Research on the Optimization Model and Algorithm for the Movement Path of the "Bench Dragon"

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Abstract. To enhance the organizational efficiency and cultural presentation effectiveness of the folk activity "Bench Dragon", this paper focuses on the dynamic optimization problem of the Bench Dragon's "panlong" (coiling) movement. A chain path optimization modeling method based on plane geometric recursion and kinematic constraints is proposed: First, an Archimedean spiral model is constructed to characterize the movement trajectory of the dragon head. The spatiotemporal coordinates and velocity distribution of each dragon body segment are systematically derived using differential-integral methods. For collision constraints during movement, a safety distance verification model in polar coordinate space is established. Combined with the bisection method and iterative algorithms, the critical coiling time is accurately solved. Under the constraint of turn space, the minimum safe pitch is determined through parameter optimization. The quantitative model system constructed in this study provides a scientific dynamic analysis framework and optimization strategy support for improving the coordination and spectacle of the dragon dance performance.

Keywords: Path Optimization, Bisection Method, Iterative Method, Arc Differential, Polar Coordinates.

1. Introduction

As the carrier of traditional folk culture, the multi-segment chain structure of ' bench dragon ' faces the problems of high risk of dragon body collision and insufficient track fluency in the performance. The current empirical path design has been difficult to meet the needs of modern cultural performance [1]. The field of path planning has formed a mature system in robot control and UAV motion planning: Hao et al. [2] summarized the dynamic obstacle avoidance algorithm in intelligent vehicle path planning, and Zhou et al. [3] systematically analyzed the topology optimization technology of UAV motion planning. However, the research on path planning in folk activities is special: Liu et al. [4] introduced the Archimedes spiral into the bench dragon trajectory modeling, but did not consider the dragon body dynamics coupling; li Yuhang et al. [5] recorded actions by motion capture, but did not construct an optimization model that integrates cultural performance and physical constraints.

The existing research is limited to the single constraint dimension and does not integrate the morphological requirements of cultural movements such as 'spiral winding'; the dynamic modeling is insufficient, and the quantitative analysis of the velocity distribution of the dragon body is lacking. Lack of cultural adaptability, the 'ornamental-coordination' quantitative evaluation system has not been established.

Based on the actual needs of bench dragon performance, this paper constructs a dragon head trajectory model based on Archimedes spiral, realizes the dynamic modeling of dragon body nodes through plane geometry recursion, establishes a polar coordinate safety distance test model, and integrates collision avoidance and cultural action norms. A closed-loop framework of ' kinematics analysis-collision inspection-parameter optimization ' is formed by combining dichotomy and iterative algorithm, which provides a quantitative optimization tool for intangible cultural heritage activities.

2. Position and Velocity Model for the Bench Dragon Handle Centers

2.1. Establishment of the Polar Coordinate Equation and Position Parametric Equation for the Archimedean Spiral

The overall movement path of the "Bench Dragon" presents a disk-like shape, coiling clockwise along an equiangular spiral with pitch c, conforming to the Archimedean spiral pattern. It is also assumed that the velocity a of the dragon head's front handle remains constant throughout the entire coiling process [6]. Therefore, determining the time and velocity of the head's front handle movement is necessary to calculate the position and velocity of the dragon body and tail. The polar coordinate equation and position parametric equation [7] for the head's front handle are thus listed:

$$r = a + b_1\theta \tag{1}$$

$$\begin{cases} x = b_1\theta cos\theta \\ y = b_1\theta sin\theta \end{cases} \tag{2}$$

where, for reasonable model simplification, the starting point distance a from the polar coordinate center in the Archimedean spiral polar equation is assumed to be $a = 0$. The pitch b controlling the distance between spiral turns is set as $b_1 = \frac{c}{2\pi}$.

