A Numerical Study on Predicting Rolling Resistance of Tires Based on the Change of Accumulated Strain Energy Density

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Abstract. The prediction of tire rolling resistance is importance in both academic and engineering. Classic computational methods are complexity and low efficiency. This paper proposes a method based on cumulative changes of strain energy density to calculate tire rolling resistance. Obtain the stress and stains states of various tire components by Finite Element Analysis (FEA), apply the Karmal's formula for rolling resistance computation by cumulative changing of strain energy density. The proposed method achieves a good tendency between prediction and measurement results. Moreover, the computational efficiency is reduced to 1/12 of that of classic methods. This method can be used for both complex and simplified tread patterns tires rolling resistance predicting.

Keywords: Tire; Rolling resistance; Strain energy density; FEA; Rubber.

1. Introduction

Tire rolling resistance consumes for approximately 20% to 30% of a vehicle's total energy [1], and it is proportional to vehicle's energy consumption [2,3]. many countries and regions have imposed regulatory constraints on tire rolling resistance [4]. With the development of electric vehicles and the increasing demand for extended mileages, the need for low rolling resistance tires has become particularly urgent. Tire rolling resistance prediction technology is an essential tool for the development of low rolling resistance tires. The rubber property: viscoelasticity, nonlinearity and their dependencies on temperature, frequency, and strain make the prediction of rolling resistance extremely challenging [5,6].

Researchers have proposed various methods for predicting tire rolling resistance. Chakko[7] proposed a simplified analytical model based on theory, Ghoreishy[8] developed a comparative method based on hyperplastic and viscoelastic FEA models, Klingbeil[9] and Wu[10] established a method based on equivalent strain harmonic response analysis, Shida[11] and Warholic[12] used hyperplastic FEA models to calculate tire rolling resistance, the difference of is the material models used: Phase Log's linear elastic model and Kelvin-Voigt viscoelastic model, respectively. Cho[13] et al. employed strain amplitude equivalence to calculate rolling resistance.

FEA methods, which can accurately predict tire rolling resistance, have been widely accepted in both academia and industry. The prediction results generally show good agreement in trends. However, existing methods essentially require processing large amounts of stress-strain data, resulting in complex computational processes. Therefore, this paper proposes a rolling resistance prediction method based on cumulative strain energy density, utilizing a new energy loss calculation formula that can more efficiently predict tire rolling resistance.

2. Tire Rolling Resistance Prediction Model

2.1. Typical Calculation Methods for Rolling Resistance

Shida [11] used static FEA to calculate the load forces on tires, obtaining energy losses by processing the stress and strain data of each element. The energy loss per rubber volume during one rotation cycle can be expressed as:

$$E_d = V \int_{cycle} \sigma(t) d\varepsilon = \pi . V . \sigma_0 . \varepsilon_0 . \sin\delta \tag{1}$$

Where σ andεare stress and strain, respectively, δ is the phase angle between stress and strain, σ_0 and ε_0 are the stress and strain amplitudes, and r_s is the static radius of the tire. Stress and strain are expanded by a finite Fourier series of order N (where N≥15):

$$\sigma(\theta) = a_0^\sigma + \sum_{n=1}^{N} \{a_n^\sigma \cos(n\theta) + b_n^\sigma \sin(n\theta)\} = a_0^\sigma + \sum_{n=1}^{N} \{A_n^\sigma \sin(n\theta + \phi_n^\sigma)\} \tag{2}$$

$$\varepsilon(\theta) = a_0^\varepsilon + \sum_{n=1}^{N} \{a_n^\varepsilon \cos(n\theta) + a_n^\varepsilon \sin(n\theta)\} = a_0^\varepsilon + \sum_{n=1}^{N} \{A_n^\varepsilon \sin(n\theta + \phi_n^\varepsilon)\} \tag{3}$$

$$\begin{gathered} A_n = \sqrt{a_n^2 + b_n^2} \\ \phi_n = \tan^{-1}(b_n / a_n) \end{gathered} \tag{4}$$

The rolling resistance of the tire can be expressed as:

$$\begin{gathered} E_D = \sum E_d \\ RR = E_D / 2\pi r_s \end{gathered} \tag{5}$$

Warholic [12] based on the viscoelastic Kelvin-Voigt model, established the following relationship between stress and strain:

$$\sigma = E\varepsilon + C\dot{\varepsilon} \tag{6}$$

where E and C are the elastic and viscous coefficients, respectively. apply the Fourier series to the stress and strain follow the circumferential direction:

$$\begin{gathered} \sigma = \sum_n (\sigma_{ns} \sin n\theta + \sigma_{nc} \cos n\theta) \\ \varepsilon = \sum_n (\varepsilon_{ns} \sin n\theta + \varepsilon_{nc} \cos n\theta) \end{gathered} \tag{7}$$

The calculation expression for the tire's strain energy loss W is derived:

$$W = \iint_A \int_0^{2\pi} \sigma d\varepsilon * rdA = \iint_A \int_0^{2\pi} (E\varepsilon + C\dot{\varepsilon}) d\varepsilon rdA \tag{8}$$

Where A is the cross-sectional area and r is the radius in the cylindrical coordinate system. Substituting Equation (7) into Equation (8) yields:

$$\begin{gathered} W = \int_A \sum_n n^2 \pi (\sigma_{ns}\varepsilon_{ns} + \sigma_{nc}\varepsilon_{nc}) \tan\delta rdA \\ RR = W / 2\pi r_d \end{gathered} \tag{9}$$

Where r_d is the rolling radius of the tire.

