Research on the Motion Problem of "Bench Dragon" Based on Computer Simulation

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Abstract. The bench dragon, composed of multiple connected benches, spirals in a manner where the dragon head leads and the body and tail follow in a helical pattern with equal intervals. This paper employs computer simulation technology to construct a mathematical model that simulates the dynamic behavior of the bench dragon. The model aims to accurately predict the position and velocity vectors of each bench unit during the spiraling motion, as well as potential collision phenomena that may occur during the spiraling process. Furthermore, the binary search algorithm is applied to conduct an in-depth analysis of the minimum helix pitch required for the bench dragon to successfully turn under different spiral spacing conditions. Through these studies, the spiral path of the bench dragon has been optimized, enhancing its aesthetic and dynamic efficiency. This motion collision model is not only of significant importance to the performance art of the bench dragon but also applicable to the analysis and optimization of dynamic systems in other fields.

Keywords: Computer Simulation, Dichotomy, L2 - Norm.

1. Introduction

The bench dragon, as an intangible cultural heritage, has profound historical significance and rich cultural value [1-2]. In this article, the bench dragon consists of 223 connected benches, including one dragon head, 221 dragon bodies, and one dragon tail. The board length of the dragon head is 341 cm, the board length of the dragon body and tail is 220cm, and the board width of all stools is 30cm. There are two holes on each bench, with a diameter of 5.5cm and a center distance of 27.5cm from the nearest board head. Adjacent benches are connected by handles. When coiling the dragon, the dragon head takes the lead, and the dragon body and tail spiral together, forming an equidistant spiral shape with a pitch of 55cm. The speed of the front handle of the dragon head remains constant at 1m/s. At the beginning, the dragon's head is located at the 16th turn of the spiral and rotates clockwise inward. This article explores the corresponding issues of bench dragon sports based on this background.

Previous research on equidistant spiral lines has mainly focused on the field of mechanics, with less attention paid to the field of motion [3-4]. Therefore, this article establishes the problem of spiral line motion in the field of kinematics. In previous studies, more research has been conducted from a mechanical perspective [5-9]. In the process of establishing the model for the bench dragon, it is difficult to obtain more mechanical data. Therefore, this article chooses to establish and analyze the model from the perspective of pure motion. In previous studies, speed was more of a variable [10]. In this article, the speed of the dragon head in the model is a constant, therefore, this model will be improved to establish a new motion model. The significance of this study lies in revealing the dynamic characteristics of the equal-spaced spiral motion of the bench dragon from a kinematic perspective, providing novel theoretical support and practical guidance for the modern scientific interpretation and inheritance of traditional cultural activities.

2. Establishment of the model

2.1. Bench dragon motion model based on computer simulation

Based on the relationship between the position, speed, and time of the dragon stool handle, establish a computer simulation based motion model for the coiled dragon. Due to the connection between the dragon head and the dragon body, the speed is correlated, and the travel speed of the front handle of the dragon head is known to always be. Next, build the model.

Step 1: Provide the formula for the position and speed of the dragon's head.

First, calculate the position and velocity of the front handle of the dragon's head. Since the velocity of the front handle of the dragon's head is fixed, we can use the formula for circular motion to determine the relationship between the change in angle and the time interval when the dragon's head is turning in. This angle represents the forward movement of the dragon's head, which means that as time passes, the dragon's head continuously turns in at a certain angle.

$$v = \omega \cdot \rho, \tag{1}$$

$$\theta_e = \omega \cdot \Delta t, \tag{2}$$

Where v is the velocity; ω is the angular velocity; ρ is the pole diameter, that is, the distance from the origin O to the position of the front handle of the dragon's head.

The initial pole angle of the front handle of the dragon's head is θ_0, and the θ_e will also change with the change of time, and the angle of the dragon's head is θ_1 after the change.

$$\theta_1 = \theta_0 - \theta_e = \theta_0 - \omega \cdot \Delta t, \tag{3}$$

Step 2: Recursive the body and tail of the dragon head by the coordinates and velocity of the dragon's head.