2.2. Establishment of the Polar Coordinate Formula for the Arc Differential of the Dragon Head's Front Handle

Since the dragon head's travel speed is constant, the linear velocity of its front handle is also constant. The change in arc length of the equiangular spiral is related to the change in θ. The relationship between θ and t_0 is obtained by establishing the polar coordinate formula for the arc differential [8]:

$$\mathrm{ds} = \sqrt{\mathrm{r}^2 + (\mathrm{r}')^2}\mathrm{d} \tag{3}$$

where $ds = v_0dt$. Integrating both sides of equation (3) yields,

$$\int_0^{t_0} av_0\,dt = \int_\theta^{32\pi} \sqrt{r^2 + (r')^2}d\theta \tag{4}$$

$$av_0t_0 = \frac{1}{2}b_1(\theta\sqrt{\theta^2+1} + \ln\left|\theta + \sqrt{\theta^2+1}\right|)\Big|_\theta^{32\pi} \tag{5}$$

thus, the calculation equation for θ with respect to t:

$$\frac{1}{2}b\left(\theta\sqrt{\theta^2+1} + ln\left|\theta + \sqrt{\theta^2+1}\right|\right) = 442.590256 - at_0 \tag{6}$$

2.3. Establishment of the Polar Coordinate Formula for the Arc Differential of the Dragon Body's Front Handle and the Dragon Tail's Rear Handle

Due to the dragon's structure being connected head-to-tail with a pulling effect, and considering the different lengths of the head and other parts, the special cases of the first body segment's front handle and the tail's rear handle need to be addressed. First, establish the polar coordinate formula for their arc differentials:

$$\frac{1}{2}b\left(\theta_n\sqrt{{\theta_n}^2+1} + ln\left|\theta_n + \sqrt{{\theta_n}^2+1}\right|\right) = 442.590256 - at_n \tag{7}$$

2.4. Establishment of the Velocity Equation for the Dragon Handles

Given the relationship between θ and t for all parts of the dragon from equation (7), establish the velocity equation for the handles by differentiating position:

$$v = \sqrt{v_x^2 + v_y^2} \tag{8}$$

where $v_x = \frac{x_{(t)}}{dt}$, $v_y = \frac{y_{(t)}}{dt}$

3. Collision Constraint Model for the "Bench Dragon"

3.1. Determination of the Dragon Head Front Handle Collision Time

If the dragon team moves to the center of the spiral, the coiling path of the head's front handle follows formula (6). Collision occurs when $\theta = 0$. Substituting $\theta = 0$ into formula (6) determines the termination time t_{max} for coiling to prevent collisions between benches. By testing whether a collision occurs at a set coiling start time t_{min}, and if no collision occurs, use the bisection method to find the head front handle collision time [9]:

$$\mathrm{t_{mid}} = \frac{\mathrm{t_{max}} + \mathrm{t_{min}}}{2} \tag{9}$$

3.2. Determination of Collision Constraint Conditions

Substitute t_{mid} into formula (6) to obtain the polar angle θ_1 of the head's front handle. Then substitute θ_1 into the position parametric equation to get the head front handle's position (x_0, y_0). Establish a circle centered at (x_0, y_0) (point O) with radius r_1 equal to the distance from the head front handle to the outer edge point of the front bench board.

Since the polar angle for an adjacent bench segment on the spiral coil is $\theta_1 + 2\pi$, substitute $\theta_1 + 2\pi$ into the position parametric equation to get the position (x_1, y_1) of a point on that adjacent coil segment. Using the arc differential formula for the body front handle and tail rear handle combined with the position parametric equation and iterative methods, obtain the positions of the front and rear handles (x_2, y_2), (x_3, y_3) of the corresponding bench on that coil segment.

Establish the formula for the distance from point O to the line formed by the front and rear handles of the adjacent bench:

$$\mathrm{l} = \frac{|\mathrm{k}x_0 - y_0 + c|}{\sqrt{\mathrm{k}^2 + 1}} \tag{10}$$

where $\mathrm{k} = \frac{y_3 - y_2}{x_3 - x_2}$, $c = y_2 - \mathrm{k}x_2$.

3.3. Establishment of the Collision Constraint Threshold

To prevent collisions between adjacent benches on the spiral coil, establish the distance formula from center O to the line formed by the adjacent bench's handles. Ensure this distance is always greater than the set safety threshold $d_{min} = r_1$. Thus:

$$\begin{cases} l > d_{min} \\ l \le d_{min} \end{cases} \tag{11}$$

4. Turn Path Optimization Model for the "Bench Dragon"

4.1. Establishment of the formula for the position of the ' bench dragon ' handle

The pitch changes, preventing the head front handle from coiling out starting from the initial point. A reasonable pitch range needs to be assumed to derive the polar coordinate formula for the head front handle. By testing for collisions when coiling into this circle:

$$\theta_2 = \frac{2r_1\pi}{p} \tag{12}$$

where p is the pitch and r_1 is the turn space radius. Substitute θ_2 into formula (2) to get the head front handle coordinate formula at this point:

$$\begin{cases} x = r_1 cos\frac{2r_1\pi}{p} \\ y = r_1 sin\frac{2r_1\pi}{p} \end{cases} \tag{13}$$

