Optimization of Planting Strategies Based on Dynamic Programming and Monte Carlo Simulation

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Abstract. This study uses a combination of dynamic programming and Monte Carlo simulation with the aim of optimizing crop cultivation strategies in a rural village in a mountainous area. First, the optimal cropping scenarios for the years 2024 to 2030 were determined using linear programming for stable economic indicators, and the returns under different scenarios (stagnant sales and discounted sales) were calculated. Secondly, the increasing trend of expected crop sales as well as the volatility were considered, and the forecast of total returns for the next seven years was obtained through dynamic programming and Monte Carlo simulation. Finally, the study introduced the substitutability and complementarity among crops, as well as the correlation between various economic indicators, modified the objective function, and further optimized the planting strategy. The planting scheme proposed through this study not only maximizes economic benefits and minimizes risks in crop cultivation, but also the results of the study will provide a reference for crop cultivation in other regions.

Keywords: Linear Programming; Planting Strategies; Dynamic Programming; Monte Carlo Simulation; Alternative and Complementary.

1. Introduction

In this paper, relevant algorithms such as linear programming [1], dynamic programming [2] and Monte Carlo simulation [3] are comprehensively applied to establish a scientific mathematical model for crop cultivation in a village in the mountainous region. The study includes the formulation of planting program under stable sales conditions, the optimal planting strategy considering the riskiness of sales volume and price fluctuation [4], and the comprehensive analysis based on the substitutability and complementarity among crops. First, planting scenarios for the period from 2024 to 2030 were formulated using linear programming methods to cope with the scenarios of crop stagnation and discounted sales. Second, dynamic programming and Monte Carlo simulation were used to re-optimize the planting strategy to ensure maximum profitability, considering the increasing trend of wheat and maize sales and the volatility of other crops. Finally, on this basis, the correlation between various economic indicators was analyzed by considering the substitutability [5] and complementarity [6] between crops, and the planting scheme was further adjusted to achieve higher economic efficiency and stable returns.

2. Planting Strategy Analysis

Under the assumption that the expected sales volume, planting cost, acreage, and sales price of crops remain stable over the next seven years, this section considers two scenarios of stagnant crops sold more than the expected sales volume and crops sold at half price, respectively, and develops planting scenarios for the years 2024 to 2030.

2.1. Modeling of Planting Scenarios

(1) Set and index

I : Set of plots, e.g. (i = A1, A2)

J : collection of seasons, e.g. (j = 1,2))

K : Crop set, e.g. (k = 1,2)

(2) Parameters

$\{Area\}i$: area of parcel i (in acres).

$\{Type\}i$: type of parcel i (flat dry land, terrace, hillside, watered land, common greenhouse, smart greenhouse).

$\{Yield\}k$: the acre yield of crop k (in pounds).

$\{Cost\}k$: the cost of growing crop k (in Yuan/acre).

$\{Price\ \}k$: unit selling price of crop k (in Yuan/pound).

$\{Season\}k$: the planting season of crop k (single season, first season, second season).

(3) Decision variables

$(x\{i,j,k\})$: the area of plot i planting crop k in season j (unit: mu).

(4) Objective function

Maximize the total return, considering the selling price and planting cost.

$$\text{Maximize } Z = \sum_{i\in I} \sum_{i\in J} \sum_{k\in K} \left(\text{Yield}_k \times x_{ijk} \times \text{Price}_k - \text{Cost}_k \times x_{ijk} \right) \tag{1}$$

(5) Constraints

1.Plot area constraint: the planting area of each plot cannot exceed its maximum usable area.

$$\sum_{j\in J} \sum_{k\in K} x_{ijk} \leq \text{Area}_i,\ \forall i \in I \tag{2}$$

2.Two-season crop cultivation constraint: watered land and greenhouses can grow two-season crops.

$$\sum_{k\in K} x_{i1k} \leq \text{Area}_i, \forall i \in I \text{ where Type}_i \in \{\text{ watered land, ordinary greenhouse, intelligent greenhouse} \tag{3}$$

