# A Study on The Precise Location of Multiple Rocket Debris Based on Simulated Annealing Algorithm

Zhaoyang Wu, Xiujuan Liu, Jinqiu Guan *

Faculty of Science, Jiamusi University, Jiamusi, China, 154007

* Corresponding Author Email: guanjinqiu@jmsu.edu.cn

**Abstract.** Rocket debris localization techniques can significantly improve the safety of the fallout zone, reduce potential damage to the environment and speed up emergency response, while promoting the sustainable development of rocket technology. To this end, this paper explored how to localize multiple airborne rocket debris through sonic boom signals captured by ground-based monitoring equipment. Firstly, this paper preprocessed the known data and transformed the position of each monitoring device into 3D coordinates, and then established a mathematical model based on time-difference localization for analysis, and determined that at least four monitoring devices are required. For a single wreck, the unknown position (*x, y, z*) and time *t* can be solved for using nonlinear least squares. For the case of multiple wreckages, it is essentially from all permutations to Due to the large amount of computation, this paper introduces a heuristic annealing model to obtain the permutations with smaller errors faster, in order to determine the corresponding wreckage of the signals measured by each device, and then to find the position and time of sonic booms of all the wreckage. After calculation, the total error of the final result is 6.17196e-07, and the time of the sonic boom generated by the four wrecks does not differ from each other by more than 5s, which is in line with the requirements, and the error is very small, and the modeling effect is better. This model can be a good solution to this type of rocket debris localization problem.

**Keywords:** Time-difference Positioning, Nonlinear Least Squares, Simulated Annealing Algorithm.

## 1. Introduction

The vast majority of rockets are multi-stage rockets in which the following stage or booster falls after completing its intended mission and is separated by an interstage separation device. Accurate prediction of the location of rocket debris is of great significance in reducing potential damage to the environment and speeding up emergency disposal [1-2]. Traditional methods for locating rocket debris are highly dependent on external conditions and are not sufficiently real-time and flexible, such as remote sensing technology, drone positioning and multi-sensor fusion technology, which may be limited in complex environments or under adverse weather conditions, affecting their accuracy and reliability. Moreover, these technologies require specialized human resources and equipment support, including operator training and equipment maintenance, which is costly and has a high demand for technical personnel.

Structure of this paper: the first chapter is the introduction, which introduces the background of the research, the current status of the research, the innovation points and the structure of the article. The second chapter is a review of related theories, outlining the application of the principle of simulated annealing algorithm in the prediction of rocket debris position. Chapter 3 is the experimental process, detailing the specific steps of data preprocessing, establishing geometric model, least squares algorithm and heuristic algorithm optimization. Chapter 4 is the analysis of the experimental results, showing the experimental results and the discussion of the results. Chapter 5 is the conclusion and outlook, summarizing the research results.

## 2. Relevant theories

Principlesof simulated annealing algorithm. The simulated annealing algorithm is a commonly used mathematical optimization method with a powerful global search capability that can adapt to optimization problems of various complexities to find global or near-optimal solutions. The algorithm is insensitive to the initial solution, easy to implement, and robust. In addition, it can increase flexibility by adjusting the cooling strategy, and due to its heuristic properties, it is suitable for problems that are difficult to build accurate mathematical models, which can help us find the optimal combination of the seven devices faster, thus obtaining a more accurate location and time of sonic booms of the four sets of wrecks, and improving the accuracy and robustness of the model [3-5].

# 3. Experimental Procedures

## 3.1. Data preprocessing

The longitude and latitude in the given data are not convenient to calculate, we will turn them into *x, y* and put them in Cartesian coordinate system to facilitate the calculation, we take the coordinates of device A as the origin of *x* and *y* axes, and calculate the coordinates of device A and the other devices by the value of distance between latitude degrees approximated to 111.263 km per degree and longitude degrees approximated to 97.304 km per degree as given in the question stem and standardize the coordinates of device A and other devices and standardize them in kilometers. See the table.1 below for details.