Because the dragon body always maintains a pulling state with the head, differentiate the head front handle coordinate formula to obtain the arc differential formula for the body front handle and tail rear handle coiling out:

$$\frac{1}{2}b_2\left(\theta'_2\sqrt{\theta'_2+1}+ln\left|\theta'_2+\sqrt{\theta'_2+1}\right|\right)-\frac{1}{2}b_2\left(\theta_2\sqrt{\theta_2+1}+ln\left|\theta_2+\sqrt{\theta_2+1}\right|\right)=t_n \tag{14}$$

4.2. Dragon Head turn Curve Optimization Model

In this study, the structure of the current turn curve is first analyzed. The turn space is defined as a circular area centered at the spiral center with a diameter of $2r_1$ meters. The turn curve consists of two tangent circular arcs. Let the radius of the first arc be R, and the radius of the second arc be 2R. The tangent point P is key, ensuring the arcs are tangent at P. To achieve this, let the center of the first arc be O_1 and the center of the second arc be O_2. The tangency condition requires that at point P, $|O_1\text{P}| = \text{R}$ and $|O_2\text{P}| = 2\text{R}$[10].

The length L of the turn curve consists of the lengths of the two arcs:

$$\text{L} = \theta_1 R + \theta_2(2R) \tag{15}$$

where θ_1 and θ_2 are the radian measures of the first and second arcs, respectively. The optimization goal is to adjust the values of R, θ_1, and θ_2 to minimize the curve length L. To maintain the tangency relationship, the distance between the center O_1 of the first arc and the center O_2 of the second arc should be:

$$|O_1O_2| = \text{R} + 2\text{R} = 3\text{R} \tag{16}$$

And the turn curve must be contained within the turn space of $2r_1$ meters.

5. Velocity Optimization Model for the "Bench Dragon" Path

5.1. Determination of the Dragon Head's Initial Position

Assume the minimum travel speed of the dragon head is v_{min}, and the travel speeds of all handles are limited to a certain range. Thus, assume the maximum travel speed of the head is v_{max}. Using the bisection method:

$$v_{mid} = \frac{v_{max} + v_{min}}{2} \tag{17}$$

When the dragon head moves to a point tangent to the spiral and the turn space, from the polar coordinate equation:

$$r_2 = b_3\theta_3 = \frac{p}{2\pi}\theta_3 \tag{18}$$

where $\theta_3 = \frac{2r_1\pi}{p}$.

Assume that when the head coils in from a certain point for t_x seconds, it reaches the tangent point. Establish the arc differential equation for the head at this point:

$$\int_0^{t_x} v_0\, dt = \frac{1}{2} b_3 (\theta\sqrt{\theta^2+1} + \ln\left|\theta + \sqrt{\theta^2+1}\right|) \Big|_{\theta_3}^{\theta_0} \tag{19}$$

From this, the value of θ_0 can be obtained.

5.2. Establishment of Position Equations for the Dragon Body and Tail

Establish the arc differential equations for the body and tail based on their relationship with the head:

$$\int_0^{t_1} v_0\, dt = \frac{1}{2} b_3 \left(\theta\sqrt{\theta^2+1} + 20\right) \ln\left|\theta + \sqrt{\theta^2+1}\right|) \Big|_{\theta_0}^{\theta_4} \tag{20}$$

Solving the system of arc differential equations and position parametric equations for the body and tail allows determining the maximum travel speed of the dragon head.

6. Simulation Verification

6.1. Solving and Analysis of Bench Dragon Handle Position-Velocity

Set the "Bench Dragon" team coiling clockwise along an equiangular spiral with a pitch of 55 cm, with all handle centers located on the spiral. The travel speed of the dragon head's front handle remains constant at 1 m/s. Initially, the dragon head is located at point A on the 16th turn of the spiral, as shown in Figure 1. Solve the positions and velocities of all 223 dragon handles from the initial moment to 300 seconds. Partial calculation results are shown in Table 1 and Table 2.