Single season crop plots are restricted to be planted only in the first season.

$$\sum_{k\in K} x_{i1k} \leq \text{Area}_i,\ \forall i \in I \text{ where Type}_i \in \{\text{ flat land, terraces, slopes }\} \tag{4}$$

3.Crop rotation requirement: ensure that the same plot cannot be planted with the same crop each season to prevent heavy cropping.

$$x_{ijk_1} \times x_{ijk_2} = 0,\ \forall i \in I, \forall j \in J, \forall k_1, k_2 \in K,\ k_1 \neq k_2 \tag{5}$$

4.Requirements for legume crops: ensure that each plot is planted with a legume crop (type legume) at least once in every three years.

$$\sum_{j\in J} \sum_{k\in K, \mathbf{Type}_k = \text{ beans}} x_{ijk} \geq 1,\ \forall i \in I \tag{6}$$

5.Sales volume and price constraints: Stagnant sales situation: if the total production of a crop exceeds the sales volume, the excess cannot be sold normally.

$$\sum_{i\in I} \sum_{j\in J} x_{ijk} \times \text{Yield}_k \leq \text{sales volume } k,\ \forall k \in K \tag{7}$$

Price reduction scenario: the excess is sold at 50% of the price.

$$Adjustedgains = \begin{cases} \text{sales volume }_k \times \text{ Price}_k \\ +\left(\sum_{i\in I}\sum_{j\in J} x_{ijk} \times \text{ Yield}_k - \text{ sales volume}_k\right) \times \text{ Price}_k \\ \sum_{i\in I}\sum_{j\in J} x_{ijk} \times \text{ Yield}_k \times \text{ Price}_k \end{cases} \tag{8}$$

6. variable non-negative constraint:

$$x_{ijk} \geq 0,\ \forall i \in I, \forall j \in J, \forall k \in K \tag{9}$$

2.2. Planting Scheme Solution

For case 1, i.e., more than part of the stagnant sales model this paper's model obtains a more reasonable planting strategy, and calculates the profit of each plot in the countryside in the first three years under the stagnant sales situation, and the profit of each plot is within the range of 0 to 600 million dollars.

For case 2, a 50% discounted sale, the model also produces a better planting strategy and calculates the profit for each plot in the village for the first three years of the discounted sale scenario.

Compared to the stagnant sales scenario, the comparison results show that the total profit from the discounted sales is significantly higher. After calculating the total profit in 2024 for the stagflation scenario is $35094085.7 while the total profit for the discounted sale is $50958694.5.

3. Uncertainty Analysis

3.1. Modeling of Risk Cultivation

(1) Setting variables

$\{i, j, t\}$ is the area (acres) of crop i grown on plot j in year t.

$Y_{i(t)}$ is the yield of crop i in year t (kg/acre).

$\mathrm{C}i\mathrm{i}(\mathrm{t})$ is the yield of crop i in year t (kg/mu).

$P_i i(t)$ is the selling price of crop i in year t (yuan/kg).

$Di(t)$ is the expected sales volume of crop i in year t (kg).

(2) Objective function

The objective of this section is to maximize the annual planting revenue, and the revenue calculation formula is:

$$R(t) = \sum_i \sum_j \left(\min\left(Y_i(t) \cdot A_{ijt}, D_i(t)\right) \cdot P_i(t) - C_i(t) \cdot A_{ijt}\right) \tag{10}$$

Explanation:

$(\mathrm{Yi(t)} * \mathrm{A\{i,j,t\}})$ denotes the total yield of crop i on plot j.

$(\min(Yi(t) * A\{i,j,t\}), Di(t)))$ denotes that the actual sales volume cannot exceed the market demand.

$((\mathrm{Y}i(\mathrm{t}) * \mathrm{A\{i,j,t\}}) - \mathrm{Di(t)}) * (\mathrm{Pi}(\mathrm{t}/\mathbf{2}))$ denotes the gain from price reduction in case 2 (50% price reduction of the exceeding portion is sold).

