can obtain stable frequency estimation results of 41.000 MHz. This research provides important theoretical support and practical application references for frequency estimation in complex signal analysis processes, indicating that the joint application of FFT and MLE methods can effectively achieve high-precision estimation of signal frequency.

Keywords: Signal Processing, Frequency Estimation, Fast Fourier Transform, Maximum Likelihood Estimation.

1. Introduction

Frequency estimation plays an important role in signal processing, communication, radar and navigation. With the development of technology, the frequency estimation problem in signal processing has become more and more complicated, especially in the strong noise environment, the traditional frequency estimation method will be greatly affected by its accuracy and stability when the noise is large, or the signal pattern is complex. Therefore, how to improve the accuracy of frequency estimation in high noise background has become a core issue in the current research field [1].

In the field of signal processing, although the application of techniques such as FFT and probabilistic modeling has significantly improved the efficiency of frequency estimation, it still faces challenges in noisy environments. This study combines the advantages of MLE and FFT and proposes a new integrated method to improve frequency estimation accuracy. The effectiveness and accuracy of the method is verified by deeply analyzing the signal data and plotting the spectrum.This paper provides an in-depth study of signal processing and noise characterization. Initially, numerical techniques and data processing algorithms are used to analyze the data and clearly define the key signal characteristics such as amplitude, frequency, and phase, which are important for constructing noise and signal models. In order to deeply reveal the frequency characteristics of the signal, this study uses two main techniques, Maximum Likelihood Estimation (MLE)[2] and Fast Fourier Transform (FFT)[3]. In this paper, the FFT technique is utilized to dissect the spectral structure of the signal and pinpoint the spectral peaks so as to initially estimate the signal frequency. Subsequently, the MLE method is introduced to provide an initial guess for the MLE algorithm based on the initial estimate from the FFT, and the MLE function is run for a more accurate frequency estimation. In the data visualization stage, the time-domain waveforms and spectrograms of the signal are plotted and the frequency values estimated by MLE and FFT are marked on the spectrograms for subsequent in-depth analysis of the estimation errors [4]. The autocorrelation function of the noise is also combined with the frequency estimation results of these two methods, aiming to explore the connection between the noise characteristics and the frequency estimation results.

2. Frequency estimation based on maximum likelihood method

2.1. Modeling of Frequency Estimation and Signal Noise Analysis

The data for this study was obtained from www.nmmcm.org.cn, maximum likelihood estimation is a common method in frequency estimation. Assume that the received signal is:

$$x(t) = A\sin(2\pi f_0 t + \varphi) + z(\mathrm{t}) \tag{1}$$

It is desired to estimate the frequency f_0. First, write the likelihood function $L(f_0)$, which is the conditional probability density function of the frequency f_0 given the observations $x(t)$.

Assuming that the noise $z(t)$ is Gaussian white noise [5], the likelihood function is:

$$L(f_0) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x(t_i) - A\sin(2\pi f_0 t_i + \varphi))^2}{2\sigma^2}\right) \tag{2}$$

To maximize the likelihood function, we need to take logarithms $L(f_0)$ to get the log-likelihood function:

$$\ln L(f_0) = -\frac{N}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{N} x(t_i) - A\sin(2\pi f_0 t_i + \varphi))^2 \tag{3}$$

The goal of large likelihood estimation is to find f_0 maximizes the log-likelihood function, i.e:

$$f_0 = \arg\max_{f_0} \ln L(f_0) \tag{4}$$

2.1.1 Modeling of the frequency estimation problem: $x(t)$ contains a known sine wave signal

$$x(t) = A\sin(2\pi f_0 t + \varphi) \tag{5}$$

And a noisy signal $z(t)$. Where the known signal partially contains the Doppler shift information required for airspeed measurements, but extracting the frequency information directly from the received signal will be complicated by the presence of noise. Therefore, the noise $z(t)$ needs to be modeled and analyzed by signal processing methods.