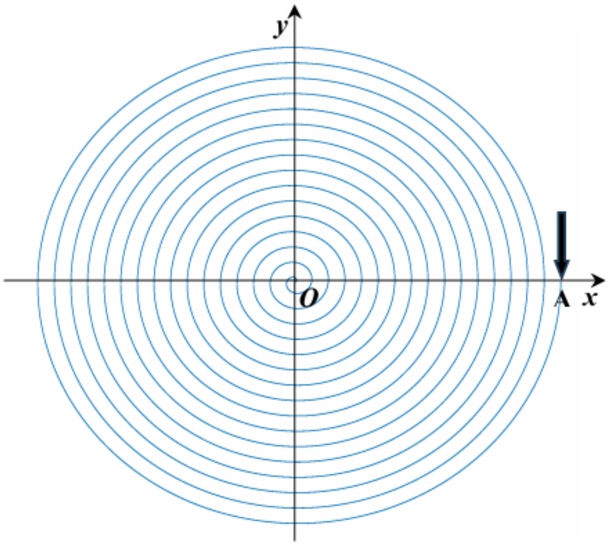

Figure 1. Equiangular Spiral Path Diagram of the "Bench Dragon"

Solve by combining the polar coordinate formula for the head front handle arc differential with the Archimedean parametric equations. Similarly, combine the polar coordinate formulas for the body front handle and tail rear handle arc differentials with the Archimedean parametric equations and the handle velocity equation. Perform recursive calculations sequentially for each handle. The position-velocity model is as follows:

$$\begin{cases} b_1 = \dfrac{55}{2\pi} \\ x = b_1\theta cos\theta \\ y = b_1\theta sin\theta \\ \dfrac{1}{2}b\left(\theta\sqrt{\theta^2+1} + ln\left|\theta + \sqrt{\theta^2+1}\right|\right) = 442.590256 - t_0 \\ \dfrac{1}{2}b\left(\theta_n\sqrt{{\theta_n}^2+1} + ln\left|\theta_n + \sqrt{{\theta_n}^2+1}\right|\right) = 442.590256 - t_n \\ n = 1, 2, 3 \ldots, 223 \\ t_n = t_0 + (341-55) + (n-1)(220-55) \\ v = \sqrt{v_x^2 + v_y^2} \end{cases} \tag{21}$$

Table 1. Dragon Handle Position Results (meters)

Handle	0 s	60 s	120 s	180 s	240 s	300 s
Head x	8.800000	5.799209	4.084887	2.963609	2.594494	4.420274
Head y	0.000000	5.771092	6.304479	6.094780	5.356743	2.320429
1st Body x	8.622662	6.966369	-2.424855	-4.517861	4.131347	3.298456
1st Body y	1.848057	-4.328007	-7.132913	5.083686	-4.322460	3.790644
51st Body x	-9.287429	-8.287606	-4.607061	3.958921	5.205986	-6.045432
51st Body y	2.439659	3.608338	7.069356	6.710003	-4.805352	1.772887
101st Body x	1.725946	4.622168	4.259952	0.575445	-5.944569	-5.275038
101st Body y	-10.181951	-8.647703	-8.215608	-8.649194	-5.412775	5.128983
151st Body x	10.991205	7.651414	3.703487	2.427648	4.362726	7.767234
151st Body y	0.535970	7.214280	9.293563	9.148161	7.748884	2.871407
201st Body x	5.779766	-5.421239	-10.328742	-9.809815	-8.385764	-8.335228
201st Body y	10.105990	9.779723	2.801072	-2.791522	-4.816659	-3.679872
Tail (Rear) x	-6.516921	6.191176	10.974688	8.415401	4.784437	3.561335
Tail (Rear) y	-9.971026	-9.646607	-0.663321	6.289748	8.774122	8.757234

Table 2. Dragon Handle Velocity Results (m/s)

Handle	0 s	60 s	120 s	180 s	240 s	300 s
Head	0.987989	0.988376	1.003551	1.001643	0.998191	1.000484
1st Body	1.003760	0.991998	1.007979	0.997440	1.005170	0.996122
51st Body	0.984434	0.994135	1.001507	1.003199	1.005158	0.994945
101st Body	0.998144	1.005560	1.000264	0.996620	0.973038	1.001152
151st Body	0.995499	0.995017	1.006321	0.986236	1.002408	1.004613
201st Body	1.011088	0.996718	1.004456	0.985461	0.995840	1.004784
Tail (Rear)	0.991975	1.007442	1.003895	0.992510	0.993507	0.989878

6.2. Collision Verification Calculation for the "Bench Dragon"

6.2.1. Determination of the Dragon Head Front Handle Collision Time

The dragon team coils along the spiral set above. If the team moves to the spiral center, the head front handle's coiling path follows formula (6). Collision occurs when $\theta = 0$. Substituting $\theta = 0$ into formula (6) yields the coiling termination time $t_{max} = 442.590256$ seconds. By testing, it is found that no collision occurs at $t_{min} = 300$ seconds.