The bench dragon consists of 223 benches, of which the first section is the dragon head, the back 221 sections are the dragon body, and the last 1 section is the dragon tail. Suppose the pole angle of the bench in section i is θ_i and the polar diameter is ρ_i, where $i = 1,2,3\ldots\ldots223$. So for each bench, there is a relationship between the pole angle between it and the front bench as follows.

$$\theta_{i+1} = \theta_i + \Delta\theta_i, \tag{4}$$

In this analysis, the polar angle $\Delta\theta_i$ represents the angular difference between each dragon bench and the front one. Given the short traversal times, the arc lengths of the spirals traversed by the dragon bodies are minimal, allowing us to approximate these spirals as circular arcs for computational simplicity. The straight-line distance from the origin O to each bench is used in preference to the spiral length, as it is typically greater and more straightforward to calculate. Accordingly, the distance between the handles is employed in place of the spiral length, and the chord length is substituted for the arc length, as illustrated in Figure 1.

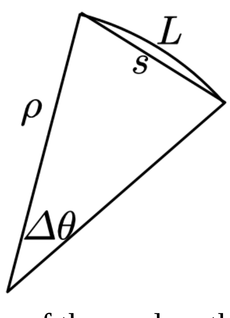


Figure 1. Schematic diagram of the arc length and chord length of a circle

The formula for the arc length of a circle is as follows.

$$L = \rho \cdot \Delta\theta, \tag{5}$$

According to the above rewording

$$s = \rho \cdot \Delta\theta, \tag{6}$$

The chord length s is the remaining length of each bench after the handles on both sides are removed, which is a known quantity, so that the polar angle difference $\Delta\theta_i$ between each bench and the previous bench can be solved. Combined with equation (4), the polar angle θ_i of each bench is calculated. And the relationship between the polar diameter and the polar angle of each bench can be solved.

$$\rho_i = a \cdot \theta_i, \tag{7}$$

Finally, the average velocity in a short period of time is approximately regarded as the instantaneous velocity of the handle at a certain moment, so through the relationship between velocity, displacement and time, the velocity formula of the handle in this paper can be calculated as follows.

$$v_i = \frac{\sqrt{\left(x_{i+1} - x_i\right)^2 + \left(y_{i+1} - y_i\right)^2}}{\Delta t}, \tag{8}$$

Since then, the movement model of the bench dragon has been completed.

2.2. Collision model based on L2-norm

In Model 1, the dragon dance team spirals inward, and it is necessary to calculate the termination time of the spiral to prevent collisions between benches. The discussion of collisions is based on the following two conditions:

(1) Collisions between adjacent benches: Since the front and back benches are connected by joint handles, collisions between adjacent benches will not occur.

(2) Collisions between benches of adjacent layers: As the dragon head spirals inward, the radius decreases and the angle increases, increasing the likelihood of collisions with the outer layer. The dragon head, being longer, is most prone to collisions with the benches of adjacent layers. If the dragon head does not collide, it is assumed that the body and tail of the dragon also do not collide.

Therefore, the collision of the dragon head is a sufficient condition for the entire dragon bench dragon to collide. By magnifying the analysis of the collision part, as shown in Figure 2, the collision conditions are determined. However, since the tilt of each dragon bench is unknown, the coordinates of the edge angle B of the dragon head cannot be solved directly, so the critical value of the collision is calculated based on the distance between AF.

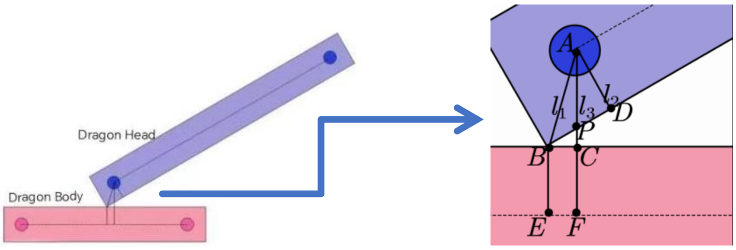


Figure 2. Enlarged schematic diagram of the part where the dragon's head collided

When the collision occurs, the edge angle of the dragon bench happens to touch the edge of the other bench, the length of AD l_2 is known, and the length of BD is also known, and the length of AB l_1 is calculated according to the Pythagorean theorem. The critical condition for collision in this problem is that the distance length from the front handle of the dragon's head to the edge of the other bench is equal to l_1. However, since the length l_1 of AB cannot be calculated, the length l_3 of AC is used to limit the conditions of exactly collision. As the actual collision process may be shown in Figure 3.

