Research on Bench Dragon Motion and Collision Risk Based on Numerical Simulation

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Abstract. ‘The Bench Dragon’, a traditional folk activity in the Heluo area, is widely present in the annual grand activities of various provinces and cities in southern China. Its winding form follows the trajectory of the helix equation. Although the winding form seems simple, it requires precise control of the speed and position of each handle to achieve coordinated and beautiful complex movement. Herein, from the perspective of establishing a polar coordinate system and helix equation, this study analyzes the relationship between the position of each handle and the speed of the bench dragon and then analyzes the results in different models. At the same time, this study further examines the limit position of disc entry without collision by the ergodic method when the time is 405.757s. Finally, it was found that the model of the research system is still accurate by adding Gaussian noise which proves that this model can be used in practical applications and has the significance of popularization.

Keywords: Equidistant spiral equation, Differential equation, Traverse, Recurrence model.

1. Introduction

As a traditional folk dance form, 'The Bench Dragon' has a long history and profound cultural heritage in China. The performance of the dragon dance team is often used to show the joy and inheritance of the festival. In the process of dragon dance performance, along with the specific path into and turn around, and with the increase of urban population size, the complexity of dragon dance performance increases, accompanied by a certain degree of danger. With the changes of the times, modern science and technology have gradually penetrated various fields of culture and art. The quantitative analysis and research of traditional art forms by using modern methods such as mathematical modeling and simulation technology not only helps to deeply understand the dynamic characteristics behind these cultural phenomena but also provides a scientific basis for the innovation and promotion of art forms.

In the existing related research, quantitative research on the bench dragon movement is relatively scarce, and most of the work is limited to the interpretation of its artistic value and cultural significance. With the advancement of computing technology, this study first attempts to establish a polar coordinate system for the space entered by the bench dragon disk to construct an equidistant spiral equation; the relationship between the position of the front handle of the dragon and the time is solved by using the constant speed of the front handle of the dragon and the spiral equations. Secondly, using the fixed length between the bench handles and the known fixed speed and position of the front handle of the dragon, the differential equations are constructed to solve the relationship between the position of the front handle of the dragon body behind the dragon head and the position of the rear handle of the dragon tail with time. Then, using the method of numerical simulation to solve the differential equation, combined with the derivation and total differential, the recursive relationship between the bench front handle and the rear bench front handle is obtained, and the speed change of all handles in 0 ~ 300 s time point is solved. Finally, the central difference method is used to numerically test the velocity obtained by the above model, and the test results are within a reasonable range. Then the theoretical model of the bench in the spiral is established, and it is concluded that the dragon head has the greatest risk of collision in theory. Secondly, it is analyzed that the faucet bench tends to collide with the bench with a shorter straight distance when it collides.

2. Construction and Solution of the Bench Dragon Spiral Motion Model

The data of this study come from https://dxs.moe.gov.cn/zx/hd/sxjm/.

2.1. Construction of coordinate system

The coiling spiral of 'The Bench Dragon' is equidistant [1], moving in a clockwise direction. Establish a polar coordinate system with point O as the origin, the polar axis is the positive half axis of the x-axis, and there is an equation:

$$r = a + b\theta \tag{1}$$

Here, ‘ r ’ denotes the distance to the pole, denoted by the polar diameter; ‘ a ’ is the initial radius (the radius when $\theta = 0$), according to the center of the spiral is located at the origin of the coordinate system, so $a = 0$; ‘ b ’is the coefficient which related to the density of the spiral, and the relationship between ‘ b ’ and pitch ‘ P ’ is $b = p / 2\pi = 0.55 / 2\pi = 0.088m$: It is assumed that ‘ θ ’ is the angle between the counterclockwise rotation and the positive direction of the x-axis starting from point O. Since the travel direction is clockwise, the ‘ θ ’ is the angle between the polar diameter rotating from the polar axis in the counterclockwise direction and the polar axis, as shown in Figure 1.

