

Dynamic Simulation of Spiral Motion in The Bench Dragon Dance Based on The SAT Algorithm

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Abstract. The Bench Dragon is a traditional folk cultural performance prevalent in the Zhejiang and Fujian regions of China. This study resolves the quantification problems of the positions and velocities of each part during the performance of the dragon dance team and the collision detection problem during the marching process. This paper established the position-velocity calculation model based on the equation of the equidistant helical curve and the binary search algorithm, as well as the collision detection model based on the SAT algorithm. Through continuous iteration, traversal, and optimization, this study ultimately obtained the position coordinates and velocity values of each part of the bench dragon per second, and detected that in the context of this study, the dragon head would collide with the ninth bench at the 412.5th second. This research can not only provide scientific references for the performances of the dragon dance team but also lay a model foundation for studying the issues of the team's turning and path planning.

Keywords: Bench Dragon Dance, Collision Detection, Spiral Motion, Dynamic Simulation.

1. Introduction

Collective behavior [1] refers to the coordinated actions of individuals within a group, often driven by a shared goal in a common environment. This concept has drawn attention from disciplines such as sociology, psychology, biology, and engineering due to its inherent complexity and its capacity for both organized and chaotic outcomes. It is particularly relevant to research on 3D scenes [2-3], as the dynamic interplay of individual actions within complex environments can lead to emergent patterns or system breakdowns. One fascinating example is the traditional Bench Dragon Dance [4] from the Zhejiang and Fujian regions of China, where participants coordinate their movements to manipulate connected benches into a spiraling, dragon-like structure. The success of this performance depends on precise synchronization, highlighting how individual contributions create a larger, collective art feast.

The Bench Dragon Dance [5] is a traditional cultural performance from the Zhejiang and Fujian regions of China. Dragon Dance Art [6] is significant in China's traditional festivals, with the unique "Bench Dragon Dance" capturing audiences' attention due to its charm. The Bench Dragon team is composed of dozens or even hundreds of participants connect bench-like structures into a winding and serpentine dragon, moving in synchronized patterns led by the dragon's head. Symbolizing the collective coordination of individuals towards a shared artistic goal. And the performance requires precise coordination, as the dragon coils and uncoils in a spiraling motion, where the smaller the space used and the faster the movement, the more visually striking the performance becomes.

Despite its immense popularity and cultural significance, the Bench Dragon Dance remains largely unexplored from a quantitative perspective in earlier studies. While traditional performances rely heavily on the subjective experience and intuition of the performers, there is a pressing need for a more rigorous and systematic approach to understand and analyze the movement patterns of the dragon.

Therefore, the quantification of the Bench Dragon Dance movement process is of great importance and urgent, so this study puts forward the first research question: **The position and velocity of each section of the Bench Dragon are quantified through the size of the Bench Dragon, the movement velocity of the dragon head, and the screw pitch.**

By quantifying the movement of each bench, this study gains valuable insights into the dynamics of the Bench Dragon Dance, providing a solid foundation for enhancing the performance's aesthetic appeal and safety. This also offers practical guidance for choreographers and performers, enabling them to fine-tune their movements and create more visually stunning performances. To address this gap, this study simulated and quantified the spiral path flow of the Bench Dragon Dance, focusing on modeling the movement of the dragon based on known parameters such as the speed of the dragon's head and the dimensions of the benches, and spiral pitch. Using mathematical modeling and computational simulations, this study predicted the position and velocity of each bench at any given time during the performance. This quantification resolves potential safety concerns.

