A Study of Bench Dragons Based on Archimedean Solenoids

Yiyi Wang *, #, Mingxi Zhou #, Haobo Wu #

College of Electrical Engineering and New Energy, China Three Gorges University, Yichang, China, 443002

* Corresponding Author Email: 17762840405@163.com

#These authors contributed equally.

Abstract. The research background of this paper is based on the dynamic movement process of intangible cultural heritage dragon dance movement during the performance. Although there are more researches on the simulation and modeling technology, there are fewer researches on the improvement of the stage choreography with the help of this technology. In this paper, we hope to bring some inspiration and improvement to the dragon dance performance through the research on the position change and velocity correlation during the displacement process of the dragon dance performance. Firstly, the Archimedes electromagnetic equation is established to simulate the motion of the dragon dance team, and then the position of the dragon head at each moment is obtained by the fixed velocity of the dragon head, and then the position and velocity of the rear point of the dragon head is gradually displaced by the known rigid-body connection condition. Finally, the data of each point at each moment of the dragon dance team were successfully obtained, In this way, the displacement process of the dragon dance performance has been successfully simulated mathematically and dynamically, to the extent that we can clearly calculate and present and visualize the approximate procession travel state at each moment. This will help to follow up with further practical problems, such as realizing the goal of process collision checking at certain spacing. This will shed some light on dragon dance performances in practical situations.

Keywords: Archimedes-screw, Finite Difference, Positioning Constant Velocity.

1. Introduction

As the nation's cultural self-confidence continues to develop, China continues to increase its emphasis on intangible cultural heritage. The Bench Dragon Performance is an intangible cultural heritage with a long history, and our research is dedicated to integrating the Bench Dragon Performance with modern technology to make its stage presentation more infectious, so that more people can experience the beauty of China [1]. The research on the fixed-speed positioning of the bench dragon is of great significance to the stage choreography and performance presentation. In order to better position each bench with the help of the intelligent creation platform, this paper simulates the huge number of bench dragons with the help of Archimedes' solenoidal equation, combined with finite difference, so as to rationalize the stage arrangement, which is of great significance to enhance the audience's viewing experience. In the current simulation research on stage performances, Boxu Han proposes to improve the data-based stage modeling process to support stage data processing and standardized stage data construction [2]. Based on this, this paper proposes for the first time to use the Archimedean difference model of locating the fixed speed for solving the stage choreography of the bench dragon. Based on this, the theoretical part of this paper will elaborate the Archimedean solenoid and other theoretical preparations, the experimental chapter will mainly introduce the model constructed in this paper and put forward a detailed solution to the problem, and finally the results obtained by means of MATLAB programming will be presented in the conclusion part, and the above content is the overall structure of this paper. This research will not only make the stage performance more beautiful and effective, and realize the rational scheduling of the stage digitalization, but also let more people know the intangible cultural heritage of the bench dragon, and then carry forward the traditional Chinese culture and the traditional Chinese aesthetics.

2. The basic fundamental of solenoidal equation

The solenoidal equation is originally a mathematical equation used to describe a spiral-shaped curve or surface formed by rotating and moving simultaneously along an axis in three-dimensional space, and it can be thought of as similar in nature to the synthesis of motion in physics. Since the movement of the dragon dance team is dished in along the spiral, the solenoidal equation can reflect the movement law of the dragon dance team more accurately. In this problem, the parametric form of Archimedes' solenoidal equation in polar coordinate system is considered to provide a theoretical basis for the study of the problem, and its basic form is:

$$r = a + b\theta \tag{1}$$

where r is the pole diameter, θ is the pole angle, a is the distance from the start point to the pole, and b is the change in pole diameter per unit angle of rotation[8]. The parametric form of the solenoidal equation can be introduced to represent the position of the dragon head, body and tail at each moment[3].