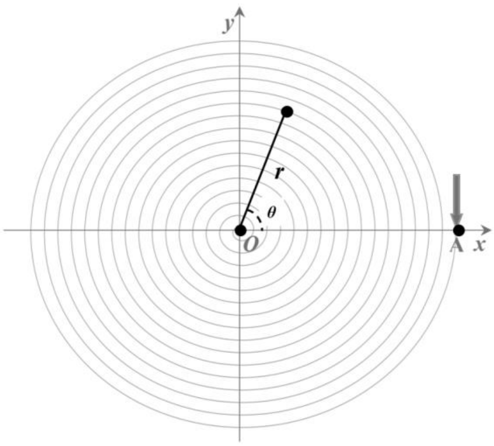

Figure 1. Schematic diagram of polar coordinate system construction

To express the position and speed of the handle more intuitively, we also establish the equation of the transformation between the polar coordinate system and the Cartesian rectangular coordinate system, such as Formula (2). Taking O as the origin of the Cartesian coordinate system, let:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \tag{2}$$

To express the change of the position of 'bench dragon' in the polar coordinate system, the differential on both sides of Eq. (1) can be expressed as:

$$\begin{cases} dx = \cos\theta dr - r\sin\theta d\theta \\ dy = \sin\theta dr + r\cos\theta d\theta \end{cases} \tag{3}$$

At this point, the above has completed the construction of polar coordinates and Cartesian coordinates and then entered the model establishment and solution board.

2.2. Solving the position of the front handle of the tap

During the marching process of 'The Bench Dragon', the front handle of the tap will enter along the tangent direction of point A in Figure 1, and the linear velocity (v) of the front handle of the front handle of the tap is constant to 1 m/s. According to the derivation of time t by Eq. (3), the differential equation of Eq. (4) is obtained:

$$\sqrt{(\frac{dx}{dt})^2+(\frac{dy}{dt})^2}=1 \tag{4}$$

After simplification, the formula (5) is obtained:

$$d^2x+d^2y=d^2t \tag{5}$$

By substituting formula (3) into formula (5), the differential equation of the relationship between angle θ, time t, and polar radius r is obtained:

$$d^2r+r^2d\theta=d^2t \tag{6}$$

Bring Eq. (2) into Eq. (6), and get:

$$b^2(1+\theta^2)d^2\theta=d^2t \tag{7}$$

Simplify to get:

$$(\frac{d\theta}{dt})^2=\frac{b^2}{1+\theta^2} \tag{8}$$

It can be seen from the title that the front handle of the tap is coiled from point A clockwise along the spiral path to point O, so θ decreases with time, that is $\frac{d\theta}{dt}<0$, and the equation is obtained:

$$\frac{d\theta}{dt}=-\sqrt{\frac{b^2}{1+\theta^2}} \tag{9}$$

After simplification, the equation is obtained:

$$\sqrt{1+\theta^2}d\theta=-bdt \tag{10}$$

Solving the integral on both sides can be obtained:

$$t=\frac{-b}{2}\cdot[\theta\cdot\sqrt{1+\theta^2}+ln(\theta+\sqrt{1+\theta^2})]+C,(C\in R) \tag{11}$$

When the front handle of the tap is located at point A, $t=0s$ $\theta=32\pi$ $C\approx 442.59$ is obtained. At the same time root. According to the formula (11), it is known that the function of time t θ concerning has an algebraic solution, and the function of time t θ concerning has no algebraic solution. Needle. For the function θ concerning time t, the numerical solution of differential equations is obtained by using Ode45 in MATLAB. It is a numerical method for solving the initial value problem of ordinary differential equations (ODE) in MATLAB [2, 3]. It is based on the variable step size Runge-Kutta method, specifically the Dormand-Prince method. Then according to Formula (1). The position of the front handle of $0\sim 300s$ the tap can be solved. By using formula (3), its position coordinates in the Cartesian coordinate system can be obtained.

2.3. The position of the front handle of the dragon's body and the rear handle of the dragon's tail is solved

In the Cartesian coordinate system, the position coordinate of a front handle in a bench is set to (x_i, y_i), where $i=1$, which means the front handle of the tap, to push back. The position coordinates of the front handle of the subsequent dragon body are (x_{i+1}, y_{i+1}), according to the length of the dragon head bench is 3.41 m, the length of the dragon body and the dragon tail bench is 2.2 m, and the distance between the front end of each bench and the center of the aperture is 0.275 m. The distance between the front handle of the dragon head and the front handle of the subsequent dragon body is $d_q = 2.86\ m$, and the distance between the front handle of the dragon body behind the dragon head and the front handle of the subsequent dragon body is $d_h = 1.65\ m$. It can be obtained that the position relationship between the front handle of the faucet and the front handle of the following dragon body is:

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = d_q \tag{12}$$

The position equation of the front handle of the dragon body behind the tap and the front handle of the following section is:

$$\sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2} = d_h, 2 \le i \le 223 \tag{13}$$

