

Analysis of the Principle of Gravitational Wave Searching and Future Applications

Yifan Gu *

Bates College, Lewiston, the United States

* Corresponding Author Email: ygu@bates.edu

Abstract. When thinking about waves, mankind will first come up with ripples of water. Similarly, gravitational waves are also ripples, but they are ripples of spacetime. Waves of water are generated due to an object dropping into the water. Likewise, gravitational waves are generated due to massive objects in space. The concept of it was announced by Einstein, which was then observed in 2015 by the LIGO collaboration, which opened up a whole new chapter of observational astronomy. Since that remarkable detection, study of gravitational waves has become incredibly important when conducting research in black hole and neutron star mergers, as well as other cosmic phenomena. By detecting gravitational waves, scientists can have a better understanding of the nature of gravity, and they can better investigate behaviors of matter under certain conditions. However, Current detectors, such as LIGO and Virgo, cannot perfectly detect and observe waves due to their sensitivity and low frequency range. Future upgrades of equipment, including next-generation detectors like the Einstein Telescope and the space-based LISA, may help detect weaker and further gravitational waves, with a larger frequency range. As gravitational wave astronomy continues to make progress, one can expect more advanced knowledge and findings of the universe, gravity, spacetime, and the most powerful events that occur in the cosmos.

Keywords: Gravitational waves; general theory of relativity; LIGO; Virgo; black holes.

1. Introduction

Gravity is important to the understanding of the universe. It provides the framework of explaining relations between masses. Issac Newton's research has greatly developed the study of gravity and gravitational waves, but Albert Einstein's theories made revolutionary progress in this field. Newton's law described gravity as not a regular force, but an attraction between two masses. For more than two centuries, it successfully explained planetary motion and a wide range of other phenomena. Einstein introduced his General Theory of Relativity in the early 20th century. He stated that gravity was not force, it was the curvature of spacetime caused by masses and energy [1]. This theory refreshed people's understanding of gravity and extended beyond what Newton had established. Newton suggested that gravity act instantaneously over distance, while Einstein believed that it propagates at the speed of light. This theory refreshed the understanding of gravitational interaction [2].

The equation below was invented in Newton's law of gravitation. It describes how gravitational forces interact between masses:

$$F = G \frac{m_1 m_2}{r^2} \quad (1)$$

It was a revolutionary formula that explained a great deal of gravitational phenomena, such as planetary motion and force that pulls the tide. However, it could not explain phenomena that required deeper understanding of gravity. In Einstein's General Theory of Relativity, gravity was defined as the curvature of spacetime instead of just forces between masses. Einstein's field equation described how masses and energy created curvature in spacetime:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (2)$$

In the equation, $G_{\mu\nu}$ is the curving spacetime represented by the Einstein tensor, Λ is the cosmological constant, $g_{\mu\nu}$ is the metric tensor and G is the well-known gravitational constant. c is another well-known constant which is the speed of light, and $T_{\mu\nu}$ is the stress-energy tensor

representing matter and energy [1]. The equation here perfectly explains how masses and energy create curvature in spacetime, and how the curvature induced motion of objects that are affected. This theory explained motion of objects in a higher accuracy. The above theorem has recast the understanding of gravity. Because of this huge progress in the study of gravitation, scientists can predict a lot of novel phenomena. The prediction of bending of light by gravitation was first experimentally verified and confirmed by Arthur Eddington during the solar eclipse of 1919. It had provided strong support for Einstein's theory [3]. The success of this experiment dramatically increased the acceptance of the theory.

