

The analytical solution for prestressed bolt (cable) support in circular roadways

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Abstract. Prestressed bolt (cable) play a significant role in the support of deep-buried roadways, effectively reducing the displacement and stress concentration of the surrounding rock, thereby enhancing the overall stability of the roadway. To further understand the mechanism of the rock-anchor system (RAS), this paper derives an analytical solution for circular roadways supported by prestressed bolt (cable) based on the theory of elasticity and verifies the solution using finite element methods. The results show a high degree of agreement between the analytical solution and the numerical results, confirming the accuracy and reliability of the model. Moreover, further analysis reveals that proper prestress levels and bolt (cable) spacing are critical in optimizing the stress distribution of the surrounding rock and reducing deformation. This study provides important theoretical guidance for the optimal application of prestressed support technologies.

Keywords: Prestressed bolt (cable); RAS; analytical solution; circular roadways.

1. Introduction

With the progressive deepening of resource extraction, underground roadways and tunnel projects are extending into deeper strata, making the stability and safety of deep-buried tunnels a critical issue in engineering design. Under deep-buried conditions, the surrounding rock is subjected to considerable self-weight and initial ground stress, leading to more pronounced stress concentration phenomena, which in turn cause a series of deformation and failure problems in the rock mass. The instability of the surrounding rock can lead to serious engineering accidents such as collapses and rock bursts, threatening the lives of construction workers and increasing project costs. Therefore, effectively controlling rock deformation and improving tunnel stability has become one of the core challenges in underground engineering [1-3].

Prestressed bolt (cable) are a commonly used support method and have been widely applied in the support of deep-buried tunnels [4-6]. By applying active support forces to the surrounding rock, prestressed bolt (cable) can effectively reduce rock displacement and stress concentration phenomena, preventing deformation and instability, and enhancing the long-term stability of the tunnel. Compared to traditional non-prestressed support technologies, prestressed support is better at controlling rock deformation and delaying the rock mass from entering the plastic failure stage.

Although prestressed bolt (cable) support technology has achieved certain results, there are still some problems and shortcomings in current engineering design and theoretical research. For example, most of the existing support designs rely on empirical formulas and numerical simulations, lacking precise theoretical analytical solutions as a basis [7]. In addition, the stress state of the surrounding rock is complex, influenced by factors such as anchor arrangement, anchor spacing, and the magnitude of prestress. How to quantitatively analyze the impact of these factors on the stability of the rock mass remains an unresolved issue. Therefore, it is necessary to conduct an in-depth theoretical study on the support mechanism of prestressed bolt (cable) and derive analytical solutions that can accurately describe the stress state of the surrounding rock, providing guidance for support design in practical engineering.

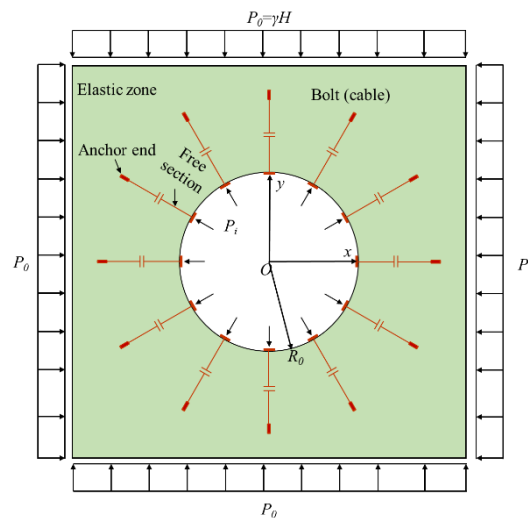
In response to the above issues, this paper derives an analytical solution for the mechanical behavior of a circular tunnel supported by prestressed bolt (cable) based on elastic mechanics. Finite element numerical simulation methods are used to verify the proposed analytical solution, and the results show a high degree of agreement between the numerical and analytical solutions, confirming

the accuracy and applicability of the proposed theoretical model. The findings of this study provide theoretical support for the long-term stability analysis of deep-buried tunnels and offer significant potential for engineering applications.