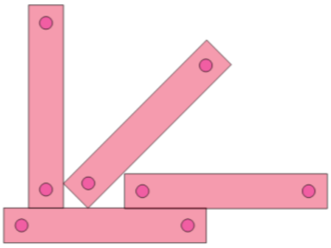


Figure 3. Schematic diagram of collision probability

According to the principle of the longest hypotenuse of a right triangle, the distance between the position of the front handle and the edge of the other bench can be solved, that is, the range of the length l_3 of AC.

$$l_2 \le l_3 \le l_1, \tag{9}$$

The straight line where EF is located is the straight line where the front and rear handles of a certain bench are located, and x, y is the horizontal and longitudinal coordinates of the bench handle in the Cartesian coordinate system. The bench dragon consists of 223 benches, of which the first section is the dragon head, the back 221 sections are the dragon body, and the last 1 section is the dragon tail. The coordinates of the front handle of the bench in section I are $(x_{2i-1},\ y_{2i-1})$; The rear handle coordinates are $(x_{2i},\ y_{2i})$. You can calculate the equation for a straight line between two points.

$$\frac{x - x_{2i-1}}{x_{2i} - x_{2i-1}} = \frac{y - y_{2i-1}}{y_{2i} - y_{2i-1}}, \tag{10}$$

It is then organized into the form $bx + cy + d = 0$. It is known that the center coordinates of the hole where the front handle of the dragon's head is located are (x_1, y_1). By limiting the length l_{AF} of AF, the solution for the exact collision of the dragon's head is realized. The length of AF can be calculated using the formula for the distance between points and lines.

$$l_{AF} = \frac{bx_1 + cy_1 + d}{\sqrt{b^2 + c^2}}, \tag{11}$$

Since then, the collision model has been established.

2.3. Minimum pitch model based on dichotomy

Dichotomy is an effective algorithm used to search for specific elements in a sorted array or optimize the objective function within a certain range. The basic idea is to gradually narrow down the search range by continuously dividing the search interval into two halves. By using the binary method, every time a smaller and satisfactory p value is calculated, the original p value is updated until there is no significant change in the p value.

In this model, the pitch size is adjusted and the initial radius is modified according to design requirements starting from the sixteenth turn. In order to ensure the accuracy of the model, strict range limitations were imposed on the adjusted pitch. The turning space is defined as a circular area with a diameter of 9 meters centered on the center of a spiral and a radius of 4.5 meters. The starting point for model construction is point A, where the dragon's head is located on the 16th turn of the spiral. This point must be outside the turning space to ensure that the dragon's head can smoothly enter the boundary of the turning space along the spiral line. Let the length of OA be l_{OA}.

$$l_{OA} = 16 \cdot p > 4.5m, \tag{12}$$

Thus, the minimum value of the pitch can be solved.

In the process of building the model, the solution results of the collision model based on L2-norm are analyzed to determine under what pitch conditions the dragon head can collide with a radius less than 4.5 meters, ensuring that the bench dragon can smoothly enter the turning space. The objective of this study is to identify the minimum pitch value that allows the bench dragon to enter the turning space and set it as the upper limit of the given pitch to ensure the practicality and safety of the model.

In the construction of the third model, the pitch is defined as a variable. Following the logic of the second model, we adopt the dichotomy strategy to develop a minimum pitch model.