6.2.2. Solving for Collision-Free Coiling of the "Bench Dragon"

According to formulas (9) and (10), substitute the collision time and collision conditions into the collision constraint model formula (11). Use the bisection method and iterative computation to make t_{mid} infinitely approach the moment $t_{max} = t_{min}$. Stop the iteration when $t_{max} - t_{min}$=10^{-6} seconds, yielding:

$$t_{mid} = t_{min} \tag{22}$$

where r_1 is calculated by the Pythagorean theorem as $r_1 = 0.31$ meters.Use the polar coordinate formula for the body front handle and tail rear handle arc differentials combined with the position parametric equation to obtain the dragon team's position and velocity at this time. Verification shows the collision-free termination time is $t_{mid} = t_{min}$ =379.4008 seconds.

6.3. Dragon Head turn Curve and Shortest Path Solution

6.3.1. Solving for the Minimum Collision-Free Pitch for Coiling Out

Set the "Bench Dragon" transitioning from coiling in to coiling out. The team switches from clockwise coiling in to counter-clockwise coiling out, requiring a turn space. If the turn space is a circular area centered at the spiral center with a diameter of 9 m, as shown in Figure 2, determine the minimum pitch p such that the head front handle can coil along the corresponding spiral to the boundary of the turn space.

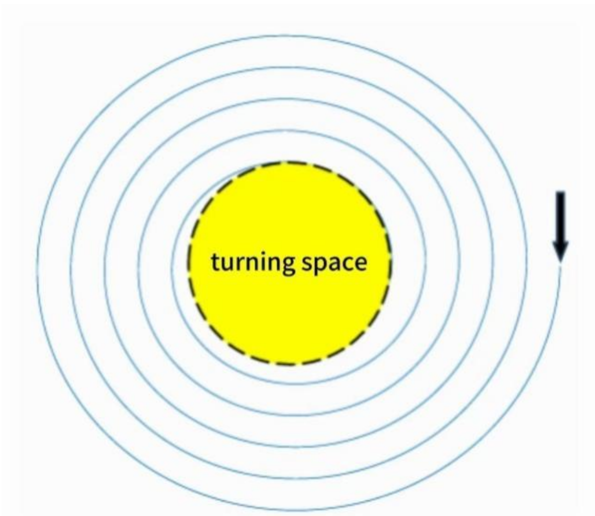

Figure 2. Schematic Diagram of the "Bench Dragon" turn Space

Determine the head front handle's position at any time using formulas (12) and (13):

$$\theta_2 = \frac{9\pi}{p} \tag{23}$$

where $p \in (30,55)$ is the pitch. Substitute θ_2 into formula (2) to get the head front handle coordinate formula:

$$\begin{cases} x = 4.5cos\dfrac{9\pi}{p} \\ y = 4.5sin\dfrac{9\pi}{p} \end{cases} \tag{24}$$

Combine the arc differential formula for the body front handle and tail rear handle coiling out with the position coordinate equations to obtain their positions.

$$\frac{1}{2}b_2\left(\theta'_2\sqrt{\theta'_2+1} + ln\left|\theta'_2 + \sqrt{\theta'_2+1}\right|\right) - \frac{1}{2}b_2\left(\theta_2\sqrt{\theta_2+1} + ln\left|\theta_2 + \sqrt{\theta_2+1}\right|\right) = t_n \tag{25}$$

Since the head's initial turn position on the turn space circle is already determined, at this point:

$$t_n = (341 - 55) + (n - 1)(220 - 55) \tag{26}$$

Based on collision constraint conditions and the threshold, using the bisection method and iterative computation, stop iteration when $p_{max} - p_{min} = 10^{-6}$ to find the minimum pitch $p = p_{min} = 0.52871\text{m}$.