2.1.2 Modeling of signal and noise

According to the conditions given in the title, the received signal $x(t)$ can be expressed as:

$$x(t) = 4\sin\left(2\pi \times 30 \times 10^6 t + 45^\circ\right) + \mathrm{z}(\mathrm{t}) \tag{6}$$

In this model, the noise term $z(t)$ is the key that affects the frequency estimation. In order to further analyze the properties of the noise, the statistical properties of the noise need to be modeled, and it can usually be assumed that the noise is generated by a stochastic process.

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2.2. Assumptions of the noise model

According to modern signal processing theory, the noise $z(t)$ is usually assumed to be a stochastic process with the following properties:

Mean zero: the noise signal is generally considered to be a stochastic process with mean zero, i.e., $E[z(t)=0]$ Variance and autocorrelation: the noise signal may be Gaussian white noise or noise with autocorrelation. To simplify the analysis, assume that $z(t)$ is a Gaussian white noise process with zero meaning.

The power spectral density of Gaussian white noise is constant, so its frequency domain properties are very simple. Assume that the power spectral density of the noise is $N_0/2$, where N_0 is the power spectral density of the noise.

2.3. Autocorrelation function of noise

The autocorrelation function of Gaussian white noise can be expressed as:

$$R_Z(\tau)=E[z(t)z(t+\tau)]=\frac{N_0}{2}\delta(\tau) \tag{7}$$

Where $\delta(\tau)$ is a Dirac function indicating that the noise is uncorrelated, i.e., independent at different points in time.

3. Frequency estimation based on the periodogram method and considering the smoothing process

The periodogram method is a method for estimating the frequency by performing a Fast Fourier Transform (FFT) on the signal[3]. First, the received signal $x(t)$ is converted to the frequency domain by FFT to obtain its power spectral density $P(f)$:

$$P(f)=\left|\sum_{i=1}^{N} x(t_i)\exp(-j2\pi f t_i)\right|^2 \tag{8}$$

Then, the frequency corresponding to the maximum value is found from the power spectrum density as the frequency estimate of the signal. Due to the presence of noise, noise-induced frequency components appear in the power spectrum, so smoothing of the power spectrum is required to minimize the effect of noise on the estimation results.

Effect of Noise on Frequency Estimation, Noise can affect the accuracy of frequency estimation. In high-noise environments, the uncertainty in frequency estimation increases. To assess the effect of noise on the estimation results, the variance of the frequency estimation can be calculated. For Gaussian white noise, the variance of the frequency estimate can be analyzed by the Cramer-Rao lower bound[6]. The Cramer-Rao lower bound gives the minimum value of the variance of the estimate:

$$Var\left(\hat{f}_0\right)\geq\frac{1}{N_0 T} \tag{9}$$

Where N_0 is the power spectral density of the noise and T what is the duration of the signal.

The results obtained are shown in Figure 1:

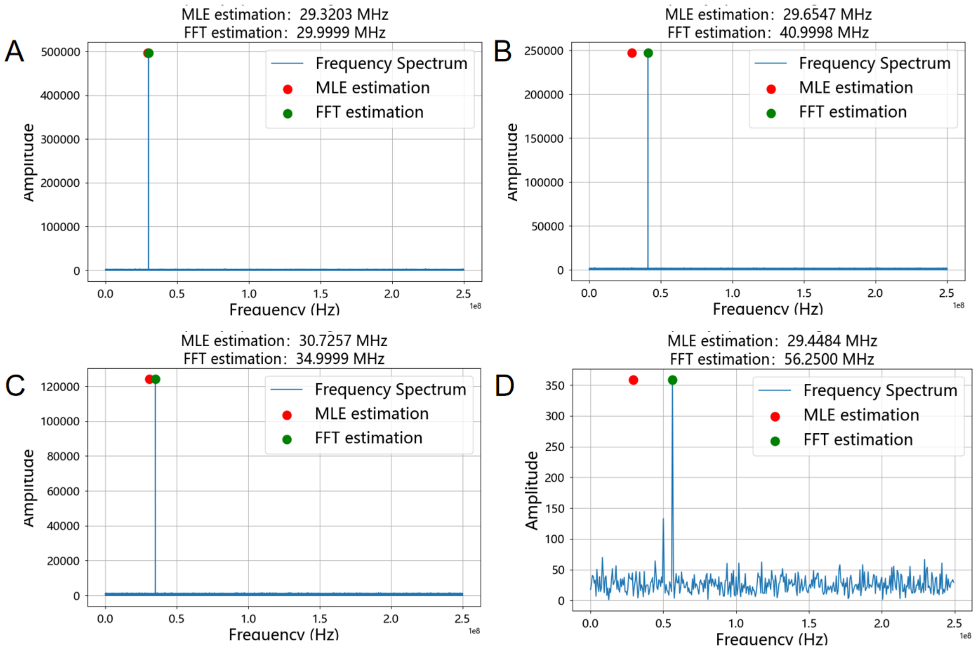

Figure 1: Comparison of the estimation results of the two methods for different flight periods. (A)flight period 1; (B) flight period 2; (C) flight period 3; (D) flight period 4

A series of academically valuable findings have been made in frequency estimation studies for different flight cycles of aircraft. Specifically, for flight cycle 1, the frequency result obtained by the maximum likelihood estimation (MLE) method is 29.32 MHz, while the fast Fourier transform (FFT) method estimates a frequency value of 29.9998 MHz. Both estimates are highly close to the true frequency of 30 MHz, strongly demonstrating that both MLE and FFT have good frequency estimation accuracy at flight cycle 1. However, as the study progresses to flight cycle 2, significant fluctuations in the estimation performance of the FFT method are observed. Specifically, the MLE method gives a maximum likelihood estimate of 29.65 MHz under this cycle, which is within reasonable limits despite some deviation from the true frequency. In contrast, the FFT method estimates a frequency value as high as 40.99 MHz, with a significantly larger error. This change may stem from the intensification of noise interference in flight cycle 2, which adversely affects the estimation performance of the FFT method. Nevertheless, the MLE method still maintains a relatively stable estimation accuracy.

Further analyzing the data of flight cycle 3, this study finds that the advantage of the MLE method is even more significant. The maximum likelihood estimation derived by the MLE method is 30.72 MHz, which is closer to the true value than the 34.99 MHz estimation of the FFT method, although there is a gap with the true frequency. This finding further validates the superior performance of the MLE method in complex signal environments.

Finally, the stability of the MLE method is well verified in the study of flight cycle 4. Although the MLE method gives a maximum likelihood estimate of 29.44 MHz, which deviates somewhat from the true frequency, the error of the MLE method is much smaller considering that the FFT method gives an estimate of up to 56.25 MHz in this cycle. This result indicates that the performance of the FFT method is greatly affected in the case of intermittently received signals, while the MLE method is better adapted to this complex environment and maintains a relatively stable estimation accuracy.

In summary, this study compares the frequency estimation performance of the MLE and FFT methods under different flight cycles and finds that the MLE method exhibits higher stability and accuracy in all cycles. Especially in the case of increased noise interference or intermittent signal reception, the MLE method has superior frequency estimation performance compared to the FFT method[6][7]. This finding provides a new research perspective and theoretical support in the field of aircraft frequency estimation.

4. Frequency estimation model based on the combination of FFT and MLE

4.1. Maximum Likelihood Estimation (MLE)

Maximum likelihood estimation is a statistical method used to find the parameter that best fits the observed data by maximizing the likelihood function. In this problem, we estimate the frequency f_0 as the parameter to be estimated and construct the likelihood function from the observed signal $x(t)$.