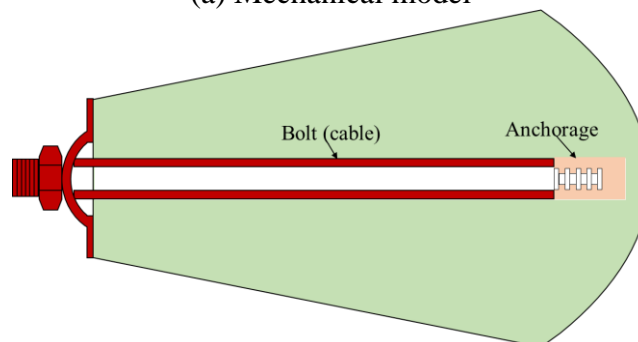
2. Model assumptions

The following basic assumptions are made to address the deformation of a circular tunnel supported by prestressed bolt (cable) [8]:

- (1) The tunnel is considered infinitely long, allowing it to be treated as a plane strain problem.
- (2) The tunnel is located at a depth sufficiently far from the surface, so the self-weight of the surrounding rock in a certain area around the tunnel can be neglected.
- (3) The tunnel is assumed to be under hydrostatic pressure.
- (4) The support provided by the bolt (cable) occurs simultaneously with the excavation of the tunnel.
- (5) The bolt (cable) are assumed to remain bonded without debonding (**Fig. 1**).



(a) Mechanical model



(b) Bolt (cable) support

Fig. 1 RAS model

3. Model derivation

3.1. Constitutive model of prestressed bolt (cable)

The synergistic coupling effect between the bolt (cable) and the surrounding rock enhances the strength and deformation resistance of the rock mass. After the application of prestress, the mechanical characteristic curve of the rock mass supported by the bolt (cable) exhibits a "stepwise" decline. This indicates that the active support provided by the prestressed bolt (cable) can effectively reduce the support force required for the surrounding rock to achieve stability[9].

The mechanical characteristic curve of the prestressed bolt (cable) is simplified into an ideal elastic-plastic model (**Fig. 2**), which is divided into an elastic phase and a plastic phase. The prestress value applied to the bolt (cable) is F_0 , corresponding to an extension of x_0 . When the extension of the bolt (cable) exceeds x_1 , the bolt (cable) enter the plastic phase, at which point the support force is F_1 . Therefore, the constitutive relationship of the prestressed bolt (cable) is given by:

$$F = \begin{cases} k(x_0 + x), & x_0 + x < x_1 \\ F_1, & x_0 + x > x_1 \end{cases} \quad (1)$$

In the equation, F represents the support force of the bolt (cable); x denotes the extension of the bolt (cable); and k is the elastic parameter of the bolt (cable).

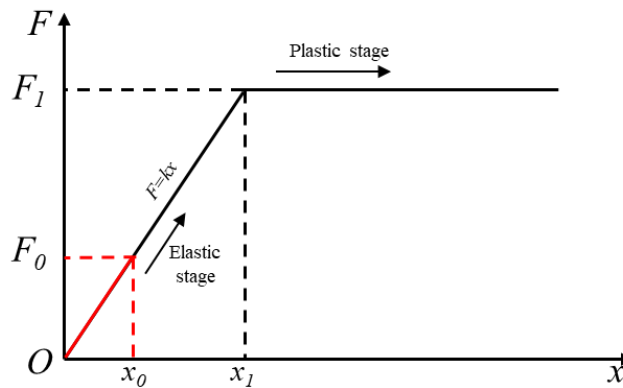


Fig. 2 Mechanical characteristic curve of prestressed bolt (cable)

The equivalent support strength of the bolt (cable) is:

$$P_i = \begin{cases} \frac{k(x_0 + x)}{S_\theta S_l}, & x_0 + x < x_1 \\ \frac{F_1}{S_\theta S_l}, & x_0 + x > x_1 \end{cases} \quad (2)$$

In the formula, S_θ represents the spacing between the bolts, and S_l represents the spacing between the rows of bolts, which is set to 1 m.