With a clear understanding of the movement dynamics in place, another critical challenge arises: ensuring that the densely packed sections of the dragon do not collide, especially during intricate maneuvers. Given the large number of participants and the complexity of the dragon's movements, there is a significant risk of collisions between benches, which could compromise both the visual spectacle and the safety of the performers. This brings us to this paper's second research question: **How can this study implements a reliable collision detection system for the Bench Dragon Dance based on the quantified movement data of each bench?**

Recognizing the potential for collisions [7] during the intricate maneuvers of the Bench Dragon Dance, this paper delves into the realm of collision detection [8]. And proposed and solved a problem that effectively avoids collisions [9] and reduces the risk of injury or accidents for Bench Dragon Dancer: **Through the establishment of a collision detection model [10] based on the SAT algorithm to calculate the time points where collisions might occur, and thereby avoid collisions.** By integrating a collision detection model based on the Separating Axis Theorem (SAT) [11-12], this study aims to identify the critical time points at which collisions may occur, thereby ensuring the safety of the performers and preserving the integrity of the performance. This dual approach of quantification and collision detection represents a significant step forward in the study of collective behavior and its application to traditional cultural performances.

By leveraging mathematical formulations, algorithmic advancements, and computational simulations, this study aims to unlock new insights into the intricate mechanics of this cultural performance. This study's major contributions consist in: (1) Quantify the details during the movement process of the dragon dance team to ensure the scientific nature of the performance. (2) Establish a new collision detection mechanism to ensure safety during the performance process. (3) Contributes to a deeper understanding of collective behavior and its manifestation in traditional cultural arts, with implications for preserving and enhancing cultural heritage worldwide.

2. Problem Formulation

This study aims to offer solutions to the two practical issues, by establishing models or applying certain algorithms. Thus, it is necessary for us to determine various indicators and parameters of a dragon dance team during the performance in accordance with the specific scenarios and real situations of the Bench Dragon Dance performance, thereby facilitating the establishment of a model to simulate the actual Bench Dragon Dance performance.

During the resolution process of the issues in this paper, the real situation simulated by us was : A certain Bench Dragon is composed of 223 bench sections. The first section is the dragon head, the subsequent 221 sections constitute the dragon body, and the last section is the dragon tail. The length of the board of the dragon head is 341 cm, while the lengths of the boards for both the dragon body and the tail are 220 cm. The width of all the boards is uniformly 30 cm. Each bench section has two holes, with a diameter (of the holes) of 5.5 cm, and the center of the holes is 27.5 cm away from the nearest end of the board (As depicted in Figures 1 and 2). Adjacent bench sections are connected by handles (As depicted in Figures 3).

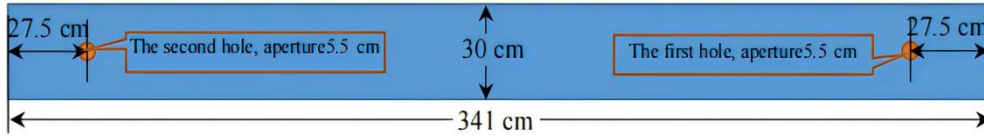


Figure 1. The top view of the dragon head

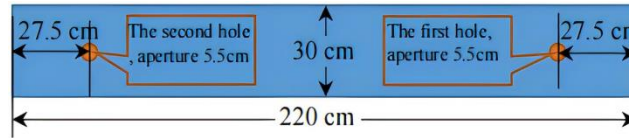


Figure 2. The top view of the dragon's body and tail

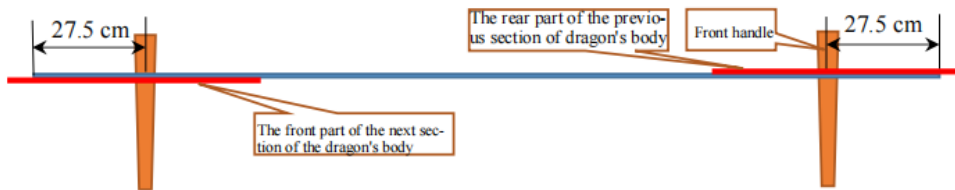


Figure 3. The frontal view of the bench

Therefore, in the problem of quantifying the movement, it is necessary for us to integrate the aforementioned dimensions of the benches and the connection mode among them to investigate when the dragon dance team spirals clockwise along the equidistant helix with a pitch of 55 cm. Each handle center is positioned on the helix, and the traveling speed of the front handle of the dragon head is consistently maintained at 1 m/s. Initially, the dragon head is located at point A on the 16th circle of the helix (as depicted in Figure 4 below). This paper is required to simulate the spiraling process of the dragon dance team and present the position and velocity of the entire team at each second during the spiraling process (namely, the positions and velocities of the centers of the front handles of the dragon head, the dragon body, and the dragon tail, as well as the center of the rear handle of the dragon tail).