**Table.1.** Processed data

| Installations | $x(km)$ | $y(km)$ | $z(km)$ | Sonic boom arrival time (s) | | | |
|---|---|---|---|---|---|---|---|
| A | 0.000E+00 | 0.000E+00 | 8.240E-01 | 1.008E+02 | 1.642E+02 | 2.149E+02 | 2.701E+02 |
| B | 5.274E+01 | 2.804E+01 | 7.270E-01 | 9.245E+01 | 1.122E+02 | 1.694E+02 | 1.966E+02 |
| C | 5.070E+01 | 6.464E+01 | 7.420E-01 | 7.556E+01 | 1.107E+02 | 1.569E+02 | 1.880E+02 |
| D | 9.730E-01 | 9.135E+01 | 8.500E-01 | 9.465E+01 | 1.414E+02 | 1.965E+02 | 2.590E+02 |
| E | 2.754E+01 | 4.595E+01 | 7.860E-01 | 7.860E+01 | 8.622E+01 | 1.184E+02 | 1.267E+02 |
| G | -1.888E+01 | 3.527E+01 | 5.750E-01 | 1.037E+02 | 1.630E+02 | 2.068E+02 | 2.103E+02 |

In order to get more accurate results, we limit the range of wreckage position and time according to the data in the above table. Through the information in the question stem, we know that the vibration wave monitoring equipment are in the theoretical fallout area of the wreckage, and we know that the theoretical fallout area of the wreckage can be accurate to within 100km, and in order to avoid losing part of the solution when the range is too small, we finally choose 200km as the theoretical fallout area, and then combine with the x, y coordinate range of all the detecting equipment to finalize the range of x as [-84,116]. The range of x is [-84,116], and the range of y is [-50,148].

We know that the wreckage was originally a rocket or booster of the following stage, so the location of its fall will not exceed 100km (the height of the atmosphere) at most, and the location of its acceleration to 340m/s under the action of air resistance is about 50km, so we choose [0,50] as the z range. According to the collinear theorem the distance between the wreckage and the detector does not exceed the sum of the squares of the maximum ranges of x, y, z, which is about 287km, we determine the range of time based on the time of all the gauges as [-70,70]. At this point, all data preparation was completed.

## 3.2. Discussion on the number of monitoring equipment units

For the sake of discussion, let's disregard the need for calculation time. Under this premise, if there is only one device detecting the range of a sphere, the planing surface is shown in Figure 1.

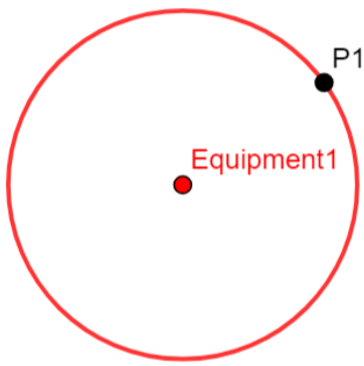


**Figure 1.** Schematic diagram of one-ball planing surface

If only one monitoring equipment measured distance as a parameter, as shown in the figure obviously, that is, can not accurately determine the location of the wreckage of the sonic boom.

If there are two monitoring equipment measured distance as a parameter, then there are three cases, if the two spheres have no intersection then no solution, if the two spheres intersect then the planer diagram is shown in Figure 2.

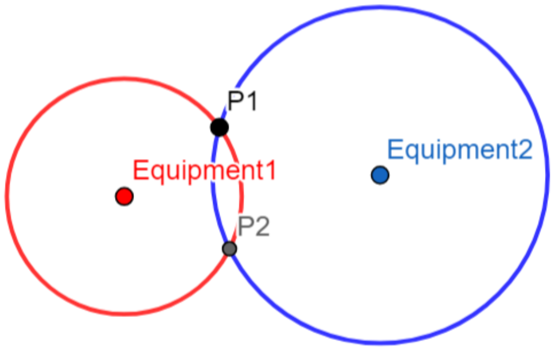


**Figure 2.** Intersecting planes of two balls

Obviously there are many solutions to this case as well, and it does not work out the exact position, which can be calculated when and only when the two balls are tangent to a point, at which point we reintroduce the time that needs to be calculated, at which point the original scope changes to two centers digging away from the smaller ball to find the point of intersection of the larger ball, which obviously does not lead to an exact solution, so with only two devices it is obvious that you can only find the position if the time is already known, the Nor can you get the position and time of the wreckage sonic boom.

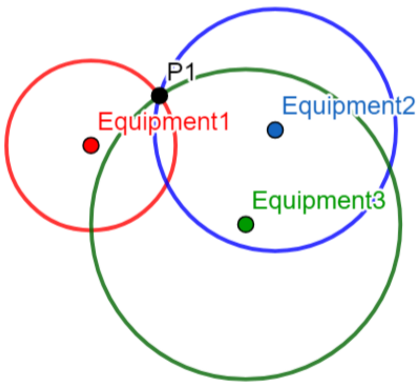


**Figure 3.** Intersecting planes of three spheres

As shown in figure 3, possible positional relationships of the surfaces of the three spheres once the distances measured by the three testing devices are available as parameters:

Completely separated, two by two tangent, tangent to the surfaces of any two spheres, the surface of the third sphere is not in contact with them, the three spheres are tangent at the common point, partially overlapped, one ball is tangent to the other internally, one ball is tangent to the other externally, and the third ball is partially overlapped. Among the cases where there is only one point of intersection: the three spheres are tangent at the common point, one ball is tangent to the other, and the third ball is tangent externally [6-7].