Both of the above algorithms require extracting six stress components and six strain components curves for a tire element over one rotation cycle, and then performing mathematical processing on the 12 curves separately, which is very time-consuming in data processing.

2.2. Calculation Method for Cumulative Strain Energy Density

According to Michelin's research, the strain range of various parts of the tire is between 3% ~ 15% [14]. Karmal [15] derived the relationship between rubber energy loss and stored strain energy under medium strain (0~100%):

$$\frac{E_{loss}}{E_{store}} = (2\pi \tan \delta)^{1/1.75} \tag{10}$$

Where E_{loss} is the energy loss, E_{store} is the elastic energy, and the total strain energy is the sum of elastic energy and energy loss, thus:

$$E_{loss} = E_{total} \frac{(2\pi \tan \delta)^{1/1.75}}{1 + (2\pi \tan \delta)^{1/1.75}} \tag{11}$$

Based on the principle of rubber material energy loss: when the strain energy of rubber material changed, the energy transform from mechanical to heat. Equations (5) and (11) establish the relationship between strain energy and tire rolling resistance.

To obtain the history of element strain energy changes, it is necessary to calculate the cumulative strain energy density:

$$w_i = \frac{1}{2} \sum_{n=1}^{K-1} abs(e_{i,n+1} - e_{i,n}) \tag{12}$$

where k is the total number of tire cross-sections, n is the cross-section number, and $e_{i,n}$ is the strain energy density value of the element numbered i in the n-th cross-section, as shown in Figure 1.

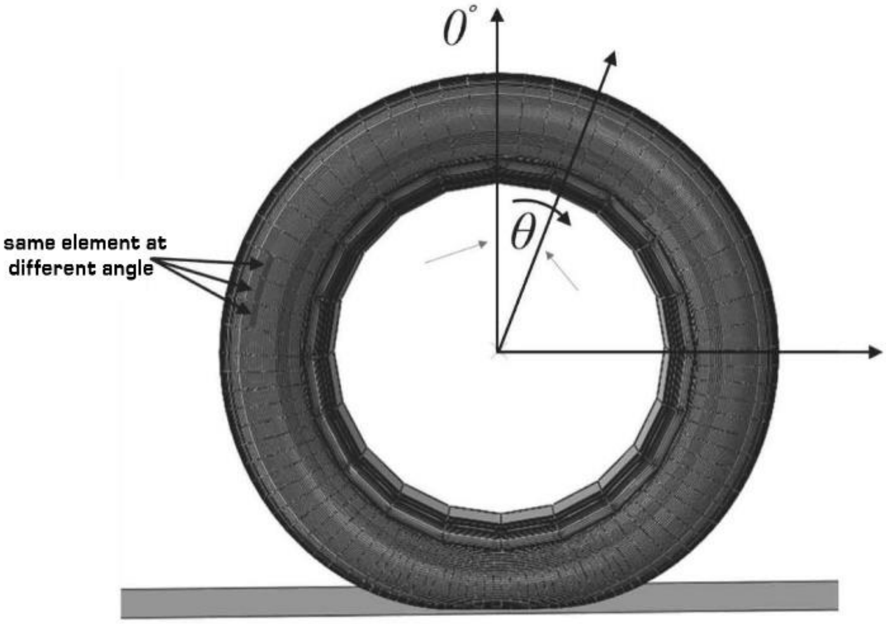

Fig. 1 tire elements and cross-section

Substituting Equation (11) into Equation (12) and rearranging, the energy loss density of the element h_i is obtained：

$$h_i = w_i \frac{(2\pi \tan \delta)^{1/1.75}}{1 + (2\pi \tan \delta)^{1/1.75}} \tag{13}$$

Thus, the rolling resistance of the tire is:

$$RR = \sum_{k=1}^{m} h_i v_i / (2\pi r_s) \tag{14}$$

3. Calculation and measurement Analysis

3.1. FEA Model

Tire 205/55R16 was applied to make the study. The geometric model of the tire was processed using in-house pre-processing software, and calculations were performed in ABAQUS software. The mesh was divided into triangular or quadrilateral elements, and the reinforce material like steel belt and carcass were divided into 2-node 1D elements, as shown in Figure 2.

Material properties were assigned to each component (all rubber materials had a Poisson's ratio of 0.49 and a density of 10^{-9} t/mm³), as shown in Table 1.

Apply internal pressure of 0.21 MPa and 4822 N loading to the tire. Based on the 2D inflation analysis, the tire cross-section was rotated 360 degrees, and the cross-section was divided circumferentially into 110 sections.