And in collision detection problem, the study also need to incorporate the aforementioned information, particularly in combination with the establishment of the equidistant helical curve model in the problem of quantifying the movement and the simulation of the entire dragon dance team's entry process, to determine the termination moment when the dragon dance team cannot continue the entry in the model for quantifying movement problem, that is, the critical moment when the dragon dance team does not encounter collisions. Based on the process of model establishment and solution in the problem of quantifying the movement, calculate the position and velocity of the dragon dance team at the termination moment.

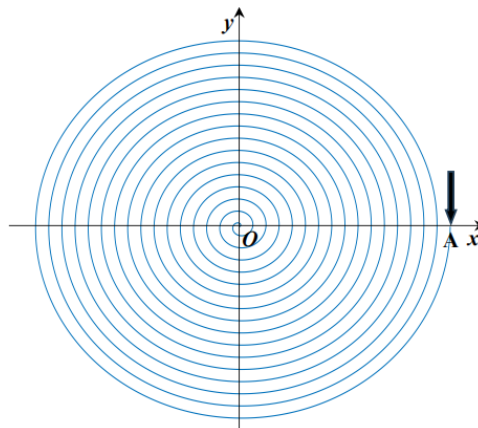


Figure 4. Schematic Diagram of Coiling into a Helix

3. Methodology

In this section, this paper introduces two methods. The first one is a position and velocity calculation model based on the equation of the equidistant helical curve and the binary search algorithm, which is employed to quantify the entire performance process of the dragon dance team. The second one is a collision detection model based on the SAT Algorithm, which is utilized to analyze and detect the moments when collisions might occur during the advancement of the dragon dance team.

3.1. A position-velocity computing model based on the equation of the equidistant helical curve and the binary search algorithm

Step 1: The Establishment of the Equation of the Equidistant Helical Curve. The polar equation of the equidistant helical curve is:

$$r = a\theta \quad (1)$$

where a is the growth rate of the helix, θ is the polar angle. Due to the pitch being 55 cm, that is, the radius increases by 55 cm for each complete revolution (2π radians). Therefore, $a = \frac{55}{2\pi}$. It is given that the initial position of the dragon head is at point A of the 16th turn of the helix. Based on the traveling speed of the dragon head, which is 100 cm/s, and the time step (0.00001s), calculate the polar angle θ_0 of the dragon head at each moment. Through the conversion from polar coordinates to Cartesian coordinates, the coordinates of any point on the helical curve can be obtained, and thus the Cartesian coordinates of the dragon's head (x_0, y_0) can be derived. The Cartesian coordinate equation of the equidistant helical curve:

$$\begin{aligned} x &= a\theta \cos\theta \\ y &= a\theta \sin\theta \end{aligned} \quad (2)$$

Step 2: Definition and Calculation of Equally Spaced Point Sequence. The starting point $P_0(x_0, y_0)$ is set to be located on the equidistant helical curve, and its corresponding angle is θ_0 . Subsequently, each point of the dragon's body and tail is required to fulfill the condition that the straight-line distance from the previous point is equal to the given length L (the length is L_0 for the first point and L for the rest). The length L is the distance between each pair of adjacent front handles of the dragon's body and tail and the rear handle of the dragon's tail. The length L_0 is the distance between the front handle of the dragon's head and the first front handle of the dragon's body. Thus, the value of L is 165 cm, and the value of L_0 is 286 cm.

