Research On Optimal Allocation of Hu Sheep Breeding Based on Stochastic Difference Equations

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Abstract. This study addresses the dynamic resource allocation problem in large-scale Hu sheep farming by establishing a production cycle model based on stochastic difference equations. Through systematic analysis of seven production stages (mating period, pregnancy confirmation, etc.), we develop a population dynamic system incorporating multi-dimensional state variables. Three critical stochastic factors—conception success rate, gestation duration variance, and lambing number variation—are innovatively integrated using composite probability models for stochastic process characterization. An improved Genetic Algorithm (GA) is employed to optimize multiple parameters including ram-to-ewe ratio, lactation duration, and sheep pen allocation, constructing a nonlinear programming model with dynamic pen utilization cost functions and production constraints. Simulation experiments demonstrate that optimized key production parameters significantly reduce spatial utilization costs while maintaining annual yield targets, validating the model's effectiveness in balancing production efficiency and cost control. This research provides a decision-making framework for large-scale breeding operations through synergistic optimization of stochastic process modeling and intelligent algorithms, offering substantial practical guidance value.

Keywords: Difference equations, Hu sheep breeding, Genetic Algorithm, Stochastic processes.

1. Introduction

As a distinctive livestock industry in China, Hu sheep breeding has progressively transitioned toward large-scale intensive production under the national Rural Revitalization Strategy [1]. Hu sheep, renowned for their multiparous trait, roughage tolerance, and strong adaptability, have become the preferred breed for industrialized farming in southeastern regions. However, the expansion of farming scale has amplified dynamic disparities in resource demands across production cycle phases (e.g., mating, gestation, lactation), resulting in periodic fluctuations in infrastructure utilization efficiency-particularly sheep pen occupancy-that directly impact spatial allocation costs [2]. Empirical studies reveal that traditional fixed-resource allocation models fail to adapt to temporal production dynamics, with pen vacancy rates peaking above 30%, substantially inflating fixed asset amortization costs per unit output [3]. Accurately quantifying stochastic production cycle impacts on resource occupancy and establishing dynamic optimization strategies have thus emerged as critical challenges for cost-effective large-scale farming.

Existing research on Hu sheep breeding predominantly focuses on genetic improvement (e.g., lambing rate enhancement) and feeding management optimization, while insufficient attention has been paid to cross-disciplinary studies integrating production system dynamics and resource allocation [4-5]. Although difference equations have been widely applied in biosystem modeling—including population dynamics simulations (e.g., epidemic transmission models) and growth process predictions-their applicability to multi-stage stochastic modeling in livestock production chains remains underexplored [6-7]. Furthermore, while agricultural engineering studies address spatial layout optimization in breeding facilities, prevailing methodologies rely on steady-state assumptions, neglecting coupled analysis of production cycle temporal fluctuations and stochastic events (e.g., conception failure, lambing number variation), thereby compromising solution robustness in real-world dynamic scenarios [8-9].

This study proposes an integrated optimization framework combining stochastic process modeling with dynamic programming for production cycle resource allocation. First, we deconstruct the sevenphase Hu sheep production chain to develop a stochastic difference equation model incorporating mating success rates, gestation duration variance, and other random factors, thereby characterizing dynamic population evolution and pen occupancy patterns. Second, we formulate a nonlinear optimization model with objectives of minimizing sheep pen leasing costs under annual production scale constraints, implementing multi-parameter coordination (ram-to-ewe ratio, lactation duration, and proprietary pen allocation) via an enhanced Genetic Algorithm. By introducing a lactation-fattening dynamic compensation mechanism, our approach effectively reduces non-productive resource retention costs, providing temporally adaptive and economically viable decision support for Hu sheep farms. Simulation experiments validate the model's optimization efficacy in stochastic dynamic environments [10].