According to the second model, the tap will stop moving in case of collision. So it is necessary to solve the coordinates of each handle and determine the linear equation between two handles on a stool. Then calculate the distance from the point to the line. The key requirement of this model is to ensure that the dragon's head does not collide outside the turning space, that is, when a collision occurs, the polar diameter ρ of the dragon's head must be smaller than the radius $r = 4.5\mathrm{m}$. According to the previous formula, the radius of the dragon's head changes as follows.

$$\frac{2\pi}{p} \cdot \rho_1 = \theta_0 - \frac{v}{\rho_1} \cdot \Delta t, \tag{13}$$

When a collision occurs, the equation for the straight line between the two handles of the stool is changed to the general formula.

$$(y_{2i} - y_{2i-1})x - (x_{2i} - x_{2i-1})y - (y_{2i} - y_{2i-1})x_{2i-1} + (x_{2i} - x_{2i-1})y_{2i-1} = 0, \tag{14}$$

The formula for the distance between the straight line between the front handle of the dragon's head and the two handles of the collided stool.

$$l_{AF} = \frac{bx_1 + cy_1 + d}{\sqrt{b^2 + c^2}}, \tag{15}$$

Under the condition that the distance satisfies the collision condition, that is, the polar diameter $\rho < 4.5\mathrm{m}$ of the dragon head, according to the binary method, p within the range of $0.28125\mathrm{m} < p < 0.55\mathrm{m}$ is solved to make the pitch p as small as possible.

3. Results

In this paper, because there are many benches that make up the bench dragon, it is inconvenient to display all the calculation results, in this paper, only the speed of the front handle of the dragon head, the front handle of the dragon head and the front handle of the dragon tail and the rear handle of the dragon tail behind the dragon head are used to show the speed of the dragon head front handle, 60s, 120s, 180s, 240s, and 300s for Model 1.

3.1. Results of the Bench Dragon Movement Model

Step 1: Establishment of coordinate systems

A polar coordinate system is established with point O as the pole and the positive direction of the x-axis as the polar axis. This is shown in Figure 4.

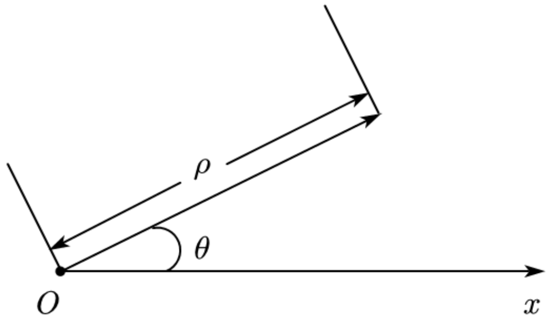


Figure 4. Schematic diagram of the polar coordinate system

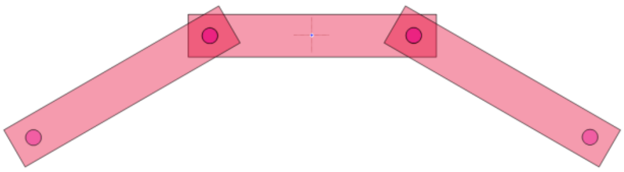
In the polar coordinate system, consider a point $P(\rho,\theta)$, where ρ is the radial distance and θ is the polar angle. The path of the dragon bench's movement is a spiral, indicating a definite relationship between ρ and θ.

$$\rho = a\cdot\theta, \tag{16}$$

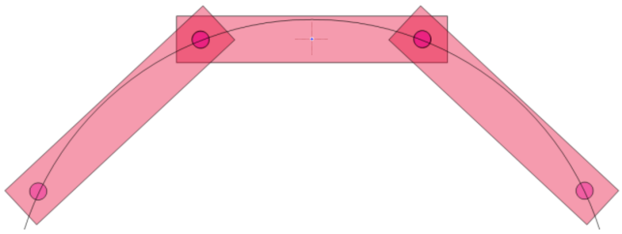
This article considers an equidistant spiral with a pitch p of 55 cm. In this case, there is a definite relationship between the constant a and p.

$$p = 2\pi\cdot a, \tag{17}$$

The dragon benches are connected by handles, each of which is located on the spiral line, as partially illustrated in Figure 5.



