Research on Smart Mine Equipment Procurement Optimization Based on Simulated Annealing and Quantum Computing

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Abstract. With the development of quantum computing, traditional combinatorial optimization problems have found new solving methods. This paper studies the excavator and mining truck matching optimization problem based on the QUBO model, aiming to optimize equipment configuration and operational plans by maximizing long-term profit. An integer optimization model is first established and converted into a QUBO model, which is solved using the simulated annealing algorithm. Furthermore, quantum computing is explored through the Coherent Ising Machine (CIM) simulator to examine its application in large-scale combinatorial optimization. The results show that the CIM simulator outperforms traditional algorithms in both accuracy and speed, significantly enhancing computational efficiency. Based on simulated annealing and quantum computing, an optimized procurement plan is provided, yielding a maximum total profit of 500 million yuan. The experiments also demonstrate the potential of quantum computing in optimizing equipment configuration in smart mining, driving the intelligent evolution of the field.

Keywords: Qubo Model, Excavator and Mining Truck Matching, Simulated Annealing, Coherent Ising Machine.

1. Introduction

Quantum computing, as a key component of quantum technology, is revolutionizing the way this study solve complex optimization problems[1,2]. Unlike traditional computers, quantum computers process information in parallel using quantum bits (qubits), allowing them to explore multiple potential solutions simultaneously. This parallelism offers significant improvements in computational efficiency, especially for large-scale combinatorial optimization problems that are difficult for classical algorithms to solve[3,4]. As quantum computing advances, it holds great promise for transforming industries that rely on complex optimization tasks, such as logistics, finance, and smart manufacturing[5,6].

One area where quantum computing can make a significant impact is in smart mining, where optimizing equipment configuration and operational plans is a typical combinatorial optimization problem. In this context, the problem involves selecting the optimal number and type of mining equipment, matching excavators with mining trucks, and ensuring the overall profitability of operations while adhering to various constraints such as budget, available resources, and operational efficiency. These types of problems are particularly well-suited for quantum approaches due to their high-dimensional solution space and complex interactions between variables. To address these challenges, the Quadratic Unconstrained Binary Optimization (QUBO) model has emerged as an effective tool. QUBO transforms combinatorial optimization problems into quadratic unconstrained optimization problems, making them amenable to solution using quantum computing techniques. By converting the problem into a format that quantum computers can process more efficiently, the QUBO model allows for the potential to solve large-scale optimization problems in a fraction of the time required by traditional methods[7].

Simulated Annealing (SA), a classic probabilistic optimization algorithm, finds near-optimal solutions in large solution spaces[8]. By incorporating quantum computing into the optimization process, this study aims to demonstrate the potential of quantum-enhanced algorithms for driving the intelligent evolution of the mining industry[9]. The ability to solve combinatorial optimization problems with greater accuracy and efficiency can lead to more effective resource allocation, cost reductions, and improvements in operational performance, ultimately contributing to the advancement of smart mining technologies.

2. Combinatorial Optimization of Excavators and Mining Trucks

The data in this paper originates from http://mathorcup.org/. This study first establishes an integer optimization model, where the service life of the excavators is not considered. In this case, the discounted long-term profit of the i-th type of excavator is represented in c_i ten thousand yuan. Let the decision variable a_i be the number of excavators of the i-th type. The objective is to maximize the total profit, as follows:

$$\max Q=\sum_{i=1}^{4} c_i a_i \tag{1}$$

The expenditure on purchasing excavators cannot exceed the available startup funds; the total number of matched mining trucks must not exceed the given number of trucks; at least three types of excavators must be included in the final configuration; and the number of matched mining trucks must obviously be greater than zero. That is, the following conditions hold:

$$\begin{cases} \sum_{i=1}^{4} P_{Ai} a_i \leqslant K \\ b_j \leqslant B_j \\ \sum_{i=1}^{4} \operatorname{sgn}(a_i) \geqslant 3 \\ a_{ij} \geqslant 0 \\ a_{ij} \ integer \end{cases} \tag{2}$$

