New Phenomena in Condensed Matter Physics: Topological Insulators and Twisted Bilayer Graphene

Tianrun Hou*

Department of Applied Physics, Faculty of Science, The Hong Kong Polytechnic University, Kowloon, Hong Kong SAR

*Corresponding author: tianrun.hou@connect.polyu.hk

Abstract. Phenomena include the Kosterlitz-Thouless phase transition, fractional quantum Hall effect, and high-temperature superconductivity have driven advances in condensed matter physics, offering a foundation for the investigation of superconductivity and the quantum Hall effect, as well as aiding in the comprehension of the behavior of strongly correlated electrons. The discovery of topological insulators and the preparation of graphene triggered important breakthroughs 20 years ago, leading to novel phenomena include topological superconductivity, the quantum anomalous Hall effect, corner graphene, and quantum spin liquids, which opened new directions in quantum materials research. Through the study of topological superconductors and flat band effects, this paper elucidates the role of topological energy band theory in describing Majorana fermions and their non-abelian statistical properties and reveals the phenomena of superconductivity and Mott's insulating state due to strong electronic interactions in magic-angle cornering graphene. The results show that the theory of topological energy bands and flat band effects can explain the strongly correlated electronic effects, providing theoretical support for understanding the phase transition from Mott insulating states to superconducting states and the Majorana zero mode in quantum computation. This paper emphasizes the importance of topological and two-dimensional materials in the study of strongly correlated electrons, providing guidance for future research.

Keywords: Topological insulators; Twisted bilayer graphene; Flat band; Quantum materials.

1. Introduction

Condensed matter physics is an important branch for studying the collective behavior of microscopic particles in macroscopic systems, and has traditionally made remarkable achievements in solid state physics, especially energy band theory and Landau phase transition theory, which have successfully explained the electronic structure and phase transitions in metals, semiconductors, and insulators. However, with the discovery of new phenomena, the traditional theories show limitations. Topological phenomena like the fractional quantum Hall effect transcend the Landau paradigm, and quantum states based on topological conservation cannot be explained by conventional phase transition theory. In addition, the BCS theory proposed in 1957, while explaining low-temperature superconductivity, fails to cover high-temperature superconductivity phenomena, which require more complex electronic interaction mechanisms. Quantum spin liquids, on the other hand, exhibit rich quantum correlation behavior different from that of conventional solid matter [1].

There are advances in quantum spin liquids and high-temperature superconductivity phenomena. Quantum spin liquid, proposed by PW Anderson in 1973, describes a state of matter without long-range magnetic order in the ground state, in which the spins remain quantum entangled in a “liquid” state. In recent years, the properties of quantum spin liquids have been experimentally verified in two-dimensional and topological materials, such as κ-$(ET)_2Cu_2(CN)_3$ and α-$RuCl_3$, where the dynamical features of no magnetic order and spin correlation have been confirmed by techniques such as neutron scattering. Quantum spin liquids are closely related to topological matter states and quantum computation, providing an important platform for studying Majorana fermions and non-abelian statistics [2].

The phenomenon of high-temperature superconductivity refers to the fact that certain materials exhibit superconductivity at relatively high critical temperatures, i.e., the material conducts electricity in a state of zero resistance and complete antimagnetism. In 1986, J. Georg Bednorz and K. Alexander

Müller discovered for the first time that copper oxide superconductors exhibit superconductivity in the temperature region of liquid nitrogen (77 K) and above, marking the birth of the high-temperature superconductivity theory. Unlike the conventional BCS theory, the pairing mechanism of electrons in the high-temperature superconductivity phenomenon is not yet fully understood, but the study suggests that strong electron correlation, antiferromagnetism, and low-dimensional structure may play a key role in its formation. The discovery of high-temperature superconductors greatly expands the scope of condensed matter physics research and opens up a whole new era in materials science and the exploration of novel quantum materials [3].