The prediction of the existence of black holes was also a revolutionary progress in physics. Black holes are regions of spacetime that have extremely great mass, leading to an extremely strong gravity that can even stop light from escaping from it [4]. The Schwarzschild solution provided the first exact description of a black hole and outlined the concept of an event horizon [5]. The Schwarzschild metric is given by:

$$ds^2 = c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \quad (3)$$

Where ds^2 is the spacetime interval, $r_s = 2GM/c^2$ M is the mass of the object (e.g., a black hole), and $d\Omega^2$ represents the angular part. The event horizon where everything goes beyond can never return, changed the understanding of the extreme gravitational environment. Over time, black holes became a representation of one of the most extreme consequences of spacetime curvature. Moreover, Einstein also predicted gravitational waves that were created due to some catastrophic cosmic events such as merging black holes. After about a century, gravitational waves were first directly confirmed by the LIGO Scientific Collaboration in 2015. This observation of gravitational waves proved that Einstein's theory was correct [4]. This discovery allows scientists to explore undetectable events. The discoveries of light bending and gravitational waves are profound implications of Einstein's general relativity, and they have changed the framework of this world from Newtonian to relativistic gravity.

Gravitational fields are essential to celestial mechanics when describing planetary orbits, the dynamics of stars, and the structure of galaxies. They also help explain the behavior of exotic objects. Recent studies of gravitational waves have provided remarkable insights into the most energetic events in the universe. LIGO science collaboration has first detected the existence of gravitational waves in 2015. This founding marked a milestone in astrophysics [6-10]. This detection was attributed to the collision of two black holes and it confirmed Einstein's prediction, opening a new era in observational astronomy. Since then, LIGO, along with its European counterpart Virgo, has detected numerous gravitational wave events, including neutron star collisions which have provided crucial information about the formation of heavy elements. Most importantly, these findings allowed scientists to explore fundamental physics in ways that were previously impossible. This paper is planned to provide an overview of gravitational fields, both in classical and relativistic descriptions. It will also illustrate their applications in different physical contexts. The paper is dedicated to solving the modern challenges to gravitational physics, like the status of dark matter and energy, and the steps being taken toward reconciliation between general relativity and quantum mechanics.

2. Concept of Gravitational Wave

Accelerating objects can create curvature in spacetime, leading to the existence of gravitational waves. Similar to how a pebble dropped into a pond creating ripples on the water's surface, these waves spread out from the center at light speed [11, 12]. Mathematically, gravitational waves are represented by perturbations in the metric tensor, which satisfies the linearized Einstein field equation:

$$h_{\mu\nu} = 0 \quad (4)$$

Where is the d'Alembertian operator, defined as [13].

$$\square = -\left(\frac{1}{c^2}\right) \frac{\partial^2}{\partial t^2} + \nabla^2 \quad (5)$$

Rather than a monopole or dipole moment, the quadrupole moment is required in gravitational waves so that it can be distinguished from electromagnetic waves. The energy carried by gravitational waves is radiated away from the source and will result in a decrease in the system's total energy, which can be observed in systems. The 2015 detection by LIGO provided direct evidence of this radiation because the waveform matched the predictions for two inspiraling black holes. The strain, which represents the fractional change in distance caused by the passing gravitational wave, is extremely small. This means highly sensitive detectors like LIGO, which use laser interferometry to measure minute changes in distance over kilometers, is required. The continued study of gravitational waves provides valuable insights into the universe's most energetic events and offers a new tool for testing the limits of general relativity under extreme conditions [7].

3. Principle and Facility

The principle of detecting is based on the concept of laser interferometry. The LIGO is designed to detect the incredibly small disturbances caused by passing gravitational waves. LIGO has two facilities: one in Hanford and another in Livingston. Each facility is set in an L-shape with two perpendicular arms, 4 kilometers in length. A high-energy laser pulse is split and sent through each arm, where it oscillates back and forth between mirrors (seen from Fig. 1) [7]. The beams are then reunited, and any difference in the length of the paths caused by a gravitational wave's passage creates an identifiable interference pattern. This setup enables LIGO to achieve such a sensitivity in length variation at an order of one ten-thousandth the size of a proton's diameter, enabling it to catch gravitational waves most accurately. The measurement of such small spacetime curvatures allows for the identification of gravitational waves, providing a direct way of observing key astrophysical phenomena. The LIGO detectors has the same function as Michelson interferometers. There are several mirrors which are used as test masses of gravitational waves. If there comes a gravitational wave, the light inside each interferometer arm will have a phase shift, causing a 180-degree phase difference between the arms. To ensure destructive interference of unmodulated light at the AS port, the arm lengths must be precisely adjusted. This setup, called the dark fringe, enables constructive interference of the modulated sideband light. In dark fringe mode, most of the power hitting the beam splitter is redirected towards the laser. Common mode signals will be reflected back to the laser, but the differential motion of the arm can be detected in the AS port [8].