Therefore, the optimization model is expressed as:

$$\max Q=\sum_{i=1}^{4} c_i a_i$$
$$s.t. \begin{cases} \sum_{i=1}^{4} P_{Ai} a_i \leqslant K \\ b_j \leqslant B_j \\ \sum_{i=1}^{4} \operatorname{sgn}(a_i) \geqslant 3 \\ a_{ij} \geqslant 0 \\ a_{ij} \ integer \end{cases} \tag{3}$$

Furthermore, in order to optimize Formula 3, a matching relationship table between different types of excavators and mining vehicles is shown in Table 1:

Table 1: Excavator and mining car matching relationship

	mining car 1	mining car 2	mining car 3
Excavator 1	1	—	—
Excavator 2	2	1	—
Excavator 3	2	2	1
Excavator 4	—	2	1

From Table 1, this paper can see that when the excavator and the mining car are matched exactly, this paper can get a matrix H:

$$H=\begin{pmatrix}1&0&0\\2&1&0\\2&2&1\\0&2&1\end{pmatrix} \tag{4}$$

The number of i-th type excavators matched with j-th type mining trucks a_{ij} can form a 4*3 matrix of the following form:

$$\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\\a_{41}&a_{42}&a_{43}\end{pmatrix} \tag{5}$$

The combinatorial optimization involved here is a type of optimization problem that seeks the optimal solution from many feasible solutions under certain constraints. Essentially, it studies the combinations of discrete variables (i.e., 0-1 variables) that maximize the objective function. For a a_{ij} matrix formed by such variables, this study use binary representations to express each integer, so that n variables can represent numbers from 0 to 2^{n-1} certain value. For example, the number of a_{11} mining trucks (7) that can be matched with excavators has 8 possible configurations (from 0 to 7). These 8 possibilities can be expressed using 3 variables, and a weighted sum of these variables gives the binary expression. Similarly, this approach can be extended to other cases.

$$\begin{cases}a_{11}=4x_1+2x_2+x_3\\a_{21}=2x_4+x_5\\a_{22}=4x_6+2x_7+x_8\\a_{31}=2x_9+x_{10}\\a_{32}=2x_{11}+x_{12}\\a_{33}=2x_{13}+x_{14}\\a_{42}=2x_{15}+x_{16}\\a_{43}=2x_{17}+x_{18}\end{cases} \tag{6}$$

$$\text{other } a_{ij}=0$$

In this way, by adding each column of the matrix, this study get the corresponding a_i :

$$\begin{cases}a_1=a_{11}\\a_2=a_{21}+a_{22}\\a_3=a_{31}+a_{32}+a_{33}\\a_4=a_{42}+a_{43}\end{cases} \tag{7}$$

By using the values a_i from Equation (6) and the given data c_i, the objective function with decision variables x_i can be determined. Similarly, by applying the values from Equation (6) and the provided data P_{Ai}, the first constraint can be calculated. To determine the number of mining trucks b_j required for an exact match, this study multiply the number of j-th type mining trucks a_{ij} that can be matched with the i-th type excavator h_{ij} by the corresponding element in matrix H (i.e., the number of j-th type mining trucks needed to match a single i-th type excavator). The total number of j-th type mining trucks is then obtained by summing the quantities for the same type of trucks. This value is used to calculate the second constraint.

$$b_j=\sum_{i=1}^{4}a_{ij}\times h_{ij}\leqslant B_j \tag{8}$$

In summary, the improved model is:

$$\max Q = \sum_{i=1}^{4} c_i a_i$$

$$s.t.\begin{cases} \sum_{i=1}^{4} P_{Ai} a_i \leqslant K \\ \sum_{i=1}^{4} a_{ij} \times h_{ij} \leqslant B_j \\ \sum_{i=1}^{4} \operatorname{sgn}(a_i) \geqslant 3 \\ a_{ij} \geqslant 0 \\ a_{ij}\ \ integer \end{cases} \tag{9}$$

3. Conversion and Solution of the QUBO Model

Equation (3) is converted into a standard unconstrained optimization QUBO (Quadratic Unconstrained Binary Optimization) model. The general approach involves transforming the decision variables into binary variables, with the objective function being a quadratic function. The standard form of this model is shown in Equation (10).