The formula for computing the distance between two adjacent handle points $P_i(x_i, y_i)$ and $P_{i+1}(x_{i+1}, y_{i+1})$ is:

$$d = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \quad (3)$$

Step 3: The calculation of positions of each handle point. In the establishment of the model for quantifying movement problem, it is initially necessary to determine the initial position of the dragon head. According to the problem conditions, the dragon head commences at point A of the 16th circle of the helix and coils clockwise along the helix at a constant speed of 100 cm/s. Through converting the polar coordinate equation of the helix into the Cartesian coordinate equation, combined with the constant speed of the dragon head and the time step, the polar angle of the dragon head at each instant is calculated using cyclic iteration, and thereby the precise coordinate position of the dragon head on the helix is obtained.

After determining the head position, the dragon dance team's remaining handle positions are iteratively calculated using the binary search algorithm. Starting from the head, a search range is set based on the fixed distance L or L_1 between adjacent handle points, and the candidate points are calculated by calculating the straight-line distance between the candidate points and the previous handle point. The candidate points are compared with the target distance to gradually narrow the

search range until a handle point position that satisfies the conditions is found. This process is repeated until the exact positions of all handle points at each time point are calculated.

Step 4: The calculation of velocities of each handle point. After the positions of each handle point have been calculated, the speeds of each point are computed by means of the differential method. Firstly, an extremely small-time interval ($\Delta t = 0.2s$) is defined. By comparing the coordinate variations of the same handle point at different time points (t and $t + \Delta t$), the displacements (Δx and Δy) in the x and y directions are calculated respectively. Then, the Euclidean distance formula:

$$= \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2} \quad (4)$$

It is utilized to calculate the resultant speed within this time period. This process is repeated for each handle point and each time point, eventually obtaining the speed data of all handle points at each time point of the dragon dance team.

3.2. The establishment of collision detection model based on SAT Algorithm

Step 1: Computation of Rectangular Region. In the model establishment of collision detection problem, this study initially employs the equidistant helical curve equation that has been established in the problem of quantifying the movement to determine the central positions of each handle point of the dragon dance team. The helical curve equation, through the conversion between polar coordinates and Cartesian coordinates, can precisely describe the process of the dragon body coiling along the helix. Subsequently, based on the central position of each handle point and in combination with the size of the bench, this study calculates the coordinates of the four corner points of each bench through translation and vertical translation, thereby determining the boundary range of each rectangular area (i.e., the bench). This process integrates the movement trajectory of the dragon dance team with specific physical positions, laying a foundation for the subsequent collision detection.

Step 2: Collision Detection Based on the SAT Algorithm. The collision detection model constitutes the core of collision detection problem, and this study employed a collision detection algorithm based on SAT. The SAT algorithm is an efficient geometric algorithm utilized for detecting whether two convex polygons intersect. Its core principle lies in the fact that if two convex polygons do not intersect, there inevitably exists a separation axis capable of segregating the vertices of the two polygons on either side of the axis. In the context of the dragon dance team, each bench is regarded as a rectangular region. This paper employs the SAT algorithm to conduct projection detection for each pair of rectangular regions. The specific steps encompass: (1) Select a separation axis, typically choosing the edges of the rectangle and their normal vectors as candidate separation axes; (2) Project the vertices of the two rectangles onto the separation axis and calculate the projection intervals; (3) Examine whether the projection intervals of the two rectangles overlap. If there is no overlap on all candidate separation axes, it is determined that the two rectangles do not intersect; otherwise, a collision is deemed to have occurred. The SAT test among the benches in the dragon dance team is shown in Figure 5. By traversing all pairs of rectangles, this study is able to detect and record collision situations in real time during the simulation process.

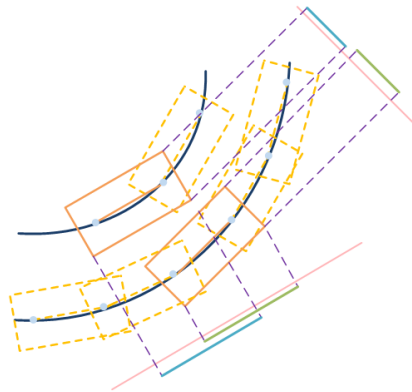


Figure 5. The Application of SAT in collision detection of Bench Dragon

Step 3: The Computation of Termination Time. The calculation of the termination moment relies on the outcomes of the aforementioned collision detection model. During the simulation of the dragon dance team coiling along the helix, this study constantly monitors the collision conditions among the benches. Once a collision between any two rectangular regions (i.e., the benches) is detected, the study immediately records the current time as the termination moment of the coiling process and cease further computations. This procedure ensures that the dragon dance team can coil along the helix for as long as possible without encountering collisions, thereby offering scientific guidance for actual performances.