(3) Constraints

1.Plot planting area constraint

$$\sum_i A_{ijt} \leq S_j,\ \forall j, t \tag{11}$$

Where S_j is the total area of plot j.

2. Expected sales volume constraint

$$Y_i(t) \cdot A_{ijt} \leq D_i(t),\ \forall i, t \tag{12}$$

3.Crop rotation requirements

$$\sum_{i \in \text{ beans}} \sum_{t' \in [t-3, t]} A_{ijt'} \geq 1,\ \forall j, t \tag{13}$$

Plant legumes at least once in three years in each plot.

4. avoid heavy cropping

$$A_{ijt} \cdot A_{ijt-1} = 0,\ \forall i, j, t \tag{14}$$

5.Crop acreage limitation

$$A_{ijt} \geq M, \text{if } A_{ijt} > 0 \tag{15}$$

M is the minimum threshold of planting area to avoid too small planting area.

(4) Uncertainty treatment

Monte Carlo simulation: used to simulate the effect of uncertainty factors such as climate, market price and yield on the results. Multiple random samples of mu yield, cost and sales price are taken each year to calculate the expectation and variance of returns under different scenarios.

(5) Solution Process

1. Linear Programming

Construct linear objective function and constraints.

2. Dynamic Programming

For the multi-year optimal solution problem, use dynamic planning to make decisions year by year and consider the impact of the previous year's decisions on the current year.

3. Monte Carlo simulation combined with optimization

Multiple stochastic scenarios are solved to select the optimal planting solution. Each simulation uses different parameters (e.g., market price, cost, yield, etc.), and the results are summarized and analyzed.

3.2. Solving the Risky Planting Model

The model successfully obtained the optimal planting strategy for the countryside for the next seven years, considering the changes of many factors, and calculated the profit for the next seven years, in which the total profit in 2024 reaches about 26 million yuan.

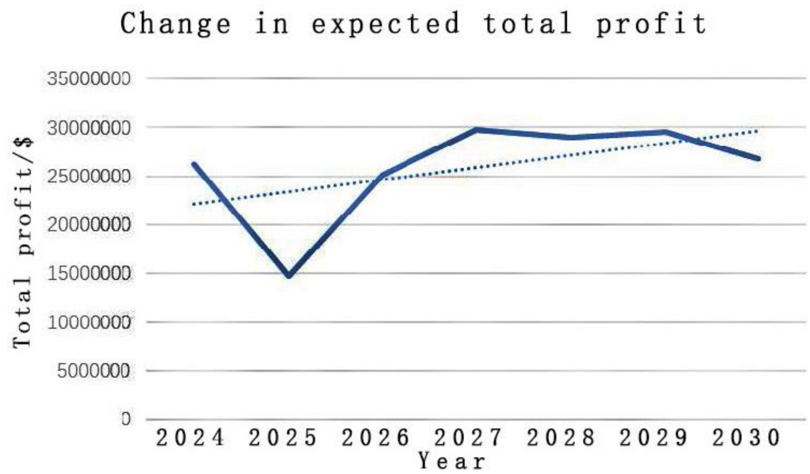

Fig. 1 Comparison of actual total profit and expected total profit

This section also simulates the trend of the expected total profit versus the modeled total profit, and from Fig. 1, it can be concluded that the direction of the model-calculated profit does not deviate significantly from the expectation, and the simulated total profit in 2025 is expected to be at the lowest level, and the total profit obtained by the model is more in line with the expectation by 2026 and beyond.

4. Analysis of Substitutability and Correlation

4.1. Modeling of Complementarity and Correlation

(1) Introduction of crop complementarity

Crop complementarity refers to the fact that certain crops can produce synergistic effects when they are rotated or planted at the same time, enhancing overall yield, or reducing planting risks. For example, legume crops (e.g., soybean, black bean, red bean, etc.) can increase soil nitrogen content after planting, resulting in increased yields of subsequent plantings of wheat, corn, etc.