In this paper, the author will introduce novel phenomena in condensed matter physics and the physical mechanisms behind them through topological insulators and corner graphene. The second part explores topological insulators, focusing on their surface-conducting and in vivo insulating properties, as well as the manipulation of electronic states by topological protection, which broadens the boundaries between insulators and conductors and provides new ideas for quantum computing and low-energy devices. The third section discusses how cornering graphene, in particular the flat-band effect under magic corners, triggers superconductivity and Mott insulating states, deepening people’s understanding of strongly correlated phenomena in two-dimensional materials. The concluding section will summarize the importance of these phenomena and explore future directions for applications in high-temperature superconductivity and quantum spin liquids.

2. Topological insulators

Topological insulators are positioned as a new quantum state that is neither a conventional insulator nor a semiconductor. Topological insulators behave as insulators in vivo and have conductivity due to the topological nature of their surface material. This topological property is mainly controlled by spin-orbit coupling and time-reversal symmetry. The unique properties of topological insulators make their electrons less susceptible to scattering of impurities as they propagate across the surface, resulting in the formation of metallic states at the surface [4]. Their topological invariants provide protection against dissipation, making them important in advanced technologies such as quantum computing. The subsequent introduction of the behavior of topological insulators in the presence of electron-electron interactions extends the study of traditionally non-interacting topological insulators [5].

2.1. Background Theory

Topological band theory (TBT) and topological field theory (TFT) are the two main theoretical frameworks upon which topological insulators are founded. Using efficient field theories like Chern-Simons, topological field theory explains the topological characteristics of topological insulators with an emphasis on topological protection mechanisms. On the other hand, topological band theory analyzes the energy band structure of electrons, particularly through the Berry phase and the topological invariants, to disclose the topological features of materials [6].

In the context of topological band theory, a wide range of effects can be separated. The quantum Hall effect shows how the trajectories of electrons in an electron plasma in two dimensions with a powerful magnetic field form discrete Landau level, the filling of which determines the Hall conductance's quantitative value. The Chern number is a topological invariant used to differentiate various topological states in two-dimensional systems, especially within the framework of the quantum Hall effect. It derives from the mathematical theory of fiber bundles and is connected to the Berry phase of Bloch wavefunctions. The formula for Chern number is described as

$$n_m = \frac{1}{2\pi}\int d^2k\, F_m \tag{1}$$

where $F_m = \nabla \times A_m$ is the Berry curvature, and $A_m = i\langle u_m(k)|\nabla_k|u_m(k)\rangle$ is the Berry connection.

Electrons in graphene have a linear dispersion relation and behave like massless Dirac fermions near low energies, which the Dirac equation can explain. It can be shown through the body-boundary correspondence principle that the topological invariants of the system's body state determine the existence or non-existence of its boundary states. For a three-dimensional topological insulator, conducting boundary states will exist on its surface which are stable without destroying the bulk topological properties. These boundary states usually behave as gapless surface electronic states, and they contrast with the energy-gap material in the body. Topological invariants $\mathbb{Z}_2$ are mainly used to describe systems with time-reversal symmetry and can be used to distinguish between phases that are topologically non-trivial and those that are topologically trivial. In time-reversal symmetric topological insulators, the $\mathbb{Z}_2$invariant determines the presence or absence of protected helical edge states at the edges of the system, which are stabilized when the symmetry of time-reversal is maintained [7].

2.2. topological Insulator: From 2D to 3D

Based on HgTe/CdTe quantum wells, the characteristics of two-dimensional topological insulators (2D TIs) can be described. In these quantum wells, high spin-orbit coupling (SOC) and bandgap inversion lead to the quantum spin Hall effect (QSH). Topological and nontrivial phases can be distinguished by the critical thickness of the quantum well. In nontrivial topological phases, time-reversal symmetry protects the formation of helical edge states. These edge states are resistant to backscattering while maintaining time-reversal symmetry [6].

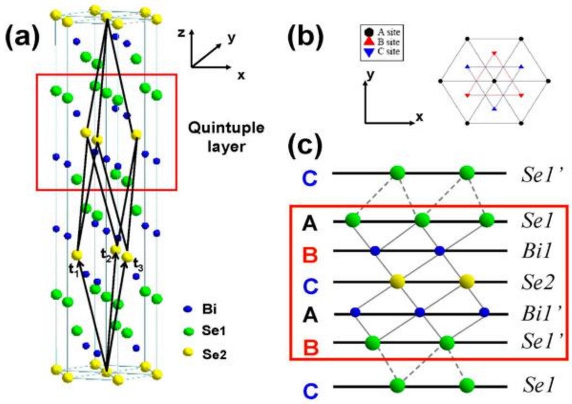

Fig. 1 (a) Bi_2Se_3's crystal structure with three primitive lattice (b) Top view along the z direction. (c) Side view of the quintuple layer structure [6].