Fig 1. Two photos of the LIGO observatories showing an overview of the buildings [7].

Two important modifications are made to a standard Michelson interferometer in LIGO to enhance sensitivity to gravitational waves. These modifications are illustrated in Fig. 2 [8]. Each interferometer arm starts with a Fabry-Perot optical cavity. The cavities are created with a partially transmitting input mirror and an end mirror that reflects most of the light. The light in the arm will be bounced back and forth multiple times between the mirrors. This contributes to making the light power stronger and allowing the phase shift amplitude to be bigger. Moreover, researchers placed some partially transmitted mirrors between the laser and the beam splitter to recycle power.

To better recycle power and enhance the power at the same time, the mirror's transmission ability is adjusted to cover the loss in the Michelson. This setup, named as the power-recycled Fabry-Perot Michelson interferometer, allows detectors of LIGO to be much more powerful than the simple Michelson [8]. As soon as LIGO and Virgo observed the first gravitation wave in 2015, their findings have refreshed the understanding of the universe. Scientists are now able to detect phenomena that were previously beyond the traditional electromagnetic observations.

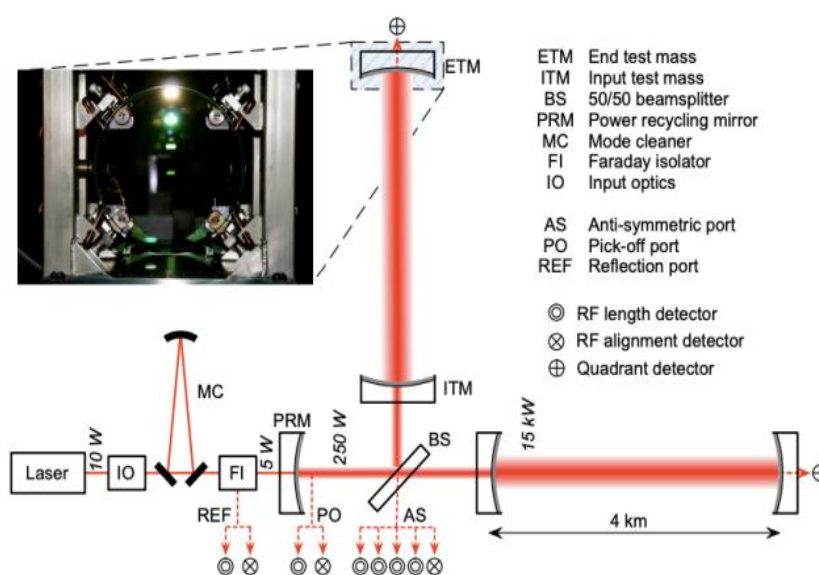


Fig 2. A diagram of the interferometer in LIGO. The structure and components of this equipment are illustrated [8].

4. Applications

Applications of gravitational wave research have gone well beyond the detection of discrete astrophysical sources. One of the important uses is to explore gravity and relativity in some serious condition. Exciting new studies using gravitational wave data have put limits on alternative gravity theories. These results provide important information about the strength of general relativity in highly nonlinear situations, such as merging black holes.

In 2019, the LIGO observatory had 11 confirmed gravitational wave events. The data span a gravitational wave mass spectrum for binary black holes between 18.6 and 84.4 solar masses, at distances between 320 and 2840 megaparsecs [9]. These findings make it possible for more precise estimation of the merger rates of binary black holes and neutron stars throughout the universe and provide insight into the mass and spin distributions of compact binary systems.