If soybeans are planted in season ton plot i $x_{isoyabeans}^{t} > 0$, then wheat or corn is prioritized for planting in the next season.

This ensures that the crop following the soybean crop receives the benefits of soil improvement. Complementarity of neighboring plots: if two neighboring plots (e.g., $A1$ and $A2$) are planted with different combinations of crops, they may reduce pests and diseases and improve yields for each other. Therefore, complementary relationships of neighboring plots can be introduced in the model. For example, if plot $A1$ grows stilt beans $x_{icowpea}^{t} > 0$, the neighboring plot $A2$ may be prioritized to grow sorghum to reduce the risk of pests and diseases:

$$x_{A1\ cowpea}^{t} \Rightarrow x_{A2\ sorghum}^{t} \tag{16}$$

(2) Introduction of crop substitutability

Crop substitutability means that different crops can fulfill the same market demand. When the price or market demand for a crop fall, farmers can hedge their risk by growing other alternative crops. For example, shiitake and white mushroom are both edible mushrooms with similar market demand. If the planted area of one crop (e.g., shiitake) increases more than the market demand, the planted area of the alternative crop (e.g., shirataki) can be automatically adjusted in the model:

$$x_{\text{imushrooms}}^{t} > D_{\text{mushrooms}} \Rightarrow x_{\text{iwhrite mushroom}}^{t} \tag{17}$$

Where $Dmushrooms$ is the market demand for shiitake mushrooms.

Inter-seasonal substitution: certain crops are substitutive at different times of the year. For example, if you grow morel mushrooms in the first season and the market is not good, you can grow similar shiitake mushrooms in the second season to avoid the market risk.

$$x_{i\ morel\ mushrooms}^{1} \Rightarrow x_{i\ mushrooms}^{2} \tag{18}$$

This means that planting morel mushrooms in the first season may affect the area and decision to plant shiitake mushrooms in the second season.

(3) Introducing correlation between sales volume, sales price and cultivation cost

The sales volume and price of a crop usually show a certain inverse relationship: when the market supply increases, the price may decrease; while when the supply decreases, the price increases. This relationship between supply and demand can be expressed by the price elasticity coefficient.

If the market price P_j of a certain crop j varies with the planted area, it can be expressed as:

$$P_j = P_j^0 - \epsilon_j \cdot \left(X_j - D_j\right) \tag{19}$$

Where:

P_j^0 is the benchmark price of the crop, ϵ_j is the price elasticity coefficient, X_j is the total planted area of crop j (the sum of the area of the crop on all plots), D_j is the market demand of the crop.

(4) Introducing a correlation between planting costs and planted area

There may be an economy of scale effect between planting cost and area. That is, the larger the planting area, the average cost may decrease, or the unit cost of planting may decrease because of the more efficient use of land, labor, and machinery and equipment. This can be expressed using the following equation:

$$C_j = C_j^0 - \lambda_j \cdot X_j \tag{20}$$

Where:

C_j^0= is the initial planting cost of crop j, and Lambda j is the reduction coefficient of planting area on unit cost.

(5) Revision of the objective function

In conjunction with the correlations described above, the original objective function is revised by replacing the new prices and costs with complementarity gains and subtracting substitution losses.

4.2. Solution of the Substitutability and Correlation Model

The simulated data is solved to obtain the value of total profit for the next seven years after the introduction of complementarity, substitutability and correlation, as shown in table 1.

Table 1. Total profit values after introducing complementarity, substitutability and correlation

Year	Total profit/$	Total profit (considering inter-crop effects/$)	Year
2024	26249577.22	22189739.41	2024
2025	14755914.47	23433666.71	2025
2026	25182863.75	22459315.13	2026
2027	29751981.49	23624455.83	2027
2028	29030245	23235090.28	2028
2029	29574681.91	23595211.6	2029
2030	26775216.91	22841557.87	2030

The model also obtained the results of the comparison of profits between substitutability, correlation and planting riskiness as shown in Fig. 2 below.