3. Experiment

The dragon is composed of 223 benches, the first one is the dragon's head, the next 221 benches are the dragon's body and the last one is the dragon's tail, the length of each bench is 341 cm, the width of each bench is 30 cm. The length of the head is 341 cm, the lengths of the body and tail are 220 cm, and the width of all the benches is 30 cm. As Figure 1 reflects, each bench has two holes of 55 cm diameter, the centers of which are 27.5 cm from the head of the nearest bench, and the two adjacent benches are connected by handles. The dragon dancers were coiled clockwise along an equidistant thread with a pitch of 55 cm, and the center of each handle was located on the thread. The traveling speed of the handle in front of the dragon is always 1 m/s. Initially, the dragon is at the beginning of the 16th turn of the thread. The position and speed of the whole dragon dance team per second from the initial moment to the end of 300 s are given by mathematical modeling.

The first step is the geometric modeling.

On the one hand, capture leading positions with differential thinking. According to the conditions, the linear velocity of the dragon's head is $1m/s$, after a period of timeΔt, the dragon's head has traveled over a section of the curve, the length s (cm) is

$$\hat{s} = 100\Delta t \tag{2}$$

Combined with the relative position of the handle on the bench, under the assumption that each section of the bench is a rigid body, the distance between the two handles is a constant value di

$$d_i = \begin{cases} 341 - 27.5 \times 2, i = 1 \\ 220 - 27.5 \times 2, i = 2,3, \cdots 223 \end{cases} \tag{3}$$

Under the condition that the pitch is 55cm, with the dragon head in front of the handle as the center of the circle, the intersection of the circle and the thread more than two, that is, at least with the adjacent circle of the thread there are intersections, at this time should be in the many intersections, priority should be given to selecting the center of the circle of the θ value of the difference in the absolute value of less than $\frac{\pi}{2}$ two intersections, and then in the two intersections to take the θ value of the larger as the handle's next position. As long as Δt to get small enough, the dragon body in the screw line have passed, in accordance with the method of taking the intersection of the circle roughly dragon dance team in the approximate location of the screw line and coordinates.

On the other hand, use cosine theorem to solve the angle between the two sides. In the above differential idea captures the dragon head before the handle on the screw line after$A_i(r_i, \theta_i)$, respectively, connect the poles and handles, can get a number of poles diameter $OA_i(i = 0,1, \cdots n)$, set OA_{i-1} and OA_i of the angle of α_i, then by the screw line equation can be obtained $|OA_i|$

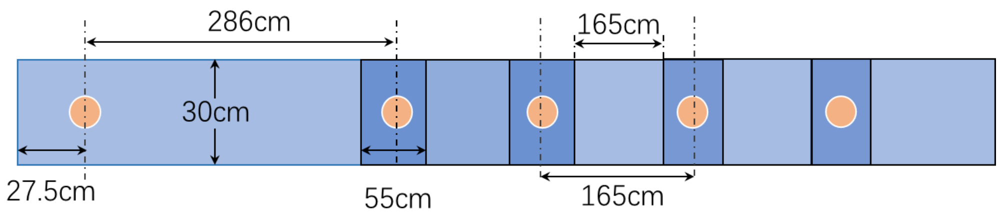

Figure 1. Bench splicing diagram

Now take the dragon head before the handle OA_{i-1}, the first section of the dragon body before the handle (dragon head after the handle) OA_i and the dragon head before and after the handle of the connecting line $A_{i-1}A_i$ (length d_i) surrounded by the triangle $\Delta A_{i-1}OA_i$ for example, to solve for the two handles of each piece of bench and the poles connected to the line of the angle α_i, the application of the cosine theorem:

$$A_{i-1}A_i^2 = OA_{i-1}^2 + OA_i^2 - 2|OA_{i-1}||OA_i|cos\alpha_i \tag{4}$$

The deformation yields α_i:

$$\alpha_i = arccos\frac{OA_{i-1}^2+OA_i^2-A_{i-1}A_i^2}{2|OA_{i-1}||OA_i|} \tag{5}$$

The second step is the treatment of time variables

Time as a key factor in this problem can be treated as either a continuous or discrete variable.