Substitute into formula (10) to obtain the position change equation of the front handles and the rear handles of the final faucet:

$$b^2\theta_{i+1}^2 - 2b^2\theta_i\theta_{i+1}\cos\theta_{i+1}\cos\theta_i - 2b^2\theta_{i+1}\sin\theta_i\sin\theta_{i+1} + b^2\theta_i^2 = d_a^2 \tag{14}$$

Where d_a is the distance between the two handles in the bench. When $i = 1$, $d_a = d_q$; when $2 \le i \le 223$, $d_a = d_h$. Among them, θ_i represents the polar coordinate angle position of each front handle and the back handle of the dragon. According to this formula, the iterative relationship between θ_i θ_{i+1} and can be obtained, and then the numerical solution of the relationship can be obtained by using the 'fsolve' function in MATLAB [4, 5]. Because the angle θ_1 of the front handle of the dragon has been solved in section 2.2, the numerical solution θ_2 can be iterated. By analogy, the θ_i of each handle of 0 ~ 300s at any time can be obtained. By using the formulas (1) and (2), the position coordinates of the front handle and the rear handle in the Cartesian coordinate system in 0 ~ 300 s can be obtained.

2.4. Solve the speed of each handle

From formula (11), it can be seen that there is a functional relationship between θ and time t, so the functional relationship can be set to $\theta(t)$. To solve the velocity relationship of each handle changing with time, the derivative of t both sides of the above equation is derived to obtain the equation [6, 7]:

$$\begin{cases} \dfrac{dx}{dt} = b\theta'(t)cos(\theta(t)) - b\theta(t)\theta'(t)sin(\theta(t)) \\ \dfrac{dy}{dt} = b\theta'(t)sin(\theta(t)) + b\theta(t)\theta'(t)cos(\theta(t)) \end{cases} \tag{15}$$

The velocity is along the direction of the spiral tangent, and the velocity equation is:

$$v = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} \tag{16}$$

According to the formula (9), for the front handle of the tap, there are the following relationships:

$$\theta'(t) = \frac{d\theta}{dt} = -\sqrt{\frac{b^2}{1 + \theta^2}} \tag{17}$$

Both sides of Eq. (14) are derived from t at the same time. After sorting out, the equation is obtained:

$$\frac{d\theta_{i+1}}{dt} = -\frac{\theta_i - \theta_{i+1}(\cos\theta_i\cos\theta_{i+1} + \sin\theta_i\sin\theta_{i+1}) - \theta_i\theta_{i+1}(\sin\theta_{i+1}\cos\theta_i - \cos\theta_{i+1}\sin\theta_{i+1})}{\theta_{i+1} - \theta_i(\cos\theta_i\cos\theta_{i+1} + \sin\theta_i\sin\theta_{i+1}) - \theta_i\theta_{i+1}(\sin\theta_{i+1}\cos\theta_{i+1} - \cos\theta_i\sin\theta_{i+1})} \cdot \frac{d\theta_i}{dt} \tag{18}$$

It can be seen from this that there is a recursive relationship between $\frac{d\theta_{i+1}}{dt}$ and $\frac{d\theta_i}{dt}$. According to the formula (9), the $\frac{d\theta_1}{dt}$ corresponding to the front handle of the lower head at any time can be obtained, and the $\frac{d\theta_i}{dt}$ of all the remaining handles at 0 ~ 300s can be derived, that is $\theta'(t)$. Combined with the $\theta(t)$ obtained in section 2.3, $\frac{dx}{dt}$ and $\frac{dy}{dt}$ can be obtained by substituting them into Formula (15), and then substituting them into Formula (16), the speed of all handles can be obtained.

2.5. Model checking and solution results

The central difference method is used to check the model. x_i and y_i represent the position coordinates of all handles. Take $\Delta t = 0.5s$, and the equation is as follows:

$$\begin{cases} v_x(t) \approx \dfrac{x_i(t+\Delta t) - x_i(t-\Delta t)}{2\Delta t} \\ v_y(t) \approx \dfrac{y_i(t+\Delta t) - y_i(t-\Delta t)}{2\Delta t} \\ v_z = \sqrt{(v_x(t))^2 + (v_y(t))^2} \end{cases} \tag{19}$$

Finally, the difference speed v_z is compared with the speed v of section 2.3, and the formula is as follows:

$$\Delta v = \{|v - v_z|\} \tag{20}$$

Taking the maximum Δv, it is found that $\Delta v < 0.001$, and the error obtained from the results is small and negligible. By analyzing the data $v\ v_z$, it can be observed that with the increase of time, the speed of each dragon's body shows a downward trend. At the same time, the farther the bench is, the smaller the speed is, and the $v\ v_z$ data in the attachment show the same trend. The rationality of the model is further verified. The velocity distribution under this trend is consistent with the geometric characteristics and velocity limits on the turning path.

