Research On Optimal Water Level of Great Lakes Based on Dynamic Optimization Model and PID Control Algorithm

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Abstract. This study aims to determine the optimal water levels of the Great Lakes by analyzing historical data from 2012 to 2022, including water levels of the lakes and flow rates of the rivers connecting them, to identify underlying trends. A multi-objective optimization model is developed, which holistically accounts for the welfare of local ecological, economic, and social stakeholders. By applying a genetic algorithm, the model calculates the optimal water levels for different times of the year. Furthermore, a dynamic relationship among the Great Lakes is modeled using differential equations, and a PID control algorithm is implemented to manage the optimal water levels under natural conditions. Compared with traditional algorithm, this approach not only enhances the precision of water level predictions but also introduces an adaptive control system that responds to real-time data. The integration of these methods allows for a more nuanced management of water levels, catering to the fluctuating demands of ecological, economic, and social stakeholders. A subsequent sensitivity analysis reveals that the proposed control algorithm demonstrates a high sensitivity to varying natural conditions and yields water levels more favorable than the actual recorded levels from the perspective of most stakeholders.

Keywords: Probability-Utility Mapping Model, Dynamic Optimization, PID Controller.

1. Introduction

The Great Lakes, situated in the central and eastern regions of North America, comprise a network of large freshwater lakes interconnected by various waterways. These lakes provide extensive ecological, economic, and social benefits that go far beyond supplying drinking water to nearly 4 million people. They are vital for numerous sectors, including fisheries, recreation, power generation, shipping, and aquatic habitats. For stakeholders, the water levels of the Great Lakes are of paramount importance, influencing a wide range of interests. Consequently, balancing the diverse needs of these stakeholders is crucial in determining the optimal water levels for the lakes. Nonetheless, due to the inherently unpredictable nature of environmental factors, managing and maintaining these optimal water levels in a constantly changing context remains a significant focus of contemporary research worldwide.

In the field of water level control, many articles have proposed various methods. Shang S (2018) proposes a simplified method for determining minimum ecological water levels in lakes and wetlands based on the lake surface area method [1]. Manli H. (2008) uses genetic algorithm to optimize the allocation of water resources [2]. The thesis of Jiangsi SUN (2021) integrates a dynamics model with a dynamic water quality model to analyze Luoyang city's water pollution control, offering a novel approach for urban water management and policy strategy development [3]. And the cases cover areas in many countries all over the world, which can be meaningful references.

However, previous literature has been limited by a focus on single objectives, neglecting the complex interplay of stakeholder needs, and has relied on static constraints that do not account for environmental variability, which can hinder the model's applicability and accuracy. Additionally, the absence of real-time control mechanisms in traditional models has resulted in less efficient and effective water management strategies. This paper's model enhances upon previous research by introducing multi-objective optimization, which balances ecological, economic, and social needs, providing a more holistic approach to water level management. It also incorporates dynamic constraint conditions to adapt to the uncertainties of environmental factors, thereby increasing the model's adaptability and robustness. Furthermore, the implementation of a PID controller enables real-time feedback control, allowing for swift responses to hydrological changes and enhancing the precision of water management.

2. Model Setting of Stakeholders and Water Controlling

2.1. Probability-Utility Mapping Model

In this section, this study establishs the probability-utility mapping model to explore how to map probability functions and probability density functions to utility functions and marginal utility functions to construct a utility function that reflects the needs of different stakeholders. Through this method, this research more accurately simulates and optimizes the impact of Great Lakes water levels on various stakeholders.

Given the parallelism observed between probability functions and probability density functions, mirroring the connection found in utility functions and marginal utility functions, coupled with the advantageous mathematical properties exhibited by prevalent probability distribution functions, it becomes plausible to consider probability functions and probability density functions as surrogates for utility functions and marginal utility functions. Consequently, the construction of the utility function can be delineated as follows：

$$\max \quad u(Q_1, Q_2, Q_3) = \sum_{i=1}^{3} (Q_i - e^{\alpha_i Q_i})\, y_i(Q_i - Q_{i0}) \tag{1}$$

$$y_i(x) = \begin{cases} 0 & (x < 0) \\ 1 & (x \geq 0) \end{cases} \tag{2}$$

Where Q_1 means the origin water volume, which will be constrained by the minimum water level, while Q_2 and Q_3 means water usage of economic goals and social goals separately. $e^{\alpha_i Q_i}$ is a penalty for the negative effects of high-water levels on the utility of different stakeholders.