Materials such as Bi_2Te_3, Bi_2Se_3 and BiSb alloys belong to the group of three-dimensional topological insulators (3D TIs). The crystal structure of Bi_2Te_3 is shown in Fig. 1. These materials are resilient against nonmagnetic impurities and have surface states represented by a single Dirac cone. The electromagnetic properties of 3D TIs are particularly exciting. For example, topological magnetoelectric effects, Kerr and Faraday rotation, and image magnetic monopole effects demonstrate how these materials can trigger new optical and magnetic phenomena [6].

2.3. Topological Superconductors

Topological superconductors (TSCs) exhibit surface states made up of Majorana fermions in the null gap, while topological insulators feature surface states composed of Dirac fermions within the null gap. The presence of Majorana fermions can be detected by tunneling microscopy. The surface states of topological superconductors can support Majorana zero modes. Since Majorana zero modes have non-abelian statistics, they can be used to implement fault-tolerant quantum computation. For representative topological superconductors (TSCs), time-reversal symmetric superconductors and $p + ip$ superconductors. Time-reversal symmetric superconductors maintain time-reversal symmetry, while $p + ip$ superconductors exhibit different topological properties, with a localized quasiparticle with precisely zero energy exists in the center of a superconducting vortex [6].

Although non-interacting models explain well the basic behavior of topological insulators, electron-electron interactions (e.g., Coulomb interactions) are always present in real materials. These interactions are sometimes so strong that they give rise to new physical phenomena such as Mott insulators, the Kondo effect and unconventional superconductivity [5]. Topological insulators in strongly correlated electron systems may behave quite differently than when they are not interacting, and may even lead to the emergence of new topological phases and phase transitions. Interactions between electrons can drive topological phase transitions. Topological phases can be triggered by electron-electron interactions rather than by the topological band structure itself, e.g., topological phases that can result from band-structure reorganization, Fluctuation, and spontaneous symmetry breaking [5]. Models of topological insulators with correlated interactions include the Kane-Mele-Hubbard model and the Haldane-Hubbard model. The Kane-Mele-Hubbard model is a typical model of a $\mathbb{Z}_2$ topological insulator that exhibits electron correlation effects in the half-full state. The introduction of electronic interactions in this model results in a shift in the topological phase of the system. The Haldane-Hubbard model is a framework for an interacting Chern insulator that demonstrates how electron correlation can lead to a new topological phase [5].

2.4. Magnetic Topological Insulators

Magnetic topological insulators, such as transition metal oxides like $\mathrm{Na_2IrO_3}$ and $\mathrm{Li_2IrO_3}$, are materials with spontaneous magnetization and topological surface states that show great potential in spintronics and quantum computing. Kitaev interactions can occur with the two materials mentioned above, and similarly with $\alpha - \mathrm{RuCl_3}$. By introducing transition metal elements into these materials, spontaneous magnetization can be induced, which interacts with topological surface states and opens the mass gap at the Dirac point. Through fine-tuning the Fermi energy, the quantum anomalous Hall effect can be achieved without an external magnetic field, providing a foundation for developing low energy spintronics, dissipation less topological electronics, and quantum computing devices. In addition, heterostructure engineering further stabilizes this effect, allowing manipulation of chiral edge states and exploring applications of Majorana edge modes to advance the development of next-generation electronic devices [8].

The Kitaev model opens up new avenues for identifying materials capable of realizing quantum spin liquids. α-$RuCl_3$ is considered an important candidate, and future research will focus on exploring other potential materials with similar properties. Transition metal oxides and those with strong spin-orbit coupling will be explored even further [9].