(1) If time is treated as a continuous variable, the solenoidal equation can be "spruced up". For polar angle θ, we have:

$$\theta = \theta_0 + \omega t \tag{6}$$

Deforming the solenoidal equation yields

$$\theta = \frac{r-a}{b} \tag{7}$$

And angular velocity is the derivative of angle with respect to time

$$\omega = \frac{d\theta}{dt} \tag{8}$$

Notice that the dragon's head is coiled in from the outside to the inside along the screw thread, so the angular velocity is negative, and substituting θ yields

$$\omega = \frac{1}{b}\cdot\frac{dr}{dt} \tag{9}$$

Thus, the modified solenoidal equation is obtained as

$$r = a + b\theta_0 + \frac{dr}{dt}\cdot t \tag{10}$$

Thus, the expression containing distance (r), time (t) and velocity ($\frac{dr}{dt}$) is obtained.

(2) If time is treated as a discrete variable, you need to set the time step to 1 second and discretize the distance traveled by the dragon's head by the distance traveled by the dragon's head:

$$\hat{s} = 100\Delta t \tag{11}$$

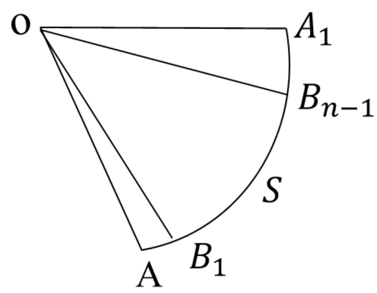

Figure 2. Separate summing schematic

As Figure 2 reflects, taking a time step, the position A_t of the dragon's head at time t corresponds to a polar angle θ_t, the value of which should be obtained by infinitely subdividing $\hat{s}$, when $\hat{s}$ is subdivided into a sufficiently large number of segments $A_{t-1}B_1$、$B_1B_2 \cdots B_{n-1}A_t$, let the polar angle of

$OB_i(i = 1,2, \cdots n-1)$ be θ_i', and the value of the angle of each infinitely equalized portion of the angle is $\frac{2\pi}{n}$, so that each portion of the curve, which is approximated as an "arc", is

$$\widehat{s_i} = \begin{cases} \frac{2\pi\theta_i' OA_{t-1}}{n}, i = 1 \\ \frac{2\pi(\theta_i' - \theta_{i-1}')OB_i}{n}, i = 2,3, \cdots n-1 \end{cases} \tag{12}$$

Accumulating these curve segments by the following formula yields

$$\widehat{s_t} = \sum_i^{n-1} \widehat{s_i} \tag{13}$$

Stop until the size of $\widehat{s_i}$ is close to or equal to $A_{t-1}A_t$ to get the polar angle θ_t corresponding to position A_t

The third step is the derivation of recursive formulas

(1) Positioning of handle coordinates

In the polar coordinate system, the position of the handle is reflected by the polar diameter and the polar angle, however, in this problem, θ is the angle between the line of the handle and the origin of the coordinates and the positive direction of the polar axis (slanting downward in the initial stage), so that $\theta_j(j = 1,2, \cdots 223)$is the angle between the line of the jth handle and the pole and the polar axis, as shown in the figure below, and it is known that OA_0 the polar angle θ_0, OA_1 the polar angle θ_1, OA_2 the polar angle θ_2 as well as α_1 and α_2 The polar angle θ_j expression can be obtained as follows:

$$\theta_j = \theta_0 - \sum_{i=1}^{j} \alpha_i \tag{14}$$

If the polar angles of the individual handles are known, substitute into the solenoid equation

$$r = a + b\theta \tag{15}$$

The polar diameter of each handle r_j can be obtained , and if in the plane right-angle coordinate system, the coordinates should be converted, the formula is as follows:

$$\begin{cases} x_j = -r_j cos\theta_j \\ y_j = -r_j sin\theta_j \end{cases} \tag{16}$$

At this point, each handle can be positioned in a planar right-angled coordinate system.