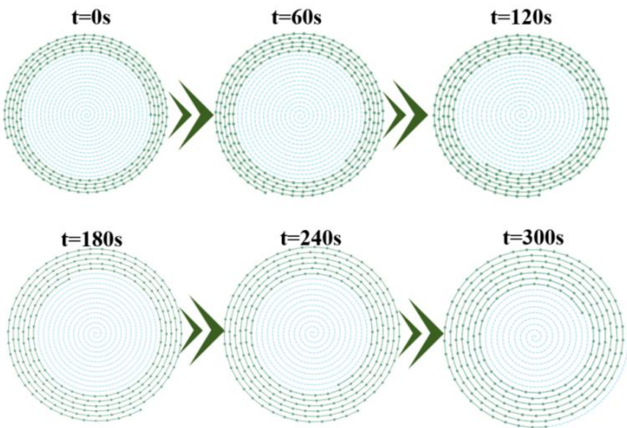

Figure 2. Chart of Insertion Process

The dynamic visualization analysis is carried out in the result of models, as shown in Figure 2. The spiral diagram drawn shows the specific insertion trajectory of the front handle of the dragon dance team's leading edge in the process of clockwise insertion. The diagram can intuitively observe the process and trajectory of the dragon dance team.

3. Collision Risk Modeling and Solution in the Bench Dragon's Spiral Movement

3.1. Premise hypothesis derivation process

In order to reduce the amount of calculation required by the model, this paper proposes two important theories and deduces them, which greatly simplifies the calculation of the subsequent collision risk model.

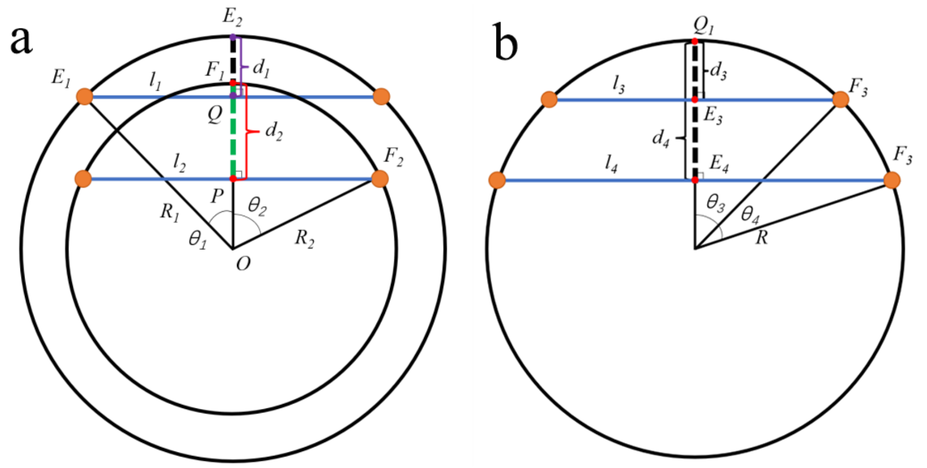

Figure 3. Schematic diagram of two model validation a. The first theory. b. The second theory

The first theory: assuming that the same $l_1\, l_2$ and are the distance between the centers of the two holes in the same bench, the position of the two handles of the same bench at a certain time is approximately located in the circle with the origin O as the center. As shown in Figure 3a, suppose $l_1 = l_2$, put it in a circle with different radii, and its two handles are positioned exactly on the same circle. Then there are:

$$d_1 = R_1 - \sqrt{R_1^2 - (\frac{l_1}{2})^2} \tag{21}$$

Continue to simplify can be obtained:

$$d_1 = \frac{(\frac{l_1}{2})^2}{R_1 + \sqrt{R_1^2 - (\frac{l_1}{2})^2}} \tag{22}$$

Similarly:

$$d_2 = \frac{(\frac{l_2}{2})^2}{R_2 + \sqrt{R_2^2 - (\frac{l_2}{2})^2}} \tag{23}$$

It is known that $R_1 > R_2$, $l_1 = l_2$ $d_2 > d_1$ can be obtained from the formula (22) and (23). It can be seen that when the distance between the two holes in different benches is the same, as the radius is larger, the distance between the center point of the line segment between the two holes and the origin O is farther, the space that can continue to be entered is larger, and the risk of collision is smaller. It shows that the smaller the distance between the handle and the origin O, the greater the risk of the first collision in the process of the 'The Bench Dragon'.