The relationship between the original water level, minimum level of water transport and the minimum level of ecological requirement is as follows:

$$H_0 = \max\{H_1, H_4\} \tag{3}$$

Where H_0 represents the original water level, H_1 represents the minimum level of water transport, and H_4 represents the minimum level of ecological requirement.

From the transportation perspective, if the water level is too shallow below the limit, ships transporting on the Great Lakes are in danger of stranding. The relevant inequality constraint is:

$$H_1 \geq c_1 \tag{4}$$

Where c_1 represents the lowest limited water level for the transportation purpose.

From the ecological requirement, water sources play an important role in ecological conservation. And the constraint is as follows:

$$H_4 \geq c_4 \tag{5}$$

Where c_4 represents the lowest limited water level for the ecological purpose.

And then, we need to calculate the minimum value of ecological demand water level c_4, The calculation procedure includes two stages: finding the target point in the S-V curve with a specified slope and finding the corresponding lake level from the H-S curve. Inspired by reference, at c_4, a relatively larger lake surface area can be reached at a relatively less water storage:

$$\frac{dS}{dV}\Big|_{V=V_e} = \frac{S_{max}}{V_{max}} \tag{6}$$

Where S_{max} represents the maximum slope, and V_{max} denotes the maximum water volume. The relationship of these three lake morphology indexes can be described by Figure 1：

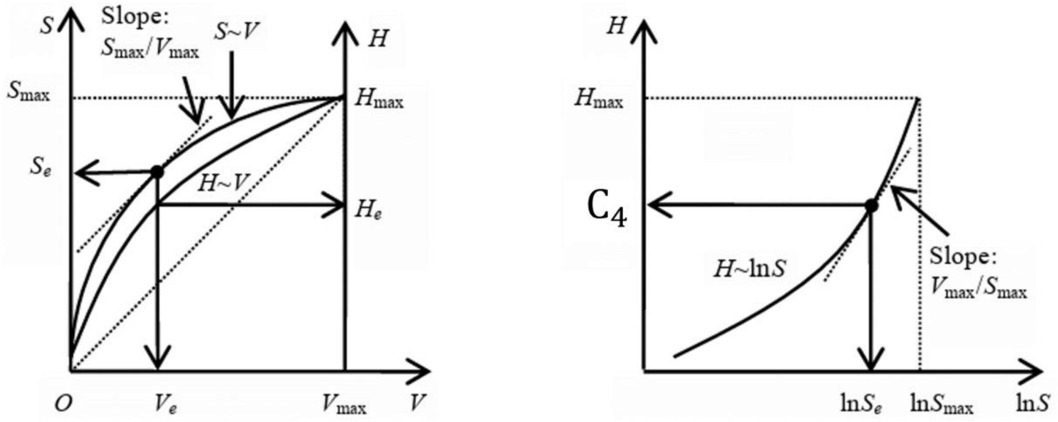

Figure 1: Illustration of The Wet Perimeter Method

For the measurement of economic benefits, we take the power generation of hydro-power generation companies into generation. Then the lower limit of power generation equals to the power consumption of society relying on hydropower (according to statistics, the proportion is about 6%) [4], which constitutes our constraint on economic goals:

$$\begin{cases} \Delta E_{total} = \Delta E_{ele} \cdot \varphi \\ \Delta E_{total} = \Delta E_G + \Delta E_P + \Delta E_T \\ \Delta E_G = 9810 \cdot \Delta H \cdot \Delta V \\ \Delta E_T = 500 \cdot (\rho_1 V_1^2 - \rho_2 V_2^2) \\ \Delta E_p = 9810 \cdot \dfrac{(p_1 - p_2)\Delta V}{\gamma} \\ \Delta V = Q_2 \end{cases} \tag{7}$$

Where ΔE_G represents gravitational potential energy, ΔE_T represents kinetic energy, ΔE_P is pressure potential energy. For other parameters, ΔH is the water conservancy height, $(p_1 - p_2)$is the pressure difference between the water of upper pool and lower pool; γ means water specific gravity; v1, v2 means the water flow velocity in the water of upper pool and lower pool; Q means the water flow used for power generation; ΔV means the volume of water that generates electrical energy, which is the same as Q_2; φ is the conversion efficiency.