$$y = x^T Q x \tag{10}$$

Among them, x is a vector of binary variables, each variable can only be 0 or 1, Q is a QUBO matrix, and there are two forms of Q matrix: the first is the upper triangular form and the second is the symmetric form, as shown in formula (11) and formula (12) respectively:

$$\begin{gathered} q'_{ij} = (q_{ij} + q_{ji}), i \leqslant j \\ q'_{ij} = 0, i > j \end{gathered} \tag{11}$$

$$q'_{ij} = \frac{(q_{ij} + q_{ji})}{2} \tag{12}$$

When considering the constraints, this study need to convert the inequality constraints into equality constraints so that the original model can be transformed into an unconstrained optimization problem by relaxing or adding penalty terms. In this way, this study can use the general algorithm of unconstrained optimization problems to solve QUBO to find the x that minimizes or maximizes y.

The whole process is divided into three steps. First, this paper need to add some variables to the left of the original constraint to turn the inequality into an equation, that is:

$$s.t.\begin{cases} \sum_{i=1}^{4} P_{Ai} a_i + s_1 = K \\ \sum_{i=1}^{4} a_{i1} h_{i1} + s_2 = B_1 \\ \sum_{i=1}^{4} a_{i2} h_{i2} + s_3 = B_2 \\ \sum_{i=1}^{4} a_{i3} h_{i3} + s_4 = B_3 \end{cases} \tag{13}$$

Where s_1, s_2, s_3 and s_4 are positive integers represented by binary. For example, the value s1 range of constraint 1 is 0~120, which can be represented by x19+2*x20+4*x21+8*x22+16*x23+32*x24+57*x25. Similarly, 15 variables are introduced to turn all inequalities into equalities.

Furthermore, the penalty function is defined according to formula (13):

$$F\left(X\right)=-\sum_{i=1}^{4}\left[P\left(L_i-R_i\right)^2\right] \tag{14}$$

Where L_i represents the left side of the i-th equation in equation (13); R_i represents the i-th side of the equation in equation (13); the parameter P represents a positive scalar penalty term, which must be chosen to be large enough to ensure that the penalty term is indeed equivalent to the classical constraint.

In the last step, let the decision variable be $X=[x_1\ x_2\ \dots\ x_{33}]^T$, then the profit is:

$$q=\sum_{i=1}^{33}x_i q_i=X^T Q \tag{15}$$

Since the variable can only take the value of 0 or 1, q can be converted into a homogeneous quadratic form:

$$q=\sum_{i=1}^{33}x_i^2 q_i \tag{16}$$

This can be expressed by a matrix. After derivation, the new objective function becomes:

$$Q'=q+F(X)=X^T QX-\sum_{i=1}^{4}\left[P\left(L_i-R_i\right)^2\right] \tag{17}$$

The matrix form is expressed as:

$$X^T Q' X=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{33} \end{bmatrix}\begin{bmatrix} q_1 & 0 & \cdots & 0 \\ 0 & q_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & q_{33} \end{bmatrix}\left[x_1 \quad x_2 \quad \cdots \quad x_{33}\right] \tag{18}$$

Finally. The QUBO model is as shown below:

$$\begin{aligned} & Q'=q+F(X)=X^T QX-\sum_{i=1}^{4}\left[P\left(L_i-R_i\right)^2\right] \\ & x_i\in\{0,1\} \end{aligned} \tag{19}$$

4. Conversion and Solution of the QUBO Model

To solve the QUBO model, this study uses the simulated annealing algorithm, which is inspired by the physical process of annealing in solids and applied to combinatorial optimization. The algorithm starts with an initial solution, randomly generates new solutions, and either accepts or rejects them based on a probability that decreases with temperature. This allows it to search a broad solution space, avoiding local optima and gradually finding the global optimum. Simulated annealing is particularly effective for QUBO problems due to its global search capability, scalability for large problems, versatility in solving NP-hard problems, and robustness in handling noisy data, which makes it well-suited for real-world applications[10].