2.5. Future Research Directions

Electron-electron interactions provide new research directions in the field of topological insulators. Modulation of the interaction strength on platforms such as quantum materials, ultracold atomic systems and optical lattices can lead to the discovery of new topological phases. In addition, experimental verification of interaction-induced new states, including topological Kondo insulators, topological Mott insulators, and fractional topological insulators is an important frontier of current research. Through in-depth study of electronic interactions, scientists are expected to develop more topological materials with practical value and promote their applications in quantum computing, quantum information processing, spintronics and other fields [5].

3. Twisted Bilayer Graphene

Since graphene's discovery and isolation in 2004, this two-dimensional carbon material, consisting of a single sheet of carbon atoms arranged in a honeycomb lattice [10], has exhibited a broad spectrum of optical, electrical, and mechanical properties. As the thinnest known material, its charge carriers possess remarkably high mobility [11].

Since graphene's charge has zero rest effective mass, the Dirac-like equation must be used to describe these quasiparticles instead of the Schrödinger equation, which defines the quantum characteristics of other materials [12]. Based on the fact that their carriers are massless, they can cover sub-micrometer distances without scattering, making them highly quantum effective even at room temperature [11]. Because graphene's electron waves can only travel via a single atomic layer, this allows various scanning probes to capture the relevant signals and is proximity sensitive to materials such as superconductors ferromagnets and high-k electrolytes [11]. Graphene's structure is also highly plastic based on its mechanical characteristics, which lead to a dramatic effect on electronic, optical, and phononic characteristics through both strain and deformation [11,13]). Since graphene typically has nanoscale corrugations [14], strain can even alter the energy band structure of graphene [15]. These studies also triggered subsequent research on twisted bilayer graphene.

The study of twisted bilayer graphene has become progressively deeper with the realization by Cao et al. (2018). They found that the critical temperature of the tunable zero-resistance state can reach up to 1.7 Kelvin when the two layers of graphene are twisted at an angle of about 1.1 degrees, called the “first magic angle”, as they also pointed out [16]. Thereafter, Tarnopolsky et al. pointed out the anomalous stability of the first magic angle in experiments and predicted the range of the second magic angle [17]. Kerelsky et al. employed scanning tunneling microscopy and spectroscopy to study the atomic structure and electronic characteristics of TBGs and discovered two van Hove bases (VHSs) in the localized density of states around the magic angle [18]. They pointed out that the critical temperature can be reached up to 1.7 Kelvin by making the ratio (U/t) of the bandwidth of each Van Hove site (VHS) reaches maximum to maximize the electronic interactions at the magic corner of the TBG. In the same year, Khalaf et. al. extended the discussion of magic corner sequences from bilayer TBGs by expanding it to triple and quadruple layers, and the results pointed out that the magic corner sequences in the triple-layer case are $\sqrt{2}$ times that of the bilayer, and the quadruple-layer case results in two sets of magic corner sequences, which are the Golden Ratio of the bilayer $\frac{1+\sqrt{5}}{2}$ and the inverse multiple of this, respectively [19].

The previous section described graphene and its significance, as well as the proposal of twisted bilayer graphene and multilayer twisted graphene, along with the discovery of superconductivity at the magic angle. Next, the theoretical foundations will be introduced, including the flat band effect, superconductivity, and the Mott insulating state.

3.1. Flat Band Effect

In TBG, two graphene layers are layered by rotating at a certain angle, forming a moiré superlattice. The "magic angle" is recognized as the twist angle between two graphene layers that is roughly 1.1°. At this magic angle, the electronic band structure undergoes significant changes, with the bands becoming nearly flat, resulting in the so-called "flat band effect." The core of the flat band effect lies in the fact that, due to the near-flatness of the bands, the electrons’ kinetic energy is minimized, allowing the electron-electron interactions to dominate the physical behavior of the system. These intense electron interactions can lead to a variety of quantum consequences, including superconductivity and the Mott insulating state [16].

The flat band effect causes the band structure to become almost flat, resulting in a very large effective mass of electrons and an extraordinarily high state density. The Van Hove singularities (VHS) in the density of states at the Fermi level greatly improve electron-electron interactions. Near a Van Hove singularity, the density of states $D(E)$ behaves as follows:

$$D(E) \sim \frac{1}{\sqrt{E-E_0}} \tag{2}$$

Here, E_0 represents the position of the Van Hove singularity. When the twist angle reaches the magic angle, the Van Hove singularity causes the electron's kinetic energy to approach zero, making Coulomb interactions between electrons extremely significant [16].