According to Bernoulli conservation of energy [5], the main sources of electrical energy include the following three parts: gravitational potential energy, kinetic energy, and pressure potential energy. We do not consider the energy losses in hydro-power conversion. Bernoulli's theory indicates that under ideal conditions, at any cross-section of the same flow tube, the sum of ΔE_G, ΔE_T and ΔE_T is a constant.

By combining the above relations, we can obtain the equation:

$$Q_2 = \frac{E_{ele}}{2648.7 \times \Delta H \times \Delta t} \tag{8}$$

For the measurement of social goals, we primarily consider the factors of society water consumption, for the Great Lakes provide domestic water for most residents in the lakeside areas. Obviously, the water volume for water consumption proceeds the basic need. And for different months, the average water consumption for the month selected from previous years is regarded as the basic need $c_3 \cdot x_1$. Since residential use accounts for most of the water use in the Great Lakes Basin, we no longer consider other water needs such as industrial water in this part. The constraint for society goals is:

$$Q_3 \geq c_3 x_1 \tag{9}$$

It is natural to have an upper bound to avoid excessive water volume as well. If the water level is too high, there are obviously potential hazards to some stakeholders and the ecological environment. Therefore, when we control the optimal water level, we should consider the upper limit control of safety factors firstly. So, the water level needs to meet the following requirements:

$$\frac{H_{max} + \overline{H}}{2} \geq H \tag{10}$$

Where H_{max} denotes the maximum water level, and $\overline{H}$ denotes the average water level in the history.

Besides, there are also some stakeholders who don’t desire a higher water level, such as the property owners along the river and recreational boaters. If the water level rises too high, they will be affected by natural factors such as floods. So, the water level needs to meet the following requirements:

$$H_{max} - \epsilon \geq H \tag{11}$$

Where ϵ is a relatively small value according to the need of stakeholders who don’t desire a higher water level.

Considering the above two equations, we can obtain:

$$\min\left\{H_{max} - \epsilon, \frac{H_{max} + \overline{H}}{2}\right\} \geq H \tag{12}$$

2.2. Lake Morphology and Hydrodynamics Model

This section introduces how to describe and calculate the hydrodynamics of lakes through lake morphological indexes. We use the Wet Perimeter Method to determine the morphological characteristics of lakes and, combined with hydrological principles, establish a dynamic model that can simulate changes in lake water levels.

The change in the water level of each lake can be expressed as:

$$\frac{dh(i)}{dt} = \frac{Q_{in}(i) - Q_{out}(i)}{A_i} \tag{13}$$

Where $Q_{in}(i)$ is the water inflow of lake i, while $Q_{out}(i)$ is the outflow of the lake.
In the forthcoming steps, we will delineate the parameterization process of our lake chain model, focusing on both the internal water flow within the chain and external factors influencing the chain.

By analyzing the meteorological data for the Great Lakes region, we can improve the model by fine-tuning the evaporation [6] and precipitation parameters according to seasonal variations. This adjustment results in a more precise simulation of seasonal dynamics within the model. The inflow rate for the i^{th} lake in the chain is defined as follows:

$$Q_{in}(i) = Q'(i-1) + Q_{rain}(i) - Q_{evaporation}(i) \tag{14}$$

By using the Manning-Strickler formula [7], we combine channel geometry, bed friction and current velocity. So, we define the outflow rate for the i^{th} lake in the chain as:

$$Q_{out}(i) = S_i \times v_i = S_i \times \frac{R_i^{\frac{2}{3}} \times E_i^{\frac{1}{2}}}{n} \tag{15}$$

$$S_i \times R_i^{\frac{2}{3}} \times E_i^{\frac{1}{2}} = N \times w^{\frac{8}{3}} \times \left(g(h_i - h_{i+1}) - f_{i,i+1} \times d_{i,i+1}\right)^{\frac{1}{2}} \tag{16}$$