3.2. Analytical solution of RAS

According to **Fig. 1(a)**, the surrounding rock is in a hydrostatic state, which results in axisymmetric stress and displacement in all directions. The equilibrium equations are established as follows [10,11]:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr}\right)\left(\frac{d^2 U}{dr^2} + \frac{1}{r} \frac{dU}{dr}\right) = 0 \quad (3)$$

To deform EQ. (3):

$$r^4 \frac{d^4 U}{dr^4} + 2r^3 \frac{d^3 U}{dr^3} - r^2 \frac{dU}{dr} = 0 \quad (4)$$

Euler's equation:

$$r = e^t \quad (5)$$

Combining EQ. (5), transform EQ.(4):

$$\frac{d^4U}{dt^4} - 4\frac{d^3U}{dt^3} + 4\frac{d^2U}{dt^2} = 0 \quad (6)$$

Solve EQ. (6):

$$U = A \ln r + Br^2 \ln r + Cr^2 + D \quad (7)$$

The stress equation:

$$\left. \begin{aligned} \sigma_r &= \frac{1}{r} \frac{\partial U}{\partial r} + \frac{1}{r^2} \frac{\partial^2 U}{\partial \theta^2} = \frac{A}{r^2} + B(1 + 2 \ln r) + 2C \\ \sigma_\theta &= \frac{\partial^2 U}{\partial r^2} = -\frac{A}{r^2} + B(3 + 2 \ln r) + 2C \\ \tau_{r\theta} &= \tau_{\theta r} = 0 \end{aligned} \right\} \quad (8)$$

Boundary conditions:

$$\left. \begin{aligned} r = R_0 &\Rightarrow \sigma_r = P_i \\ r \rightarrow \infty &\Rightarrow \sigma_r = P_0 \end{aligned} \right\} \quad (9)$$

Substituting EQ. (9) into EQ. (8), we get:

$$\left. \begin{aligned} A &= (P_i - P_0)R_0^2 \\ B &= 0 \\ C &= \frac{P_0}{2} \end{aligned} \right\} \quad (10)$$

In the equation, R_0 represents the radius of the tunnel excavation.

Substituting EQ. (10) into EQ. (8), the elastic stress analytical solution is given by:

$$\left. \begin{aligned} \sigma_r &= P_0 \left(1 - \frac{R_0^2}{r^2}\right) + P_i \frac{R_0^2}{r^2} \\ \sigma_\theta &= P_0 \left(1 + \frac{R_0^2}{r^2}\right) - P_i \frac{R_0^2}{r^2} \\ \tau_{r\theta} &= 0 \end{aligned} \right\} \quad (11)$$

The geometric physical equation is:

$$\frac{u}{r} = \frac{1-\nu^2}{E} \left(\sigma_\theta - \frac{\nu}{1-\nu} \sigma_r \right) \quad (12)$$

Substituting EQ. (12) into EQ. (11) yields:

$$u_r = \frac{1-\nu^2}{E} \left[P_b \left(r + \frac{R_0^2}{r} \right) - P_a \frac{R_0^2}{r} \right] - P_b \frac{\nu(1+\nu)}{E} \left(r - \frac{R_0^2}{r} \right) \quad (13)$$

Let $R_0=0$, the initial displacement u_0 is:

$$u_0 = \frac{1-\nu^2}{E} P_0 r - \frac{\nu(1+\nu)}{E} P_0 r \quad (14)$$

Thus, the radial relative displacement Δu is:

$$\Delta u = u_r - u_0 = \frac{(1+\nu)R_0^2}{Er} (P_0 - P_i) \quad (15)$$

4. Model validation

4.1. RAS numerical model

A finite element model (FEM) of the RAS was constructed using ABAQUS software to verify the analytical solution. As shown in Fig. 3, the model dimensions are 100m × 100m, and the tunnel excavation radius is 5m. A 10 MPa overburden pressure was applied to the top of the model, and normal boundary constraints were imposed on the left, right, and bottom boundaries. A plane strain model was adopted, assuming that the surrounding rock material is homogeneous and isotropic. The surrounding rock mesh element type was CPE4R. The anchors were installed immediately after tunnel excavation, and their cross-sections were set as truss-type with the mesh element type being T3D2. The specific mechanical parameters of the surrounding rock and anchors are listed in Table 1.