3.2. Superconductivity

In magic-angle TBG, the flat band effect enhances electron interactions, promoting the formation of electron pairs, which is a key mechanism for superconductivity. By doping, the system can be transitioned from an insulating state to a superconducting state, forming a "dome-shaped" superconducting phase diagram like that of high-temperature copper oxide superconductors. The superconductivity in the system typically results from electron pairing mediated by Coulomb interactions, rather than the conventional phonon-mediated pairing.

Theoretically, the formation of superconductivity can be explained using the ratio of "Coulomb interaction to bandwidth," U/t. The formula is

$$\frac{U}{t}=\frac{\text{Coulomb interaction energy}}{\text{Bandwidth}}. \tag{3}$$

When the ratio U/t reaches a certain threshold, the strong electron-electron interactions become sufficient to induce superconductivity. At the magic angle, the U/t ratio is maximized, indicating that the strong electron-electron correlations enable electrons to effectively form superconducting pairs [16].

3.3. Mott Insulator State

The Mott insulating state is an insulating phase driven by strong electron interactions, even though traditional band theory predicts that the system should be conductive. In magic-angle TBG, when the electron density approaches "half-filling" of the moiré superlattice, the system exhibits a Mott insulating state. This occurs because Coulomb repulsion between electrons prevents them from moving freely, turning the system into an insulator. A key characteristic of the Mott insulating state is the energy gap Δ, which arises due to electron correlations. This gap is not a result of the band structure, but is caused by the repulsive interactions between electrons.

The Mott insulating state can be described using the Hubbard model, where the Hamiltonian is:

$$H=-t\sum_{<i,j>,\sigma}\left(c_{i\sigma}^{\dagger}c_{j\sigma}+H.c.\right)+U\sum_{i}n_{i\uparrow}n_{i\downarrow} \tag{4}$$

Here, t represents the electron's kinetic energy, U represents the Coulomb repulsion energy, representing the interaction between electrons; $c_{i\sigma}^{\dagger}$ and $c_{j\sigma}$ are the creation and annihilation operators of electrons, respectively; $n_{i\uparrow}$ and $n_{i\downarrow}$ are the number operators for electrons with spin-up and spin-down, respectively. When U is much larger than t, the electron kinetic energy is suppressed, and the system enters the Mott insulating state [16].

3.4. Relationship Between Superconductivity and Mott Insulating States

In the phase diagram shown in Fig. 2, superconductivity and the Mott insulating state are often closely linked. When the system is in the Mott insulating state, strong electron-electron interactions prevent the free movement of electrons, making the system behave as an insulator. However, doping allows it to shift from the Mott insulating state to the superconducting state. The proximity of superconductivity to the Mott insulating phase suggests that the same strong electron correlation that leads to insulating behavior contributes to the formation of the electron pairing required for superconductivity. The similarity of this correlation to that of high-temperature superconductors suggests that superconductivity may originate from a physical mechanism of strong electron correlation [16].

The magic corner phenomenon in twisted bilayer graphene causes a number of quantum effects, particularly in the presence of enhanced electronic interactions. The flat-band effect, superconductivity, and Mott insulating states are essential processes for studying its physical behavior. The flat-band effect results from the flattening of the energy bands at the magic angle, which boosts electron interactions. Superconductivity is due to the formation of electron pairs through strong electron interactions, which exhibits a non-resistive superconducting state at low temperatures. Mott's insulating state is an insulating state due to strong mutual repulsive forces between electrons, although energy band theory predicts that the system should be conducting. The appearance of these phenomena is closely related to the flat-band effect at the magic angle, revealing the importance of strong electron correlations in quantum materials.