Where R_i denotes the length of the wetted perimeter length along the cross-section of the i^{th} flow and S_i represents the cross-sectional area of the river. The energy gradient drop E is mainly associated with the river slope, which is determined by utilizing the difference in mean heights of the two lakes. N is constant coefficient, $d_{i,i+1}$ signifies the distance between the i^{th} and the $(i+1)^{th}$

lake, and w stands for the river width. $f_{i,i+1}$ denotes the flow friction cost parameter between the two lakes, which is primarily influenced by the topography and geomorphology of the lake basin and the friction coefficient of the riverbed.

2.3. PID Feedback Control and Water Management Model

Next, we set up PID Feedback Control and Water Management Model to detail how to use a PID controller to achieve real-time feedback control of Great Lakes water levels [8-10]. By adjusting the parameters of the controller, we can ensure that water levels are maintained within the optimal range while meeting the needs of ecological, economic, and social objectives.

For controller, we use the PID controller, which is a classical feedback control algorithm aimed to achieve stable and accurate terms:

$$Q(t) = \left\{K_{p1} \times \left[h_{target} - h\right] + K_{i1} \times \int e(\tau)d\tau\right\} \times A \tag{17}$$

Where Q(t) represents the current control variable, which is the discharge of the compensation project. K_{p1}, and K_{i1}, are the proportional and integral gains, respectively. And h_{target} is the targeted water level determined by our model. A is the lake area, and $\int e(\tau)d\tau$ is the cumulative water level error of Lake Huron-Lake Michigan.

When $\int e(\tau)d\tau > I_0$(the control threshold), the controller switches to:

$$Q(t) = K_{p1} \times \left[h_{target} - h\right] \times A \tag{18}$$

3. Result

3.1. The result for Probability-Utility Mapping Model

In Model 1, our objective is to approach Pareto optimality to the greatest extent possible, subject to the imposed constraints. To achieve this, we employed a genetic algorithm to solve the multi-objective optimization model, aimed at determining the optimal monthly water levels for the Great Lakes. The outcomes of this analysis are presented in Table 1 and 2 below:

Table 1: The Optimal Water Levels of The Great Lakes from January to June

	Jan.	Feb.	Mar.	Apr.	May.	Jun.
Lake Erie	174.35	174.35	174.35	174.35	174.35	174.35
Lake Ontario	74.619	74.619	74.619	74.619	74.619	74.619
Lake Superior	183.459	183.459	183.459	183.459	183.459	183.459
Lake St. Clair	176.44	176.44	176.44	176.44	176.44	176.44
Lake Michigan and Lake Huron	176.23	176.23	176.23	176.23	176.23	176.23

Table 2: The Optimal Water Levels of The Great Lakes from July to December

	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Lake Erie	174.73	174.73	174.73	174.73	174.73	174.73
Lake Ontario	75.49	75.49	75.49	75.49	75.49	75.49
Lake Superior	183.101	183.101	183.101	183.101	183.101	183.101
Lake St. Clair	176.81	176.81	176.81	176.81	176.81	176.81
Lake Michigan and Lake Huron	176.79	176.79	176.79	176.79	176.79	176.79

3.2. Sensitive Analysis

In practice, statistics can often be inaccurate, and biases may exist in the data inputs to our models. These biases can potentially influence the outcomes. To evaluate the robustness of our model, we will conduct a sensitivity analysis in this section. The findings from the analysis indicate that our model demonstrates excellent stability.

We can set a disturbance rate (φ) to intentionally induce abnormal fluctuations in the outflow of the dam. At that time, the control algorithm will adjust the abnormal outflow. So, we can measure the robustness of our model by the duration it takes for the outflow to return to normal after the occurrence of abnormal fluctuations. Meanwhile, we can use the different values of φ to measure the robustness of our control algorithm in different intensity of perturbations.

$$O_i^{t*} = O_i^t(1+\varphi) \tag{19}$$

where O_i^t refers to the original outflow of lake i, while O_i^{t*} is the fluctuated outflow.