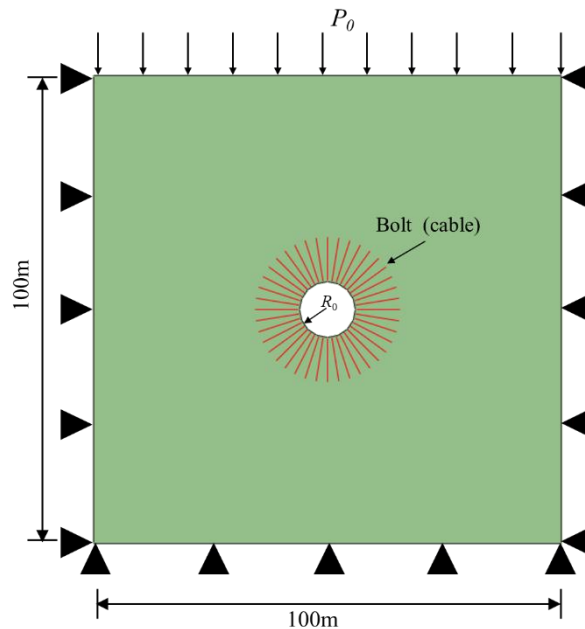


Fig. 3 RAS numerical model

Table 1. Mechanical parameters of RAS

Parameters	Bolt (cable)	Surrounding rock
E	-	2.24 GPa
ν	-	0.3
P_0	-	10 MPa
x_0	0.23mm	-
E	452kN/mm	-
F_0	320kN	-
Prestress	200kN	-
S_0S_l	0.7m×1m	-
E	-	-

4.2. Results validation

The proposed analytical solution was verified through numerical results. The findings show that as the depth of the surrounding rock increases, the radial stress gradually rises, while the tangential stress first increases and then decreases. At a distance of $10R_0$ from the tunnel wall, the stress in the surrounding rock approaches the initial in-situ stress. The maximum error between the stress curves of the surrounding rock obtained from the numerical and analytical solutions is 5.3% (Fig. 4(a)). Fig. 4(b) illustrates the displacement changes in the surrounding rock, with displacement gradually approaching zero as depth increases. At the tunnel wall ($r = R_0$), the displacements from the analytical and numerical solutions are 220 mm and 222 mm, respectively, with an error of 0.9%.

In conclusion, the analytical solution closely matches the numerical results, accurately reflecting the stress distribution and displacement trends in the surrounding rock.

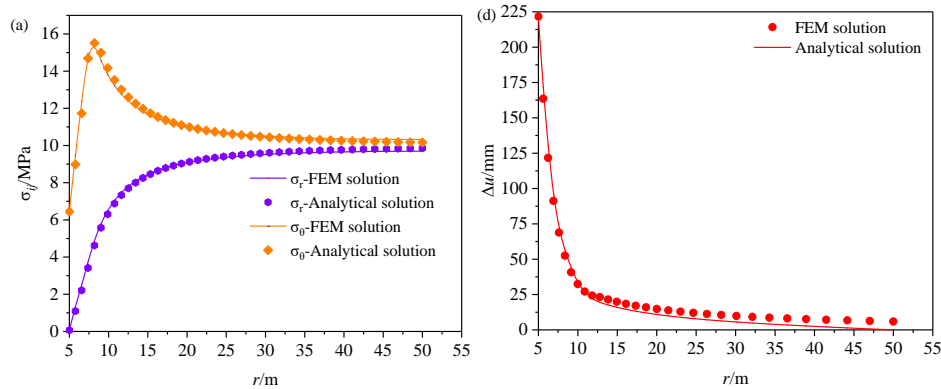


Fig. 4 Model validation (a) Verification of surrounding rock stress (b) Verification of surrounding rock displacement

5. Discussion

5.1. Effect of prestress

As shown in Fig. 5, there is a clear negative correlation between the prestress and the displacement of the tunnel wall. As the prestress increases, the wall displacement gradually decreases. Specifically, when the prestress increases from 0 MPa (without prestress) to 200 MPa, the displacement of the tunnel wall decreases from 0.433 m to 0.275 m, a reduction of 36.4%. This indicates that increasing prestress significantly enhances the restraining effect on the surrounding rock, improving the support performance and reducing deformation. Therefore, a reasonable increase in prestress helps effectively control the displacement of the surrounding rock, improving the stability and safety of tunnel engineering projects.