Astronomers have improved their distance measurements to sources with the use of detections and catalogs of the Milky Way. This has provided more information about their physical properties and cosmological significance. A few candidates of possible gravitational waves were identified but not considered definitive detections. This indicates room for more research and breakthroughs down the road. Gravitational waves enable the study of the formation and evolution for binary systems, including black hole pairs and neutron star pairs. These insights represent the implications for

astrophysics and cosmology. In 2021, M Zevin conducted a study that explored the formation mechanisms of binary black hole systems. The research unveiled the difference between those parts from the underlying population and the detectable population. It seems that a lot of pathways for the detected system are involved. It is much more likely that the detected black hole population is a combination of different channels rather than a single channel. The authors evaluated different assumptions and the findings rejecting highly inefficient common-envelope models. The author highlights the importance of considering multiple formation channels, as neglecting these can lead to biased conclusions on branching fractions and physical models. This study helps people to answer some of the most critical questions in stellar evolution [10].

Gravitational wave astronomy also allows scientists to study nuclear physics in environments that are far more extreme than anything one can create on Earth. By observing neutron star collisions, such as the GW170817 event, researchers have gained a unique opportunity to learn about the behavior of matter at ultra-high densities. This helps understand the properties of neutron stars and describes how matter behaves at nuclear densities. Neutron star (NS) mergers have provided insights into gravity and dense matter physics in nuclear physics. The detection has had a large impact on the understanding of these phenomena [14]. Gravitational wave observations provide a tool to investigate the dynamical and physical processes of binary neutron star mergers, and is a key tool in this field [11]. These insights are of great value because they make physical conditions that are beyond the scope of replication in Earth-based laboratories possible, which significantly expand the understanding of astrophysics, fundamental physics, and the universe.

5. Limitations and Prospects

There are some challenges and problems that need to be faced and solved to fully explore the potential of this field. For example, the current gravitational wave detectors like LIGO and Virgo are too sensitive. Although this allows detectors to measure very small changes, they cannot exclude different types of noise, e.g., seismic vibrations, and quantum noise. These sources of noise can interfere with the gravitational signal and make it very hard to pick up weaker or more distant events. Moreover, the current network of detectors doesn't cover the entire sky, which makes it harder to find the exact source of a gravitational wave. Therefore, adjusting sensitivity and expanding the network of detectors could help us make more accurate and precise measurements.

Another limitation is the frequency range that these detectors can handle. LIGO and Virgo are optimized for a specific range of frequencies, usually between 10 Hz and a few kilohertz. However, other sources like the merging of supermassive black holes emit signals at much lower frequencies that current detectors are not able to detect. To access a wider variety of cosmic events, future detectors like the space-based LISA are being developed where LISA will be capable of detecting these low-frequency waves.

Much more progress can be made with upgraded technology and apparatus. For example, the future detectors may be able to greatly enhance the ability to detect fainter and more distant sources. The upcoming launch of LISA will allow us to observe events like supermassive black hole mergers by enabling us to study low-frequency gravitational waves. As one continues to improve the sensitivity of the instruments and expand the global network of detectors, the astronomy will become an even more powerful tool for exploring the universe and allow us to discover the deepest aspects of physics and the nature of spacetime itself.

6. Conclusion

In the end, gravitational wave investigation has a great impact on the understanding of the universe. Scientists have been gradually making progress in GW astronomy from the confirmation of the predictions by Einstein to revealing hidden dynamics of black holes and neutron stars. While there are still obstacles to overcome with improved detector sensitivity and extended frequency range in

future upgraded detectors. The progress to date is nothing short of remarkable and the growth in knowledge will go on with new generation detectors and space-based observatories. Further exploitation of these ripples means further probing into the nature of gravity and spacetime, as well as some of the most extreme events in the cosmos, while one builds an expanded understanding of the universe and a nuanced view of the place in it.

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