In most cases, only one intersection point on the surface of the three spheres is relatively rare, so in the case of known time three devices can also only part of the case to calculate the position, which does not meet the requirements. So only three devices can obviously only be used to find the position in the case of already know the time, also can not get the position and time of the wreckage sonic boom. The equations that need to be solved to accurately measure the location and time of the sonic boom in this problem are as follows:

$$v\ (t_i - t) = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 \quad i = 1,2,3 \cdots n \tag{1}$$

Equations contain four unknowns, and in the mathematical theory of linear equations, the relationship between the number of equations and the number of unknowns is crucial. For a system of linear equations containing four unknowns, at least four equations are needed to ensure the certainty of the solution. This is because each individual equation provides an additional constraint on the system, thus reducing the degrees of freedom of the solution. If there are only three equations, then it is usually not possible to uniquely determine the values of all the unknowns, i.e., where and when the sonic boom occurred [8].

So to get to get the location and time of the sonic boom of the wreckage, at least four devices at specific locations (such that the corresponding equations have a solution) are required. The schematic is shown in Figure 4.

**Figure 4.** Schematic diagram of the four testing devices

### 3.3. Simulated annealing algorithm optimization

We optimize this complex problem through MATLAB using a simulated annealing algorithm to find a solution that minimizes the sum of squares of the residuals through continuous iterations and adjustments. There are three main functions involved: createH, createK and createQ [9].

The function createH calculates the residuals between the given data and the desired unknown quantity. The residuals are the difference between the actual value and the predicted value. The function calculates the sum of the squares of these residuals through a series of calculations and returns this function [10]. The calculation of residuals is based on the following formula:

$$f_{\mathrm{i}} = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 - v\,(t_i - t)\,,\quad i = 1,2,3 \cdots 16 \tag{2}$$

$$\mathrm{h} = \sum_{i=1}^{16} f_i^{\,2} \tag{3}$$

In this paper there are four groups of unknowns to be solved, four in each group, so i should be 16. by minimizing the sum of squares of the residuals for functional purposes. The resulting fitted curve is the best-fit nonlinear model that best describes the nonlinear relationship between the data points.

The create K function writes the code of the first question of the way to extract from seven sets of data and finally take the minimum residual sum of squares as the optimal solution as a new function create K for easy calling.

The create Q function implements the simulated annealing algorithm for finding the optimal solution in a given search space. It first defines the parameters of the simulated annealing, including the initial temperature, the final temperature, and the rate of cooling. Then, the function enters a loop in which it keeps perturbing the current solution data to generate a new solution new Data and calculates the sum of squares of the errors of the new solution through the create K function, and if the sum of squares of the errors of the new solution is less than the current minimum sum of squares of the errors or if, according to the probabilistic acceptance criterion of the simulated annealing, the new solution is accepted, then the minimum sum of squares of the errors and the optimal combination are updated. The loop continues until the final temperature is reached. The function finally outputs the minimum error sum of squares and the corresponding best combination [11-12].

We use tshe model to get the desired optimal combination faster by simultaneously designing the annealing algorithm function create Q, initially setting the initial temperature to 10000, the minimum temperature to 1, and the cooling rate to 0.9, and according to the actual data, we further improve the model's effect by debugging the initial temperature, the minimum temperature, and the cooling rate [13].

## 4. Results

We update the ranges of each variable obtained in the data preprocessing stage into the function to obtain a more accurate function create. the optimal permutations obtained by the annealing algorithm are shown in the table.2.

**Table.2.** Data from monitoring stations corresponding to wreckage

| Installations | Wreckage 1 | Wreckage 2 | Wreckage 3 | Wreckage 4 |
|---|---|---|---|---|
| A | 1.008E+02 | 1.642E+02 | 2.149E+02 | 2.701E+02 |
| B | 1.694E+02 | 9.245E+01 | 1.122E+02 | 1.966E+02 |
| C | 1.569E+02 | 1.880E+02 | 1.107E+02 | 7.556E+01 |
| D | 2.590E+02 | 1.414E+02 | 1.965E+02 | 9.465E+01 |
| E | 1.184E+02 | 8.622E+01 | 7.860E+01 | 1.267E+02 |
| F | 2.669E+02 | 1.663E+02 | 1.755E+02 | 6.727E+01 |
| G | 1.630E+02 | 1.037E+02 | 2.068E+02 | 2.103E+02 |

The position and time data we obtained for the four wrecks are shown in Table.3.