(2) Quantification of Handle Speed

By the conditions of the topic, now know the initial position of the dragon head, pitch and the traveling speed of the dragon dance team, from the starting point of the thread of the screw as the origin of the coordinates, can be obtained

$$a = 0 \tag{17}$$

Since the dragon dance team is in solenoidal motion, with the dragon head as the object of study, the linear velocity satisfies the following relationship:

$$v = \omega r \tag{18}$$

where t is the polar diameter, ω is the angular velocity, and the angular velocity defining equation

$$\omega = \frac{d\theta}{dt} \tag{19}$$

Also, by the solenoidal equation

$$r = b\theta \tag{20}$$

From the curve segment of the spiral in the polar coordinate system calculated by the formula

$$l = \int_a^b \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2} d\theta \tag{21}$$

This variable upper limit can be integrated out, i.e., the length of the curved segment on the solenoid is related to the angle by the equation

$$s(\theta) = \frac{b}{2}\left(\theta\sqrt{1+\theta^2} + ln\left(\theta + \sqrt{1+\theta^2}\right)\right) \tag{22}$$

To get the linear velocity, just take the derivative of the length with respect to the time

$$v = b\sqrt{1+\theta^2}\frac{d\theta}{dt} \tag{23}$$

In the actual calculation, the time discretization is considered to be processed, so the difference method is used to express the recursive relationship equation between speed and angle, and the Euler two-step format expression form is applied here as follows[4]:

$$v_i = b\sqrt{1+\theta_i^2}\frac{\theta_{i+1}-\theta_{i-1}}{2\Delta t}, i = 1,2,\cdots 222 \tag{24}$$

where v_i is the velocity at time point i, θ_i is the angle at time point i, and Δt is the time step[5].

4. Results

According to the above related theoretical and experimental part of the meticulous study of Archimedes solenoid and finite difference model [6], the position of the marked bench derived from MATLAB programming is shown in Table 1 and the speed of the marked bench is shown in Table 2, and based on the data in the table, the result of data visualization is shown in Figure 3. [7]:

Table.1. Positioning results for marking benches

	0s	600s	1200s	1800s	2400s	3000s
Dragon’s head x(m)	8.8000	5.7976	-4.0873	-2.9615	2.5921	4.4214
Dragon’s head y(m)	0.0000	-5.7726	-6.3028	6.0958	-5.3579	2.3182
Dragon's body 1 x(m)	8.3701	7.44897	-1.4631	-5.2224	4.8096	2.4881
Dragon's body 1 y(m)	2.8074	-3.4568	-7.4022	4.3768	-3.5772	4.3957
Dragon's body 51 x(m)	-9.3695	-8.4070	-4.8583	3.7476	5.3728	-6.1212
Dragon's body 51 y(m)	2.1162	3.32856	6.9029	6.8334	-4.6228	1.5064
Dragon's body 101 x(m)	1.4726	4.29237	3.9258	0.0991	-6.2982	-4.9234
Dragon's body 101 y(m)	-10.2190	-8.8124	-8.3766	-8.6629	-4.9871	5.4597
Dragon's body 151 x(m)	10.9929	10.7494	4.6555	3.5098	5.4097	8.0915
Dragon's body 151 y(m)	-0.3178	-0.6426	8.8447	8.7785	7.0422	1.6997
Dragon's body 201 x(m)	6.9827	8.45249	-9.7554	-10.1320	-9.1730	-8.8972
Dragon's body 201 y(m)	9.3019	7.96811	4.3667	-1.0283	-3.0052	-1.8777
Dragon's tail x(m)	-7.8377	-9.2177	10.6935	9.4583	6.5986	5.4647
Dragon's tail y(m)	-8.9538	-7.5096	-2.4916	4.5347	7.4795	7.6893

Table.2. Velocity results of the marking benches

	0 s	60 s	120 s	180 s	240 s	300 s
Mixer (m/s)	1	1	1	1	1	1
Dragon's body 1(m/s)	0.99995	0.99971	0.99974	0.99993	0.99999	0.99982
Dragon's body 51(m/s)	0.99730	0.98497	0.98664	0.99649	0.99987	0.99085
Dragon's body 101(m/s)	0.99466	0.97024	0.97354	0.99304	0.99974	0.98188
Dragon's body 151(m/s)	0.99202	0.95551	0.96044	0.98959	0.99961	0.97291
Dragon's body 201(m/s)	0.98938	0.94078	0.94735	0.98615	0.99949	0.96395
Dragon's tail (m/s)	0.98821	0.93430	0.94158	0.98463	0.99943	0.96000

The next step is to analyze the speed data.

From the surface observation of the data change trend and size relationship can be summarized as three distribution laws [8].

(1) At any moment from the dragon's head to the back point of the recursive back point velocity is always less than 1.