The second theory: assuming that the same l_1 l_2 and are the distance between the centers of the two holes in the same bench, the position of the two handles of the same bench at a certain time is approximately located in the circle with the origin O as the center. As shown in Figure 3b, suppose $l_4 > l_3$, put it in a circle with the same radius, and its two handles are exactly on the same circle. Then there are:

$$d_3 = R - \sqrt{R^2 - (\frac{l_3}{2})^2} \tag{24}$$

$$d_4 = R - \sqrt{R^2 - (\frac{l_4}{2})^2} \tag{25}$$

It is known that R the same $l_4 > l_3$ $d_4 > d_3$ is obtained from the formula (24) and (25). It can be seen that the radius is the same, the greater the distance between the two holes in the bench, the closer the distance between the center point of the line segment between the two holes to the origin O, the smaller the space that can continue to be entered, and the greater the risk of collision. It shows that the greater the distance between the two holes, the greater the possibility of the first collision in the process of 'The Bench Dragon' disc entry.

Based on the above two theories, the smaller the distance between the handle and the origin O, the greater the probability of collision; the greater the distance between the two holes, the greater the risk of the first collision. It is known that the distance between the two holes of the dragon head bench is larger than that of the dragon body and the dragon tail, and the position of the front handle of the dragon head is the smallest from the origin O, that is, the position of the front handle of the dragon head is the most prone to collision. It can be understood that if there is no collision at the tap, the dragon dance team can continue to enter, and once the collision occurs at the tap, the dragon dance team can no longer continue to enter.

3.2. Model to test whether the bench dragon collides

Point A of the front handle of the tap on the 16th circle is entered along the tangent steering wheel, and the position of the tap is checked one by one by using the traversal method [8, 9] until the collision occurs, that is, it stops. Using the position model formula in section 2.2-2.3, it is assumed that the tap only changes 0.1 radian each time until it collides with a certain bench, and stops traversing after it cannot continue to be turned in.

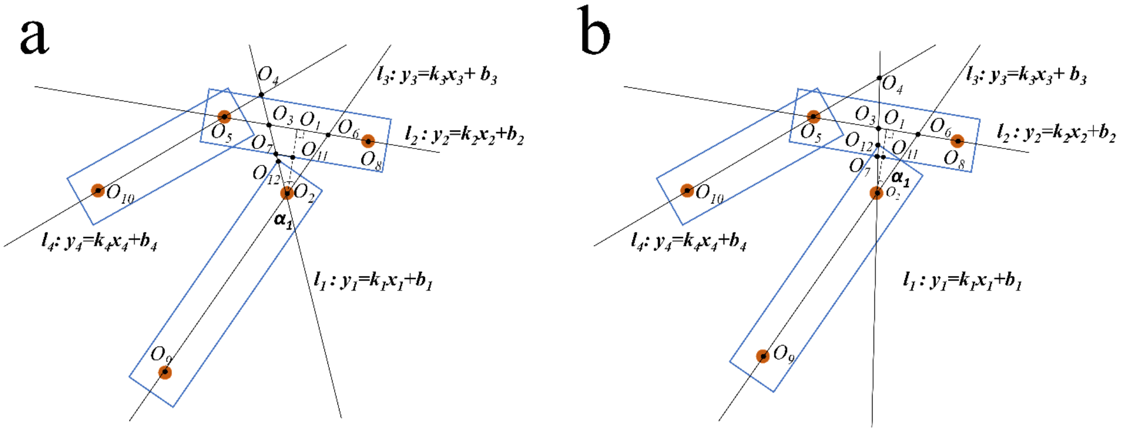

Figure 4. Schematic diagram of two cases: a. No collision. b. Collision

To simplify the operation, the process of collision is visualized as shown in Figure 4, Figure 4a is the model sketch before the collision of the tap, and Figure 4b is the sketch when the collision of the tap occurs. From sections 2.2-2.3, it can be known that the position of each handle on the spiral changes with time, that is, the coordinates of point O_5, point O_{10}, point O_8, point O_2, and point O_9 are known. A straight line is determined between the two points and a straight line equation can be obtained: $y_4 = k_4 x + b_4$ $y_2 = k_2 x + b_2$, $y_3 = k_3 x + b_3$.