(a) Schematic diagram of the part connected by the handle of the dragon bench



(b) A partial schematic diagram of the dragon bench movement

Figure 5. Partial dragon bench

By utilizing the transformation between the Cartesian and polar coordinate systems, establish the relationship between the x and y coordinates of the dragon bench handles in the Cartesian system and their radial distance and polar angle in the polar system.

$$\begin{cases} x = \rho\cdot\cos\theta \\ y = \rho\cdot\sin\theta \end{cases}, \tag{18}$$

Step 2: Calculate the relevant values

Let s_h be the length of the dragon head bench with the handles on both sides removed, and s_b be the length of the dragon body and tail bench with the handles on both sides removed. $s_h = 341 - 27.5\times 2 = 286\text{cm}$, $s_b = 220 - 27.5\times 2 = 165\text{cm}$.The pitch is a fixed value $p = 55\text{cm}$. The initial angle of thedragon's head is the initial angle of the 16th turn of the spiral, so the result is $\theta_0 = 16\times 2\pi = 32\pi$.

The initial position of the front handle of thedragon's head corresponds to the spiral radius $\rho_1 = 16\times 55 = 880\text{cm}$. According to Eq. (17), a can be calculated.

$$a = \frac{p}{2\pi} = \frac{55}{2\pi}, \tag{19}$$

After ρ_1 is calculated, the angular velocity of thedragon's head is calculated according to the relationship between velocity and angular velocity (1).

$$\omega_1 = \frac{v_1}{\rho_1} = \frac{1}{0.88} \approx 1.136 m/s, \tag{20}$$

This model calculates the position and velocity in the time range from 0 to 300 s, and sets the total time. The model needs to solve the position and velocity corresponding to 0 s, 60 s, 120 s, 180 s, 240 s, and 300 s, and the time interval is 1 s to solve the required answer.

Step 3: Solve the dragon body and tail step by step

It is known that the initial position of the dragon head is located at point A, the velocity is v_1 and the initial pole angle is θ_0, the pole angle θ_1 of the dragon head changes with time according to Eq. (3), and the angular relationship between adjacent dragon benches is Eq. (4). Using s_h, s_b to solve the relationship between the polar angle difference $\Delta\theta_i$ and ρ_i between each bench and the previous bench, it is known that there is a relationship between ρ_i and the polar angle θ_i of each bench as shown in Eq. (7). The result is calculated as follows.

$$\rho_{\mathrm{i+1}} = a \cdot \left(\theta_i + \frac{s}{\rho_i}\right), \tag{21}$$

$$\theta_1 = \theta_0 - \omega_1 \cdot \Delta \mathrm{t}, \tag{22}$$

Since a change in time causes a change in angle and radius, the final result can be solved by calculating the position and velocity of each point at different time intervals. The image at 300s is shown in the Figure 6.

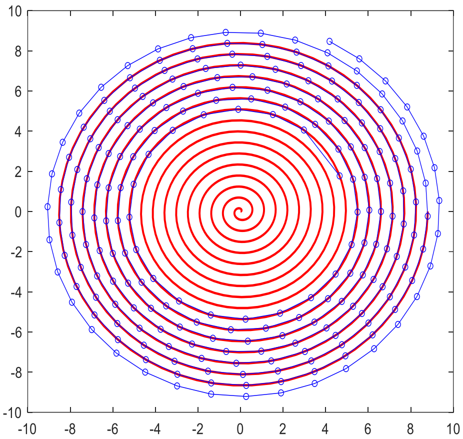


Figure 6. Image of the dragon bench at 300s

The final results are shown in Table 1.