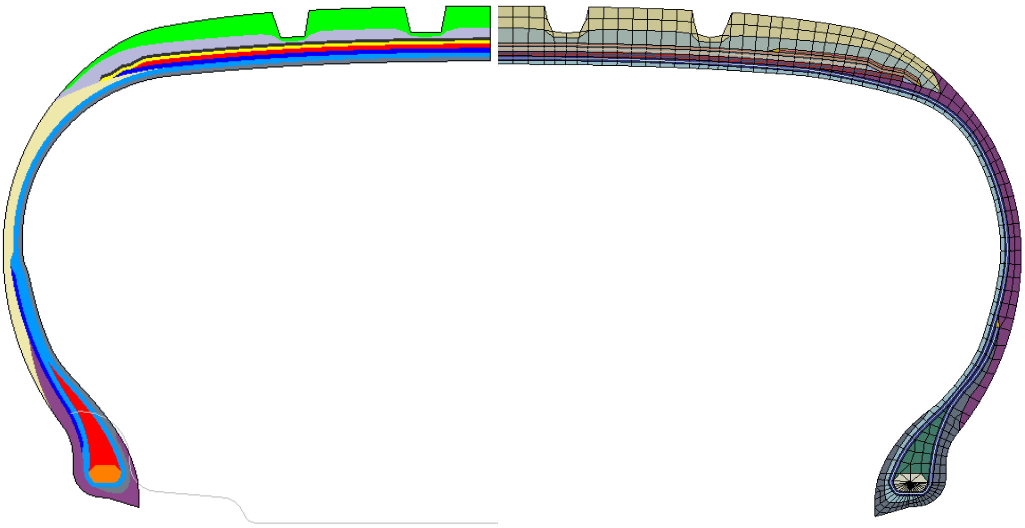

Fig. 2 tire's CAE model (a) geometry (b) mesh

Loading analysis was performed, and print out the strain energy density values (SENER values) for all the rubber material elements.

Table 1. material property

Part name	modulus (MPa)	tanδ
carcass	2	0.1
Cushing	3	0.12
inner liner	2	0.14
Base rubber	3.5	0.08
Belt rubber	3	0.12
Tread	4	0.16
Sidewall	3	0.14
Apex	8	0.15

A Python program was carry out to get the FEA results and process the data according to Equations (5) and (11~14) to calculate the rolling resistance of the tire.

3.2. Result Analysis

According to the standard testing method ISO28580, the calculation model set the same air pressure and loading. The comparison between the calculated and measured rolling resistance are shown in Figure 3. Repeat above procedure for 3 more tire sizes for CAE model and algorithm verification.

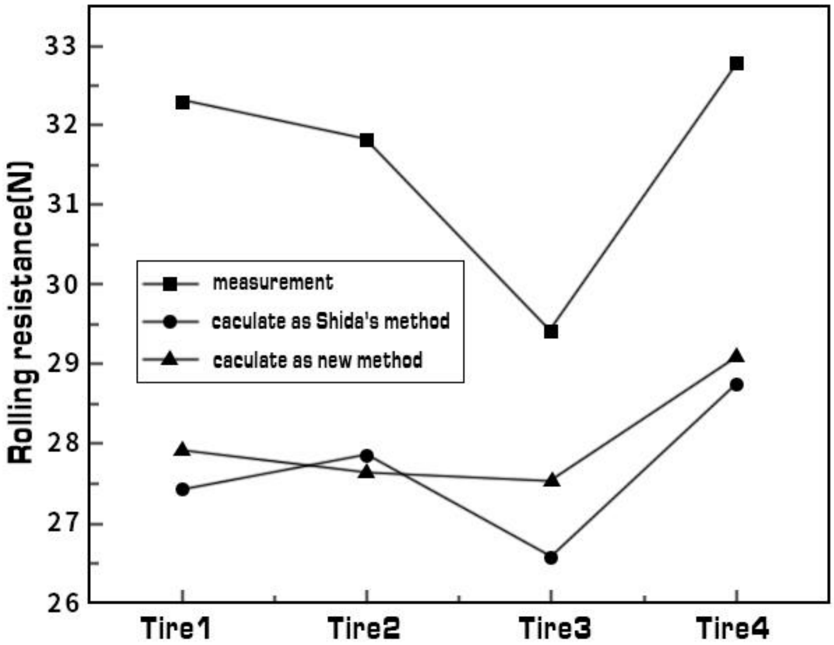

Fig. 3 comparison of calculation and measurement

According to Figure 3, the predicted rolling resistance results of this paper show a good tendency with the measured results, validity of the prediction model. The proposed method is more accurate in trend compared to typical calculation methods.

The difference between the prediction and measurement is mainly due to the neglect of the frequency, temperature, and strain dependence of the hysteresis loss.

The method proposed in this paper significantly reduces the calculation time for rolling resistance, only approximately 1/12 of that of typical calculation methods, pretty much improve computation efficiency.

4. Summary

(1) A tire rolling resistance prediction model based on cumulative strain energy density was established.

(2) Experimental results confirmed the accuracy in trend and validated the effectiveness of the model.

(3) The computational complexity of the proposed method is approximately 1/12 that of typical methods, significantly improving computational efficiency.

(4) The proposed method can be used for both complex and simplified tread patterns tires rolling resistance predicting.

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