Determine the dangerous distance of collision [10]: First of all, to determine which bench between the leader and the adjacent two benches may collide first, the center of the bench handle is regarded as a point, and the point O_{12} and the point O_2 are used as a straight line. At the intersection points O_3 and O_4 of the straight lines y_4 and y_2, the position of the point is calculated by MATLAB, and then the distance between the point O_2 and the intersection point of the front handle of the leader is calculated according to the distance formula between the two points. The shorter the bench is, the more likely it is to collide and the more dangerous the position is.

Determine the collision conditions: After determining the bench that is more likely to collide, then this study establishes the conditions for judging the occurrence of collisions. The point O_2 intersects with the vertical line of the straight line y_2 at point O_1, and the distance formula between the point and the straight line is used:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \tag{26}$$

The length of $|O_1O_2|$ is obtained, and the point O_3 is determined by intersecting two straight lines with one point. The length between $|O_2O_3|$ is obtained by using the distance formula between two points. If the bench width is 30 cm, then $|O_1O_{11}|$ is 15 cm, then $|O_2O_{11}|$ is $|O_1O_2| - |O_1O_1|$. It is easy to know $O_5O_8 // O_7O_{11}$, according to the similar triangle theorem:

$$\Delta O_3O_1O_2 \sim \Delta O_7O_{11}O_2 \tag{27}$$

$$\frac{|O_3O_{12}|}{|O_2O_7|} = \frac{|O_1O_2|}{|O_2O_{11}|} \tag{28}$$

According to the above formula, $|O_2O_7|$ can be obtained. It can be known from the figure that if the collision happens, the point O_7 coincides with O_{12}. According to the analysis of the top view of the faucet bench given by the title, it is known that:

$$|O_2O_{12}| = \sqrt{(27.5)^2 + (15)^2}\, m \approx 31.324910 m \tag{29}$$

As shown in Figure 4a and Figure 4b, if $|O_2O_7| > |O_2O_{12}|$, it is safe, no collision occurs, and the dragon dance team can continue to move forward; if $|O_2O_7| < |O_2O_{12}|$, the collision has occurred, and the dragon dance team cannot continue to enter.

Based on the above conditions, the traversal method can be used to solve the termination time of the turn-in and turn-out is 405.757 seconds. At the same time, the model in section 2.3-2.4 is used to solve the position and speed of the dragon dance team at this time.

3.3. Test and promotion of the model

The horizontal and vertical coordinates in the rectangular coordinate system of the front handle of the faucet are added to obey the normal distribution. The mean value is 0, and the standard deviation is half of the aperture, that is, 0.0275 Gaussian noise [11-13]. Finally, the time is 396.0680 s, which further verifies that the model is less affected by the error and still has rationality in practical applications. It also further demonstrates the generalization and practical guiding significance of the model.

4. Conclusion

In this study, the polar coordinate system is established by establishing the space of the bench dragon disk, and the equidistant spiral equation is constructed. The relationship between the position of the front handle of the dragon body and the change of time is solved by using the constant speed of the front handle of the dragon head and the spiral equations. Secondly, using the fixed length between the bench handles and the fixed speed and position of the known dragon front handle, the differential equation group is constructed to solve the relationship between the position of the dragon front handle and the dragon tail rear handle with time. The results show that the speed of the bench dragon from the dragon head to the dragon tail is gradually decreasing; Secondly, using the method of numerical simulation to solve the differential equation, combined with derivation and total differential, the recursive relationship between the bench front handle and the rear bench front handle is obtained, and the speed change of all handles in 0 ~ 300s time point is solved. Finally, the central difference method is used to numerically test the velocity obtained by the above model, and the test results are within a reasonable range. Then the theoretical model of the bench in the spiral is established, and it is concluded that the dragon head has the greatest risk of collision in theory. Finally, the critical condition of the collision is calculated according to the geometric model of the collision with the bench. It is found that when the time is 405.757 s, it just meets the critical condition of no collision.

Genarally, this study provides a research idea and framework applied to the field of kinematics analysis. At the same time, the feasibility of the model is proved by using the central difference method and the method of adding Gaussian noise. At the same time, the model is not only suitable for 'The Bench Dragon' performance but also for other large-scale activities [14], such as flower car parade, collective dance performances, and so on. It is necessary to consider the cooperation between people speed control, and performance. These factors are particularly important during the performance.

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