4. Result analysis and discussion

In this article, the position and velocity calculation model based on the equation of the equidistant helical curve and the binary search algorithm, as well as the collision detection model based on the SAT algorithm, which this paper has proposed, are intended to address the issues of the difficulty in quantifying the motion state during the performance of the dragon dance team and the difficulty in detecting and warning of collisions among the benches. Given these models and methods, the solutions to the problems are as follows:

During the solution process of the problem of quantifying the movement, this paper employed the binary search algorithm to ascertain the exact positions of each handle point when the dragon dance team spirals in along the equidistant helical curve. The binary search algorithm precisely calculates the points at which the distance between each handle point and the previous one is a specific value at each time point by continuously narrowing the search range. Through continuous iteration and optimization of the algorithm, this paper derived the precise positions of each handle point on the helical curve per second.

After computing the precise locations of each handle point, this paper established a velocity calculation model by means of the differential approach to further determine the velocities of each handle point at diverse moments. This model subdivided the time intervals and computed the displacements of each point in the x and y directions within each small-time segment, thereby deriving the component velocities of each point. Subsequently, the combined velocities of each point were calculated using the Euclidean distance formula. Through the application of this model, the study obtained the velocity distributions of the entire dragon dance team at different times.

Through integrating the position and velocity calculation model based on the equation of the equidistant helical curve and the binary search algorithm, the study successfully addressed all the requirements put forward in Question 1. These outcomes not only offer the precise positions and velocities of each handle point of the dragon dance team per second, but also present the overall coiling state and velocity distribution of the dragon dance team at different time points through detailed data records (as indicated in Figure 6, Table 1 and Table 2).