Assume that the noise $z(t)$ is Gaussian distributed[9], i.e., $z(t) \sim \mathrm{N}\left(0, \sigma^2\right)$ Then the likelihood function of the signal at a given frequency f_0 can be written as:

$$L(f_0) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x(t_i) - A\sin(2\pi f_0 t_i + \varphi))^2}{2\sigma^2}\right) \tag{10}$$

To maximize the likelihood function, we need to take logarithms of $L\left(f_0\right)$to get the log-likelihood function:

$$\ln L(f_0) = -\frac{N}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{N} x(t_i) - A\sin(2\pi f_0 t_i + \varphi))^2 \tag{11}$$

The goal of maximum likelihood estimation is to find f_0 maximizing the log-likelihood function, i.e:

$$f_0 = \arg\max_{f_0} \ln L\left(f_0\right) \tag{12}$$

Due to the complexity of this optimization problem, the gradient descent method is used in this paper to find the maximum likelihood solution within the given frequency search range.

4.2. Fast Fourier Transform (FFT)

FFT is an efficient frequency analysis method that can convert a time domain signal to the frequency domain, thus facilitating the identification of the main frequency components. The FFT method is suitable for smooth signals and can quickly find the main frequency components of the signal.

For the signal $x(t)$ of flight period 2, by the following steps:

(1) Perform the FFT transform on the signal $x(t)$ to obtain the frequency domain signal $X\left(f\right)$.

FFT transform: let the sampling points of x(t) be $x(t_1), x(t_2), \ldots, x(t_N)$, then the expression of FFT transform is:

$$X\left(f_k\right) \sum_{i=1}^{N} x(t_i) e^{-j2\pi f_k t_i} \tag{13}$$

(2) Calculate the spectral amplitude $\left|X\left(f\right)\right|$ and find the frequency component with the largest amplitude[8].

Maximum likelihood estimation: the frequency is estimated by maximizing the following log-likelihood function over the search interval f_0:

$$\ln L(f_0) = -\frac{N}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{N} x(t_i) - A\sin(2\pi f_0 t_i + \varphi))^2 \tag{14}$$

(3) The frequency obtained from the FFT estimation is used as the initial value for the maximum likelihood estimation to improve the solution efficiency.

$$X(f)=\sum_{i=1}^{N}x(t_i)e^{-j2\pi f_k t_i} \tag{15}$$

$$\hat{f}_0^{MLE}=\arg\max_{f_0}\ln L(f_0) \tag{16}$$

4.3. Model results

Bringing the preprocessed data into the above model in this paper using Python, the Python code was written to solve the problem FFT estimation results and MLE estimation results are 41 MHz, indicating that in-flight cycle 2, the main frequency characteristics of the signal is more obvious, the noise on the estimation of the impact of the smaller.The results indicate that,MLE estimated frequency: 41.000 MHz.FFT preliminary estimated frequency: 41.000 MHz

4.3.1 Time-domain diagrams:

Observing the change of the observed signal with time in the time domain graph, we can understand the general shape of the signal, amplitude change rule, etc. The signal shows more regular periodic fluctuations. This signal shows more regular periodic fluctuations, it can be initially judged that the signal may have certain periodic frequency components, and by observing the fluctuations of the cycle length to roughly estimate the frequency (the reciprocal of the period is the frequency) of 41 MHz, as shown in Figure 2.