Table 1. Velocity results of Model I

	0 s	60 s	120 s	180 s	240 s	300 s
Tap (m/s)	-1.000×10⁻¹	-5.840×10⁰	-6.249×10⁰	6.136×10⁰	-5.397×10⁰	2.234×10⁰
Section 1 Dragon Body (m/s)	2.724×10⁰	-3.538×10⁰	-7.383×10⁰	4.444×10⁰	-3.653×10⁰	4.340×10⁰
Section 51 Dragon Body (m/s)	1.484×10⁰	2.684×10⁰	6.480×10⁰	7.183×10⁰	-3.984×10⁰	6.843×10⁻¹
Section 101 Dragon Bod y(m/s)	-9.965×10⁰	-8.106×10⁰	-7.669×10⁰	-8.514×10⁰	-6.227×10⁰	4.173×10⁰
Section 151 Dragon Body (m/s)	1.628×10⁰	7.995×10⁰	9.668×10⁰	9.392×10⁰	8.298×10⁰	4.102×10⁰
Section 201 Dragon Body (m/s)	10.633×10⁰	9.165×10⁰	1.618×10⁰	-3.984×10⁰	-5.928×10⁰	-4.948×10⁰
Dragon's Tail (Rear) (m/s)	-10.568×10⁰	-8.956×10⁰	5.717×10⁻¹	7.278×10⁰	9.352×10⁰	9.215×10⁰

3.2. Results of the Bench Dragon Movement Model

The length l_{AF} of AF is the vertical distance from the point to the straight line. The width of the bench dragon is 30cm, and the length of the CF l_{CF} =15cm can be solved. The length l_3 of AC is an uncertain value, which changes with the angle at the time of collision, and is located in the range of the length l_1 of AB and the length l_2 of AD, $l_2 \le l_3 \le l_1$.l_1 is half the width of the bench dragon, that is, 15 cm.

The length l_{BD} of the BD is the distance from the bench dragon handle to the edge, l_{BD} =27.5cm. According to the Pythagorean theorem, the length of AB $l_1 = \sqrt{l_{\mathrm{BD}}^2 + l_2^2} \approx 31.32$cm can be calculated, but since only the length l_3 of AC can be solved, the specific value of it is not known now, so the estimation method is used. Suppose AC is the bisector of the angle of angle δ_{BAD}, i.e., $\delta_{\mathrm{BAC}} = \delta_{\mathrm{CAD}}$.

$$\tan\delta_{BAD} = \frac{l_{BD}}{l_2} = \frac{27.5}{15} \approx 1.83, \tag{23}$$

$\delta_{\mathrm{BAD}} \approx 6\ 1.39°$, $\delta_{\mathrm{BAC}} = \delta_{\mathrm{CAD}} \approx 3\ 0.69°$, $tan\,\delta_{\mathrm{CAD}} \approx 0.594$, $l_{\mathrm{DP}} = l_2 \cdot tan\,\delta_{\mathrm{CAD}} = 15 \times 0.594 = 8.91$cm.

Then according to the Pythagorean theorem, the length of AP is $l_{\mathrm{AP}} = \sqrt{l_{\mathrm{DP}}^2 + l_2^2} \approx 17.45$cm.

The length of BD is l_{BD} =27.5cm, the length of DP is l_{DP} =8.91cm, the length of BP is$l_{\mathrm{BP}} = l_{\mathrm{BD}} - l_{\mathrm{DP}} = 27.5 - 8.91 = 18.59$cm. $\delta_{\mathrm{BPC}} = \delta_{\mathrm{APD}}$, and then

$$\cos\delta_{BPC} = \cos\delta_{APD} = \frac{l_{CP}}{l_{BP}} = \frac{l_{DP}}{l_{AP}}, \tag{24}$$

$$l_{CP} = \frac{l_{DP}}{l_{AP}} \cdot l_{BP} = \frac{8.91}{17.45} \times 18.59 = 9.49cm, \tag{25}$$

The length of AC is $l_3 = l_{\mathrm{AP}} + l_{\mathrm{CP}} = 9.49 + 17.45 = 26.9$cm. The length of AF is $l_{\mathrm{AF}} = l_3 + l_{CF} = 26.9 + 15 = 41.9$cm.