The time required for each lake to recover to the optimal water level by the established control algorithm under different fluctuation levels is shown in Figure 2:

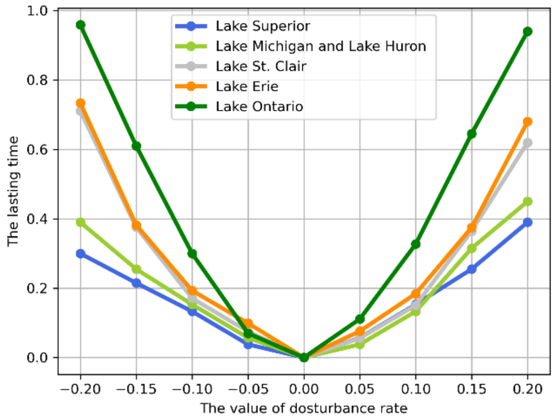

Figure 2: Sensitivity analysis of the control algorithms under daily fluctuations

The water level of each lake takes a short time to return to the normal water level under the interference of the abnormal input. This can prove that our control algorithm has strong robustness.

To measure the sensitivity of our algorithm to changes in environmental conditions, we can use different values of γ to simulate different levels of weather extremes. In the presence of extreme weather, we can use our algorithm to calculate the cumulative impact on subsequent lakes, as shown in Figure 3:

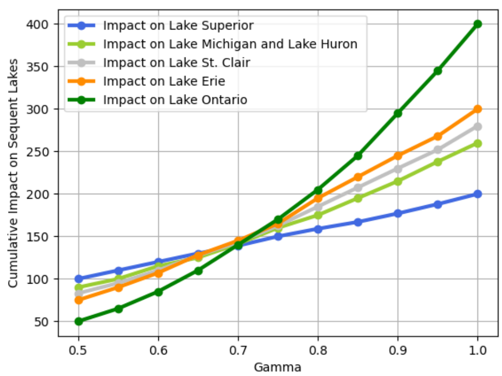

Figure 3: Sensitivity analysis of control algorithms under extreme weather conditions

Our control algorithms are designed to minimize inter-lake impacts during extreme weather events, demonstrating that the algorithm maintains a high degree of robustness when managing individual lakes under such conditions.

4. Conclusion

The optimal management of water resources is a pressing issue in contemporary society, with increasing concerns surrounding sustainability and resource allocation. The Great Lakes, one of the largest freshwater reserves globally, are central to this discussion. In conclusion, this study successfully employs a multi-objective optimization model and a PID control algorithm to manage the Great Lakes' water levels, striking a balance among ecological, economic, and social interests. The model's sensitivity analysis confirms its robustness, providing a reliable basis for future water resource management strategies in the face of environmental uncertainties.

The study begins by analyzing the utility of various stakeholders. A probability-utility mapping model is constructed to maximize the utility for all parties, accommodating their different needs and priorities. These include water usage for agriculture, municipal drinking water, industrial processes, and ecological preservation.

The interconnected dynamics of the Great Lakes system are modeled using differential equations. These equations capture the interactions between lakes, considering factors such as water inflow, outflow, evaporation, and precipitation. This framework allows for a comprehensive understanding of how fluctuations in one lake's water levels can influence others, providing insight into the system's overall behavior.

To regulate water levels, PID control algorithm is applied. The PID controller is selected for its robustness and effectiveness in responding to both routine fluctuations and extreme weather events. This control method enables precise adjustments, ensuring optimal water levels are maintained across the system, thereby promoting long-term sustainability and minimizing adverse effects on stakeholders.

Future research presents opportunities to develop policies aimed at sustaining these optimal water levels. Policymakers could craft regulations and frameworks that maximize stakeholder utility while preserving the ecological integrity of the Great Lakes. Additionally, further investigation is needed into the potential impacts of climate change on water levels, with a focus on refining control strategies to adapt to evolving environmental conditions. Ultimately, the goal is to establish a comprehensive management system that balances human needs with environmental stewardship, ensuring the Great Lakes remain a critical and sustainable water resource for future generations.

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