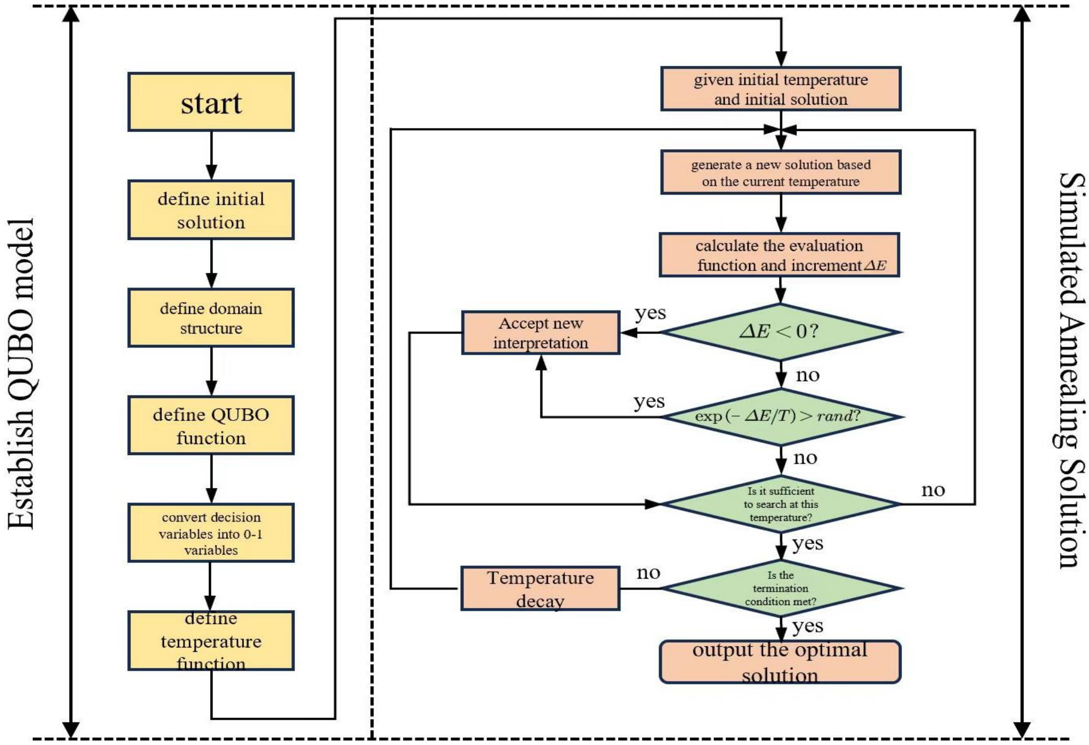

Figure 1: Approach and process of solving QUBO models using the SA

Based on the maximum total profit and the number of excavator types obtained, the objective function is formulated in QUBO form and solved using the simulated annealing algorithm, as shown in Figure 1. The solution yields the following procurement plan: 3 units of excavator type 1, 2 units of excavator type 2, 4 units of excavator type 3, and 2 units of excavator type 4. The maximum total profit is 440 million yuan.

Furthermore, this study also explores the use of quantum computing for solving the problem. Quantum computing, leveraging quantum technology, offers faster and more efficient solutions for large-scale combinatorial optimization problems. Based on the principles of quantum mechanics, it can perform multiple computations simultaneously, enabling parallel exploration of various solutions. The CIM, as a type of quantum computer, has been applied in a range of fields, including compressed sensing, multi-puzzle solving, and job scheduling. To demonstrate the performance of the CIM simulator, the CIM physical machine is compared with the SA algorithm. An efficiency ratio is defined to evaluate the capability of CIM, reflecting the time-saving effect and improvement in accuracy.

$$\gamma_{SA} = \frac{f_{CIM}}{f_{SA}} = \frac{t_{CIM}}{t_{SA}} \tag{20}$$

Where $t_{CIM/SA}$ is the running time of the corresponding algorithm, and $f_{CIM/SA}$ is the objective function value of the corresponding algorithm. Substituting the experimental data into the efficiency change with the number of bits is shown in Figure 2:

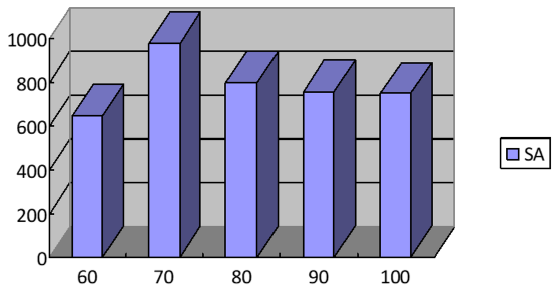

Figure 2: Solving the relationship between efficiency and number of bits

As shown in the figure, the CIM simulator outperforms classical heuristic algorithms both in terms of accuracy and speed, significantly enhancing computational performance. To calculate the maximum total profit and the number of excavator types, the objective function is formulated in QUBO form and input into the CIM simulator. The results are shown in Figure 3.

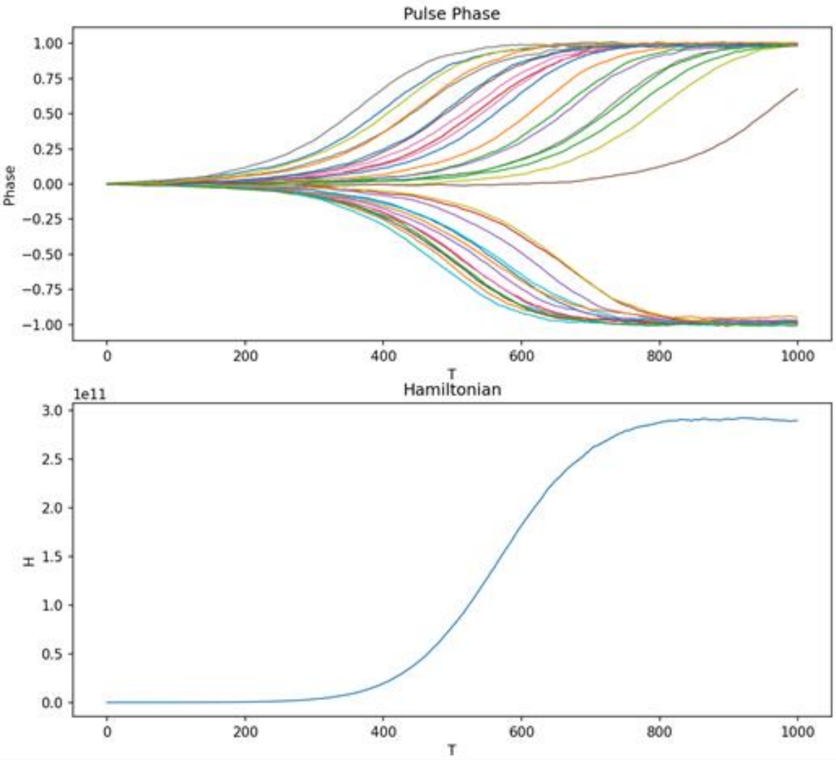

Figure 3: CIM simulator solution results

The solution yields the following procurement plan: 1 unit of excavator type 1, 1 unit of excavator type 2, and 9 units of excavator type 3. The maximum total profit is 500 million yuan.

5. Conclusion

This paper presents a combinatorial optimization model for excavator and mining truck procurement, aimed at maximizing long-term profit while meeting budget and operational constraints. The optimization problem is formulated as a QUBO model, which is solved using both the SA algorithm and quantum computing via the CIM. The results show that while the SA algorithm yields a total profit of 440 million yuan, the CIM simulator achieves a higher profit of 500 million yuan, demonstrating its superior performance in terms of both accuracy and computational efficiency. This comparison underscores the potential of quantum computing in solving large-scale combinatorial optimization problems more effectively than traditional methods.

In conclusion, this study highlights the effectiveness of QUBO modeling in optimizing mining equipment procurement and illustrates the advantages of quantum computing in enhancing solution accuracy and speed for complex optimization tasks. Future work could focus on exploring hybrid quantum-classical approaches and extending the model to incorporate dynamic variables, allowing for more flexible and adaptive optimization in real-world mining operations.

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