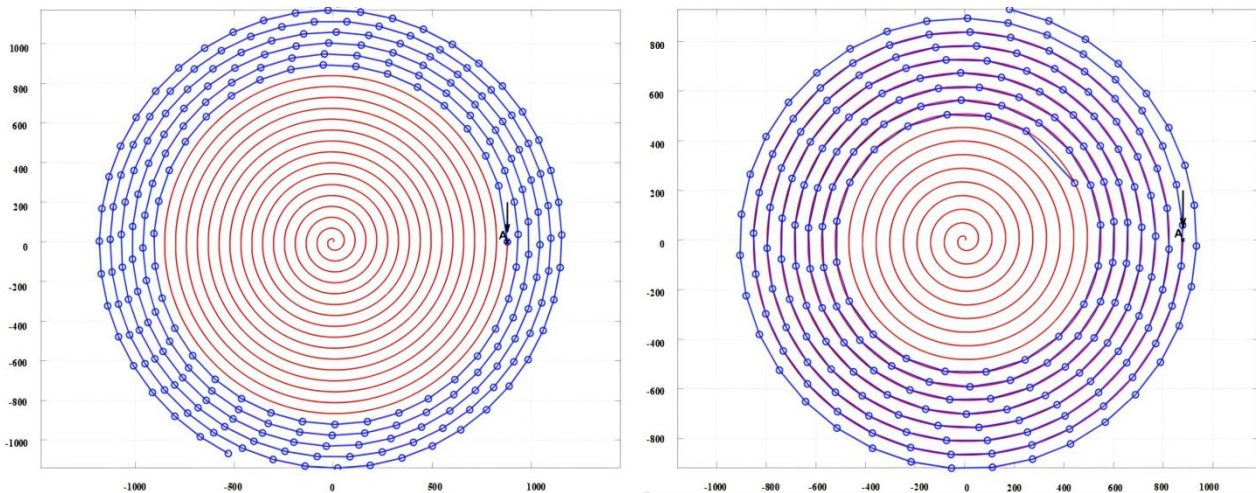


Figure 6. The simulation diagram of the Dragon Dance team's weaving-in process

Table 1. The coordinates of each point at different time points

	0 s	120 s	180 s	240 s	300 s
Dragon Head x(m)	8.800000	-4.090661	-2.953251	2.578964	4.431368
Dragon Head y(m)	-0.000010	-6.300638	6.099641	-5.363957	2.298227
The First Dragon Body x(m)	8.363827	-1.452262	-5.229679	4.810828	2.480906
The First Dragon Body y(m)	2.826534	-7.404472	4.368319	-3.575554	4.389948
Dragon Tail x(m)	-5.305452	10.974822	7.391961	3.257163	1.809269
Dragon Tail y(m)	-10.676580	0.836561	7.484280	9.463648	9.296247

Table 2. The speed of each point at different time points

	0 s	120 s	180 s	240 s	300 s
Dragon Head (m/s)	0.000000	1.000038	1.000047	1.000061	1.000087
The First Dragon Body (m/s)	0.000000	0.999984	0.999964	0.999921	0.999797
Section 101(m/s)	0.000000	0.999323	0.999024	0.998519	0.997424
Dragon Tail(m/s)	0.000000	0.998944	0.998553	0.997914	0.996619

These simulation results demonstrate that the established model accurately predicts the position and velocity of each bench section throughout the Bench Dragon performance.

In the solution to collision detection problem, this paper adopted a collision detection model based on SAT to determine the termination time of the dragon dance team's spiral entry. After calculating the precise coordinates of each handle point of the dragon dance team along the helix entry, this study obtained the four corner points of each bench by translating the handle point coordinates. Then, this paper applied the SAT algorithm to perform collision detection on all bench rectangular areas. Through the steps of selecting the separation axis, calculating the projection, and checking for overlapping, this paper traversed all time steps and detected whether any two rectangular areas would overlap. Finally, a collision was detected at 412.5 seconds, determining the termination time of the dragon dance team's spiral entry. At this time, the front handle of the dragon head collided with the ninth bench, and the spiral entry had to be stopped. This paper recorded the positions and velocities of each handle point at this time (as indicated in Table 3).

Table 3. The coordinates and velocities of the points at the termination moment

	x (m)	y (m)	(m/s)
Dragon Head	1.279543	1.893730	1.000733
The First Dragon Body	-1.579164	1.807741	0.992261
Section 51	1.360257	0.026773	0.977551
Dragon Tail	1.038376	0.074969	0.972968

Through collision detection, this study not only successfully identified the longest time point for avoiding collisions among benches—412.5 seconds, but also effectively prevented the occurrence of collisions, ensuring the safety. This outcome validates the effectiveness of the SAT algorithm in complex motion scenarios.

In summary, the methods and results of this research are conducive to an in-depth understanding of collective behaviors in traditional culture, laying a model foundation for further studies on the turning and path planning of the dragon dance team in the future and holding great significance for the protection and development of traditional culture.

5. Conclusions

This study addressed the challenges of quantifying the position and speed of each segment of the Bench Dragon during performances, as well as detecting potential collisions during the procession. By developing an innovative position and speed calculation model based on the equation of an equidistant helical curve and implementing a binary search algorithm, alongside a collision detection model utilizing the SAT algorithm, eventually this study successfully obtained the per-second position coordinates and speed values of each handle point of the dragon dance team. This study analyses revealed that the dragon head is projected to collide with the ninth bench at the 412.5th second. These findings significantly contribute to enhancing the safety and visual appeal of the Bench Dragon performance by providing a scientific basis for choreography and movement planning. Moreover, this research lays a solid foundation for future explorations into complex maneuvers such as U-turns and advanced path planning for the dragon dance team. However, this study is based on specific performance parameters and conditions. Future research should aim to generalize these models to accommodate varying performance settings and team configurations. Additionally, integrating real-time data acquisition and machine learning algorithms could further optimize the Bench Dragon's movements, adaptively enhancing the cultural experience for performers and audiences alike.

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