Then, according to problem 1, the coordinates of the handles of each bench are solved, and then the equation of the straight line where the two handles of each bench are located is calculated. Then, by equation (14), the length of AF is solved using the formula of the center coordinates x_1, y_1 of the small hole where the dragon's head front handle is located and the distance between the two handles of each bench so that the length l_{AF} of AF is 41.9cm.

The end time of the dragon dance team is 0.4976s.

At this time, the position and speed of the front handle of the dragon head, the front handle of the dragon head and the front handle of the dragon tail and the rear handle of the dragon tail behind the dragon head are shown in Table 2.

Table 2. Velocity results

The moment at which the inventory is terminated	Speed
Tap (m/s)	1.000×10^0
Section 1 Dragon Body (m/s)	9.971×10^{-1}
Section 51 Dragon Body (m/s)	9.868×10^{-1}
Section 101 Dragon Body (m/s)	9.852×10^{-1}
Section 151 Dragon Body (m/s)	9.846×10^{-1}
Section 201 Dragon Body (m/s)	9.842×10^{-1}
Dragon's Tail (Rear) (m/s)	9.841×10^{-1}

3.3. Settlement of the minimum pitch model

Similar to the previous two models, the steps are shown in Table.3.

Table 3. Solution steps

Step	Content
1	Define parameters
2	Initialize matrices and variables
3	Binary search algorithm
4	Calculate the initial radius and angular velocity of thedragon's head
5	Simulation calculations
6	Update the search compartment

When the pitch is a variable, the specific coordinates of the position of thedragon's head cannot be solved, so in the process of solving, the result is related to the pitch $\boldsymbol{p}$, and the parameters are finally expressed in the form of pitch $\boldsymbol{p}$, and the distance $\boldsymbol{l_{AF}}$ at the time of collision has a specific value, which is the value of 41.9cm in problem 2. At this point, the value of the pitch is calculated.

By adjusting the pitch, and then changing the initial radius of the dragon's head, the distance from the dragon's head to the bench is less than a certain value, that is, the dragon's head collides with other benches, and the bench dragon will no longer continue to coil in. A binary search algorithm is used to gradually optimize the initial radius to ensure that the distance between the dragon's head and the bench meets the requirements within a given time frame. This is achieved through simulations and distance calculations, and the optimal solution is continuously approached by narrowing the search interval.

Finally, the size of the pitch $\boldsymbol{p}$ is 0.497697m.

4. Conclusions and outlooks

This paper successfully addresses the challenges of position tracking, velocity analysis, collision prediction, and minimum pitch optimization in the dragon-chair dance motion around an equidistant spiral through the establishment of a computer simulation-based dragon-chair motion model, an L2-norm collision model, and a binary search-based minimum pitch model. These models not only hold significant importance in theoretical research but also demonstrate vast potential for application in multiple fields. Firstly, the dragon-chair motion model provides scientific support for dragon dance performances and training, enhancing training efficiency and performance quality through simulation-based optimization of dance movements. Secondly, the collision model possesses potential value in aerospace collision warning, capable of real-time monitoring and predicting collision risks to ensure flight safety. Lastly, the minimum pitch model contributes to mechanical manufacturing and optimization design by refining pitch design, thereby improving the machining precision and performance of components. Consequently, the research findings in this paper not only enrich the theoretical frameworks of related fields but also provide robust technical support for practical applications, boasting extensive promotional value.

While the research presented in this paper showcases widespread application prospects and significant practical value across multiple domains, it is imperative to acknowledge its limitations. Firstly, regarding data acquisition and processing, the dragon-chair motion model exhibits a strong dependence on data and faces challenges in obtaining high-quality data, which, to a certain extent, restricts the model's accuracy and scope of application. Secondly, the binary search-based minimum pitch model's search results may be influenced by initial conditions, posing a risk of falling into local optimal solutions rather than global ones. Consequently, future research endeavors should focus on optimizing data acquisition and processing methods to enhance the model's adaptability and robustness. Additionally, exploring more efficient global optimization algorithms is crucial to overcoming the limitations of existing models.

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