**Table.3.** Initial data for four wrecks

| Digital | Wreckage 1 | Wreckage 2 | Wreckage 3 | Wreckage 4 |
|---|---|---|---|---|
| $x$(km) | 4.195E+01 | 5.741E+00 | 3.661E+01 | 2.520E+01 |
| $y$(km) | 2.259E+01 | 4.962E+01 | 4.696E+01 | 8.300E+01 |
| $z$(km) | 2.287E+01 | 1.148E+01 | 1.165E+01 | 1.153E+01 |
| $t$($s$) | 1.483E+01 | 1.400E+01 | 3.686E+01 | 1.300E+01 |
| sum of squared errors | 4.250E-09 | 3.310E-09 | 2.430E-09 | 6.070E-07 |

We reduced the resulting Cartesian coordinate system to latitude and longitude to obtain position and time-accurate data for the four wrecks in the table .4.

**Table.4.** Four wreckage reduction data

| Digital | Wreckage 1 | Wreckage 2 | Wreckage 3 | Wreckage 4 |
|---|---|---|---|---|
| longitude(°) | 1.107E+02 | 1.103E+02 | 1.106E+02 | 1.105E+02 |
| latitude(°) | 2.741E+01 | 2.765E+01 | 2.763E+01 | 2.795E+01 |
| altitude (m) | 2.287E+04 | 1.148E+04 | 1.165E+04 | 1.153E+04 |
| $t$(s) | 1.483E+01 | 1.400E+01 | 1.686E+01 | 1.300E+01 |
| sum of squared errors | 4.250E-09 | 3.310E-09 | 2.430E-09 | 6.070E-07 |

According to the calculation, the total error is 6.17196E-07, which is a very small value. And the four wrecks produce sonic booms at a time different from each other no more than 5s, in line with the question. In order to visualize the feeling, we obtained the data through python to draw a three-dimensional graph as Figure 5.

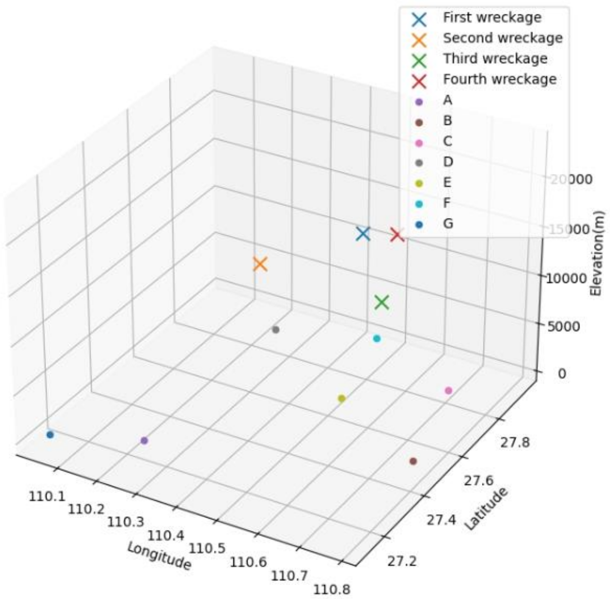


**Figure 5.** Schematic of the position of the four wrecks in three-dimensional space

From the figure, it can be seen that all the required positions are within the limited range, and at the same time, the error obtained from the calculation of this model is very small, and the distance between it and the actual position is very close, which meets the requirements.

## 5. Conclusions

This paper develops a comprehensive model of a localization system designed to accurately locate airborne rocket debris from sonic boom signals captured by ground-based monitoring equipment. The model combines Time Difference Orientation (TDOA), a simulated annealing algorithm to monitor the data and optimize the accuracy and robustness of the solution. Through a series of complex calculations and algorithm optimization, the model successfully achieves the required accuracy of rocket debris localization and is more credible with relatively small errors in simulation tests. This study provides important support for environmental protection and emergency response rescue work, which fully guarantees the safety of people and property, and has great potential for application. Meanwhile, the geometric model, algebraic equation solution, least squares optimization model and heuristic algorithm to optimize the equipment combination used in this paper have high generality and feasibility. The future direction of development is to combine artificial intelligence, machine learning and other technologies, using satellite remote sensing data, meteorological data, ground monitoring data and other multi-source data, to carry out multi-source data fusion research, to further improve the accuracy, real-time, and reliability of the prediction of the rocket debris fall position.

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