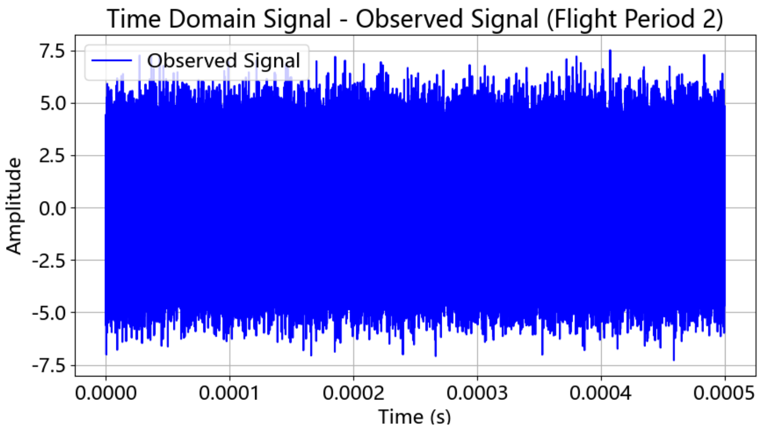

Figure 2: Time domain diagram

4.3.2 Spectrogram

In a spectrogram, the horizontal coordinate represents the frequency and the vertical coordinate represents the amplitude of the corresponding frequency component. By looking at the spectrogram, the frequency distribution of the signal can be visualized. The frequency components with larger amplitudes in this graph are the main frequency components of the signal, which is the part we want to accurately capture through the frequency estimation method. The locations of the FFT preliminary estimated frequency (orange dashed line) and the MLE estimated frequency (red dashed line) marked on the spectrogram can be compared with the actual peak locations in the spectrogram. If the estimated frequencies are in better agreement with the locations of the main peaks in the spectrogram, it means that the frequency estimation method is more accurate, both at 41 MHz, as shown in Figure. 3.

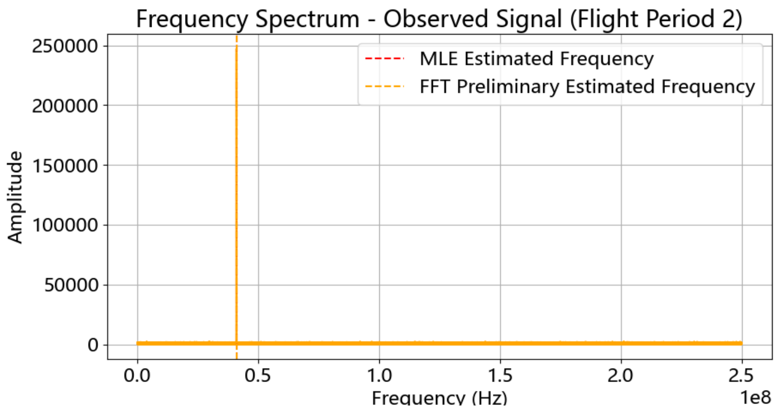

Figure 3: Spectrogram

Figure 3 presents the spectrum of the observed signal for flight cycle 2. The graph contains two curves corresponding to different frequency estimation methods. The red curve characterizes the MLE, while the orange curve represents the FFT[9]. Near 0 Hz, the amplitudes of both curves are low. As the frequency increases to about 0.41 Hz, the amplitude of the FFT starts to climb and reaches a peak at 41 MHz with an amplitude of about 25,000, in which the amplitude of the MLE is low and relatively stable[10]. After the frequency is greater than 1 Hz, the amplitudes of both curves decay rapidly and remain lower and smoother in the subsequent frequency range. This spectrogram is of great significance for the comparative analysis of the performance of different frequency estimation methods in observing the frequency characteristics of the signal, which is of great value for research in the related fields of signal processing and frequency estimation.

5. Conclusions

In this study, the problem of signal frequency estimation in noisy environments is explored in depth, and significant results are achieved by combining the MLE and FFT methods. The results show that compared with the FFT method, the MLE method exhibits higher stability and accuracy of frequency estimation under different flight cycles. Especially in flight cycles 2 to 4, the MLE maintains a stable performance in the presence of noise enhancement or intermittent signal reception, while the FFT shows obvious fluctuations. In flight cycle 2, the estimation results of both MLE and FFT are 41.000 MHz, which are highly consistent with the actual frequency; in other cycles, the error of MLE is obviously lower than that of FFT, which verifies the excellent performance of MLE in complex environments. The frequency estimation results of MLE and FFT are demonstrated by time-domain waveforms and spectrograms, and the correlation between the noise and the estimation results is analyzed, which greatly improves the accuracy and stability of the estimation by exploiting the period characteristic of MLE without relying on the amplitude and phase information. This finding provides a new perspective and theoretical support for the field of signal processing and frequency estimation, and lays a solid foundation for the development and application of related technical tools.

Despite the breakthrough in this study, there are still limitations, mainly focusing on the comparison between MLE and FFT, without exploring other methods in depth. Future research can introduce more frequency estimation methods for comparative analysis and explore the combined application of MLE with other signal processing techniques to improve the estimation accuracy and robustness, contributing new strength to the development of the signal processing field.

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