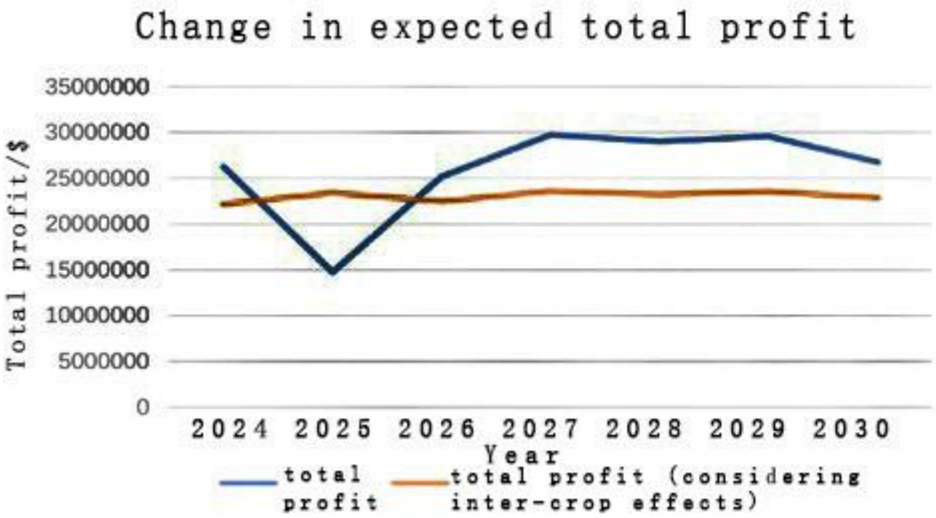

Fig. 2 Comparison of total profit between substitutability, correlation and planting riskiness

It is easy to find that the future total profit calculated by the model after considering the correlation, substitutability and complementarity is much smoother.

5. Conclusion

The study used computer algorithms, such as dynamic programming and Monte Carlo simulation, to propose the optimal planting scheme for the next seven years under the influence of uncertainties to maximize the economic benefits and minimize the risks. First, the model sets multiple types of constraints for the fluctuation of economic indicators of different crops. Through linear programming, the model obtained the profit prediction in 2024 under two scenarios of stagnant and discounted sales. Further, the study considered the substitutability and complementarity among crops, updated the constraints, and optimized the objective function so that the model could reflect the market dynamics more accurately. Ultimately, based on the proposed strategy, the simulation results show a stabilization of total future profits with reduced volatility, demonstrating greater economic sustainability.

References

[1] Xue Yi. History and development of linear programming[J]. Mathematical Modeling and its Applications, 2024, 13(03): 100-105. DOI: 10.19943/j.2095-3070.jmmia.2024.03.12.

[2] Li Chaobing, Bao Weimin, Li Zhongkui, et al. Multi-stage rocket full mission decision-making based on approximate dynamic programming[J]. Journal of Astronautics, 2024, 45(08): 1251-1260.

[3] Deng Shuo. Practical exploration of Monte Carlo method in cultivating students' computational thinking [J]. China Modern Education Equipment, 2024, (16): 45-48. DOI: 10.13492/j.cnki.cmee.2024.16.014.

[4] Feng Wei. Research on planting activity program under the perspective of home-home cooperation[J]. Henan Education (Basic Education Edition), 2024, (01): 93-94.

[5] Ding Zizhao, Xiao Changfeng, XU Yunying, et al. Progress in the application of alternative non-conventional feed ingredients in poultry farming [J]. Shanghai Journal of Agriculture, 2024, 40(05): 142-152. DOI:10.15955/j. issn1000-3924.2024.05.19.

[6] Nong Vin shou. Research on Agricultural Complementarities and Agricultural Trade Strategies between China and Vietnam [D]. Guangxi University, 2013.