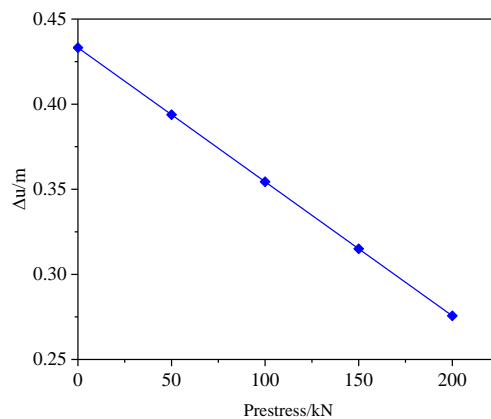


Fig. 5 Prestress effect on tunnel wall displacement

5.2. Influence of bolt (cable) spacing

As shown in Fig. 6, the spacing of bolt (cable) has a positive correlation with wall displacement; as the spacing increases, the wall displacement gradually increases. Specifically, when the anchor spacing increases from 0.6 m to 1.2 m, the wall displacement increases from 0.222 m to 0.358 m, showing a significant upward trend. This indicates that increasing the anchor spacing weakens the support effect on the surrounding rock, leading to greater wall displacement. Therefore, to effectively control wall displacement, it is essential to design the anchor layout spacing reasonably to enhance the stability of the surrounding rock.

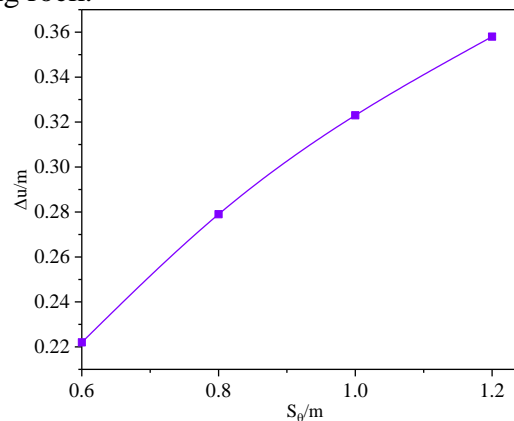


Fig. 6 Influence of bolt (cable) spacing on tunnel wall displacement

6. Conclusions

This study focuses on the mechanical characteristics of prestressed bolt (cable) support for circular roadways. It first derives relevant analytical solutions and validates them through numerical simulations. The aim of the research is to explore the effects of prestress and bolt (cable) spacing on the stability of roadway surrounding rock, providing a theoretical basis for roadway support design.

(1) The study successfully derives analytical solutions for prestressed bolt (cable) support of circular roadways and validates the accuracy of these solutions through numerical simulations, demonstrating that the proposed analytical solutions can effectively reflect the distribution characteristics of surrounding rock stress and displacement.

(2) The results indicate that applying prestress significantly enhances the support effect of roadways, effectively reducing the displacement and stress concentration of surrounding rock, thereby improving the overall stability of the roadway. Specifically, applying prestress can reduce the deformation of the surrounding rock after roadway excavation, lower the peak stress within the surrounding rock, and mitigate the risk of plastic deformation and failure, providing important theoretical support for roadway support design.

(3) When discussing bolt (cable) spacing, a reasonable arrangement of bolt (cable) significantly optimizes the stress state of the surrounding rock, indicating that bolt (cable) spacing has a crucial impact on the deformation and stress distribution of the surrounding rock. An appropriate spacing can reduce the interaction forces between bolt (cable), optimize the stress transfer path in the surrounding rock, and minimize localized stress concentrations. Properly designing bolt (cable) spacing not only enhances the stability of the surrounding rock but also improves the overall efficiency of the support structure and reduces engineering risks.

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