(2) The velocity from the dragon's head to the dragon's tail at any moment shows a roughly monotonous decreasing trend [9].

(3) As the moment goes backward, the extreme difference between the dragon's head and the dragon's tail will gradually become bigger and bigger [10].

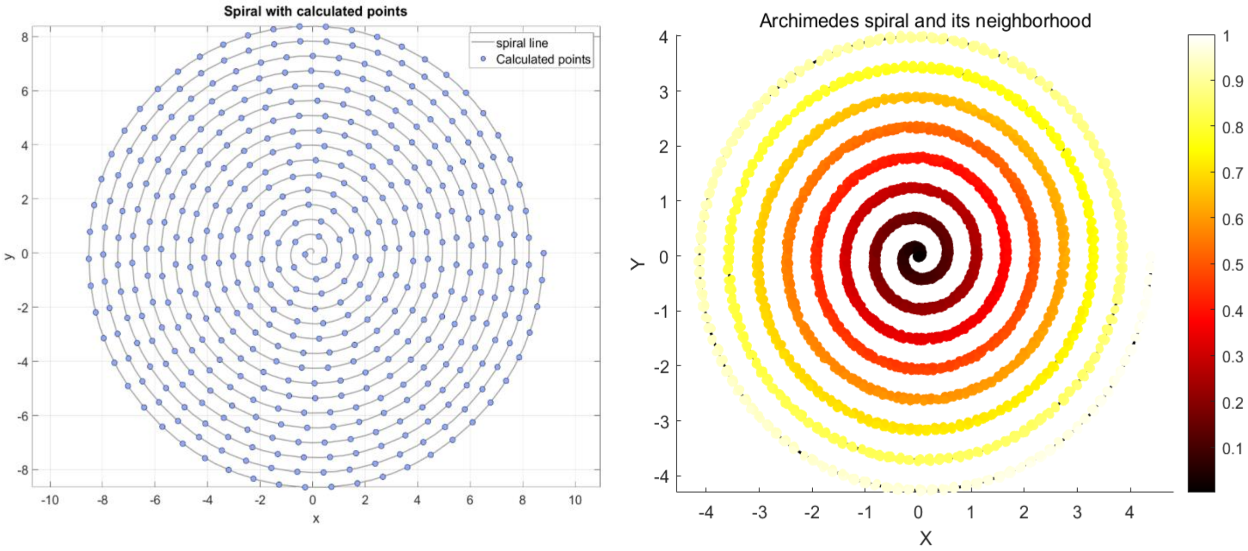

Figure 3. Heat map of dragon's head points and speed magnitude

Next, from the physical level to analyze the distribution of the phenomenon of the law of the reason: according to the definition of physics associated with the speed, combined with any two handles here between the fixed distance between the mass (here can be understood as a rigid body) remains unchanged, which leads to each of the two neighboring points have a common associated with the speed, but due to the connection between the line and the two tangential speed of the two points there is a certain angle and in general, the two are not equal, which makes the two points have different velocities. At the same time, it is easy to prove that in the Archimedean equidistant helix when such a situation occurs, there is always the front point angle is greater than the back point angle, which leads to the overall speed from the dragon's head to the dragon's tail shows a decreasing trend.

In summary, this paper rigorously considers the rigid-body connectivity problem between the connection points when carrying out the layout study, and analyses the critical velocity aspect of physics, handles the connection problem and obtains the position and velocity information at any moment, which well makes up for the lack of consideration of this aspect in the previous studies.

5. Conclusions

The above experiment mainly realizes the numerical simulation of the dragon dance team's movement process, with the help of finite difference, Archimedean spiral and other mathematical knowledge, it realizes the calculation of the team's position and speed at any moment, and simulates the team's movement pattern in the process of coiling into a spiral line through visualization technology. The law that can be found is: when the team travels to the inner circle, the speed of the whole team shows a trend of gradually increasing from the dragon's head to the dragon's tail and the difference of the speed of the dragon's head and the dragon's tail will become bigger with the increase of time, which is due to the connection of the rigid body of the whole team, which is a regular manifestation of the correlation of speed, and this brings certain inspirational significance to the team's mobile choreography.

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