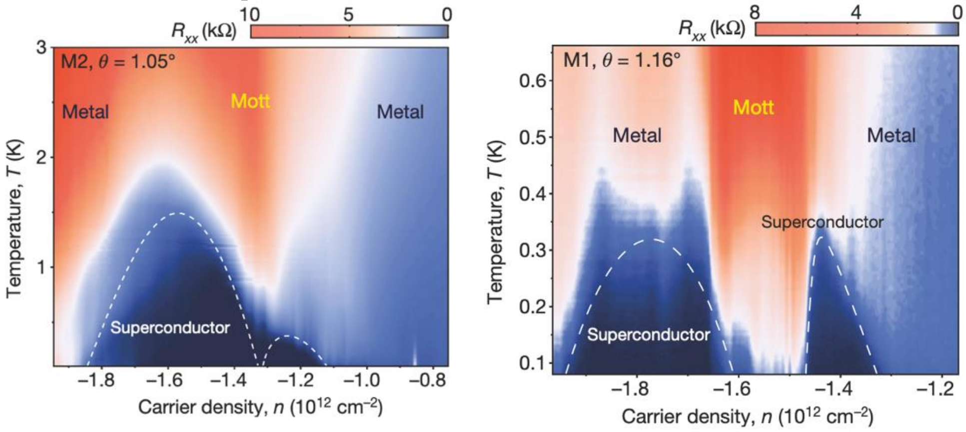

Fig. 2 (a) Phase diagram that contains Metal, Mott insulator, and Superconductor. (b) Similar to (a) but for device M2, which shows two asymmetric and overlapping superconducting domes [16].

3.5. Future Research Directions

There are several important directions for future research on TBG. First, by tuning the torsion angle and applying a perpendicular electric field, the electronic interactions in TBG can be further fine-tuned to gain insight into its physical properties. Second, using pressure to increase interlayer hybridization or linking different magic-angle TBG structures to induce Josephson coupling may aid to raise the critical temperature Tc of superconductivity [16]. Furthermore, analogous magic angle superlattices and flat band electronic structures are likely to be achieved in various two-dimensional materials or lattices, which provide new opportunities to explore strongly correlated electronic systems of different nature. To better understand the superconducting mechanism, future experiments (e.g., tunneling effects and Josephson heterojunctions) need to further investigate superconducting pairing symmetries in TBGs, including symmetries such as predicted $(d + id')$ waves, $(p_x + ip_y)$waves, and spin-triple-state s-waves [16].

Further expansion of the Moore family of superconductors is also a key direction. Researchers have demonstrated robust superconductivity in magic-horn twisted three-layer graphene and are exploring four- and five-layer graphene to build a family of multilayer magic-horn superconductors. Understanding the role of flat bands in these multilayer systems is also crucial, as flat band structures play a central role in superconductivity. Future research will focus on how to utilize flat-band structures to improve the robustness and properties of superconducting states [20]. Finally, the effect of external fields on TBGs is likewise an area that deserves in-depth study. In particular, the application of in-plane magnetic fields can help to further reveal the mechanism of superconductivity and investigate the effect of external fields on superconducting behavior [20].

4. Conclusion

This paper explores the novel phenomena of topological insulators and corner graphene in condensed matter physics and their physical mechanisms. The study of topological insulators reveals their unique surface conductivity and in vivo insulation dependent on spin-orbit coupling and time-reversal symmetry, which not only broadens the understanding of the boundaries between insulators and conductors, but also provides new ideas for the development of quantum computing and low-energy devices. The magic-angle down-flat-band effect in corner graphene, on the other hand, reveals how strong electronic interactions trigger superconductivity and Mott insulating states, providing a theoretical basis for understanding the strong correlation phenomena in 2D materials. It is demonstrated that the idea of topological energy bands and flat-band effects provides a useful foundation for explaining the behavior of tightly coupled electronics. Majorana fermions and non-abelian statistics in topological insulators, as well as Mott insulating states and superconductivity in corner graphene, demonstrate novel phenomena in quantum phase transitions. This not only extends the Landau phase transition theory, but also provides new perspectives and methods for zero-state physics. This study provides a new understanding of quantum processes in tightly coupled electronic devices, especially in phase transition processes dominated by topological conservation effects and electronic interactions. Future research should further explore quantum spin liquids and KT phase transitions, especially to verify their properties in two-dimensional and topological materials, and to expand their applications in quantum computing and novel quantum materials.

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