6.3.2. Solving for the Shortest Path of the Dragon Head turn Curve

Set the pitch of the coiling-in spiral to 1.7 m. The coiling-out spiral is centrally symmetric to the coiling-in spiral about the spiral center. The dragon team completes the turn within the defined turn space. The turn path is an S-shaped curve composed of two tangent circular arcs. The radius of the first arc is twice that of the second. It is tangent to both the coiling-in and coiling-out spirals. By adjusting the arcs while maintaining tangency, solve for the shortest turn path.

According to formulas (15) and (16), adjust the value of R within a certain range. Verified, it is reasonable to assume $R = [1, 3]m$ and $\theta_1 = \theta_2 = [0, \pi]$. This paper adopts a grid search method to traverse all combinations of R, θ_1, and θ_2. For each parameter set, calculate the turn curve length L. Record the curve length for each set and find the minimum length and its corresponding parameter combination: L_{min}=14.1372m, achieving the shortest path optimization for the turn curve.

6.4. Solving the "Bench Dragon" Path Velocity

In the bench dragon performance, it is very important to determine the maximum speed of the dragon head, which not only ensures that the speed of each handle does not exceed the limit, avoids the dancer 's out of control due to excessive centrifugal force or the disconnection of the dragon body, but also allows the dragon shape to maintain a smooth curve when marching and turning, fits the rhythm, and at the same time, unifies the rhythmic beauty and safety of traditional folk performances. Set the travel speed of the dragon team's head front handle to remain constant at 1 m/s. Determine the maximum travel speed of the dragon head such that the speed of all handles does not exceed 2 m/s.

Assume the minimum travel speed of the head is $v_{min} = 1m/s$. Stipulate that the travel speed of all handles does not exceed 2 m/s. Therefore, assume the maximum travel speed of the head is $v_{min} = 3m/s$.

When the head moves to the tangent point between the spiral and the turn space, from the polar coordinate equation:

$$r_2 = b_3\theta_3 = \frac{1.7}{2\pi}\theta_3 \tag{27}$$

where $\theta_3 = \frac{9\pi}{1.7}$.

Assume the head coils in from a certain point for 20 seconds to reach the tangent point. Establish the head arc differential equation at this point:

$$\int_0^{20} v_0\, dt = \frac{1}{2} b_3 \left(\theta\sqrt{\theta^2 + 1} + \left|\theta + \sqrt{\theta^2 + 1}\right|\right)\Big|_{\theta_0}^{\theta_4} \tag{28}$$

Differentiating the position parametric equation obtained by combining the arc differential equations and position parametric equations for the body and tail yields the head's velocity. When:

$$\begin{cases} v_{mid} > 2 & v_{max} = v_{mid} \\ v_{mid} \le 2 & v_{min} = v_{mid} \end{cases} \tag{29}$$

where $t_1 = 20 + (341 - 55) + (n - 1)(220 - 55)$. Use the bisection method and iterative computation to find the head's maximum travel speed. Stop iteration when $v_{max} - v_{min} = 10^{-12}$, yielding the head's maximum travel speed $v = 1.957533m/s$.

7. Conclusions

Simulation verification demonstrated that the model can precisely calculate the position and velocity distribution of a 223-section dragon body from the initial state up to 300 seconds. To address collision constraints, a safety distance verification model was constructed within the polar coordinate space. Combining the bisection method and an iterative algorithm, the critical coiling time was solved, determining the coiling termination time at 379.4008 seconds, effectively preventing body collisions. For the turning space constraint, the minimum safe spiral pitch of 0.52871 meters was determined through parameter optimization. Based on a two-segment tangent circular arc model, the turning curve was optimized, achieving a shortest path reduction of 14.1372 meters. The velocity optimization model employed the bisection method to constrain the dragon head's speed range, ultimately determining the maximum safe travel speed as 1.957533 m/s.

The research established a closed-loop framework of "geometric modeling-constraint verification-algorithm optimization". This framework pioneers the quantitative dynamic analysis of folk performance movements, providing a path planning solution for bench dragon performances that balances scientific rigor and cultural adaptability. The results not only enhance the coordination and spectacle of the dragon dance performance but also advance the integration of traditional intangible cultural heritage activities with modern optimization algorithms. This work provides quantifiable technical support and a scientific analytical paradigm for the innovative preservation of intangible cultural heritage.

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