2. Dynamic System Theoretical Framework for Hu Sheep Production Cycle

2.1. Production Cycle Characteristics of Large-Scale Hu Sheep Farming

Hu sheep, a unique Chinese indigenous sheep breed, exhibit distinct phased characteristics and biological rhythms in industrialized farming. As shown in Figure 1, the production cycle of Hu sheep is decomposed into seven interconnected stages: Non-Mating Period, Mating Period, Pregnancy Identification, Gestation Period, Lactation Period, Recovery Period (for non-pregnant ewes), and Lamb Fattening Period. These stages form a coupled system influenced by multiple stochastic factors.

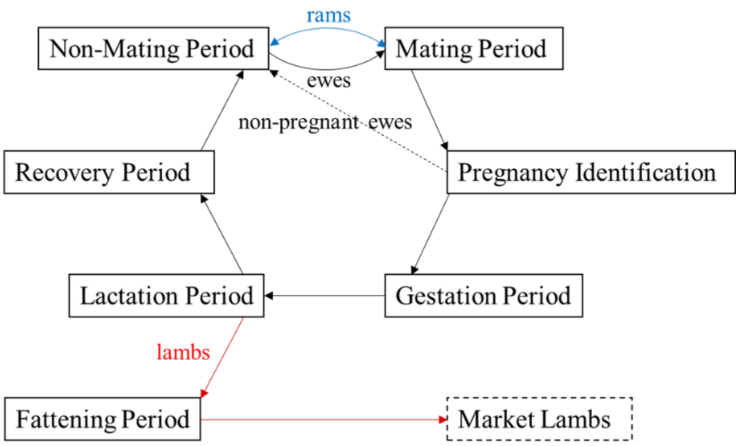

Figure 1. Production Cycle of Hu Sheep

Standard sheep pen specifications in Hu sheep farms are configured as follows: Recovery Period (for non-pregnant ewes) and Pregnancy Identification (for pregnant ewes): Up to 14 base ewes per pen. Non-Mating Period (rams): Up to 4 breeding rams per pen. Natural Mating Period: 1 breeding ram and up to 14 base ewes per pen. Gestation Period: Up to 8 pregnant ewes per pen. Lactation Period (postpartum): Up to 6 lactating ewes and their lambs per pen. Fattening Period: Up to 14 lambs per pen.

To simplify the model, maternal abortion, mortality, lamb mortality during lactation/fattening, and individual growth rate variations are excluded. Three non-negligible stochastic perturbations are identified:

(1) Conception Success Rate: A conception probability of 85% per mating cycle, resulting in approximately 15% of ewes entering the recovery queue per batch.

(2) Gestation Duration Variance: A 3-day uncertainty window in predicted parturition dates, complicating pen scheduling.

(3) Lambing Number Variation: Typical parturition yields 2 lambs per ewe, with occasional cases of 1 or ≥3 lambs. Current methods lack practical control or pre-detection of litter sizes. Neonatal lamb mortality averages 3%, yielding an estimated mean of 2.2 lambs per birth.

2.2. Mathematical Characterization of Hu Sheep Breeding Dynamics

Considering rams, ewes, lambs, and their production stages, the Hu sheep population is classified into 10 states. The population size of each state is defined by state functions $x_i(t)$ $(i = 1,2,\cdots,10)$, The data on the production cycle of Hu sheep breeding and sheep pen usage presented in this study are sourced from Reference [11], with their definitions, spatial occupancy, and durations detailed in Table 1:

Table.1. Correspondence Table of State Function Definitions and Hu Sheep Production Cycle

State Function	Production Stage	Standard Pen Capacity	Cycle Duration
$x_1(t)$	Non-Mating Period (rams)	4	arbitrary
$x_2(t)$	Non-Mating Period (ewes)	14	arbitrary
$x_3(t)$	Mating Period (rams)	1	20
$x_4(t)$	Mating Period (ewes)	14	20
$x_5(t)$	Pregnancy Identification	8	30
$x_6(t)$	Gestation Period (ewes)	8	150~127
$x_7(t)$	Lactation Period (ewes)	6	$T_{Lactation}$: 35~45
$x_8(t)$	Lactation Period (lambs)	6	
$x_9(t)$	Fattening Period (lambs)	14	$210 + 2(40 - T_{Lactation})$
$x_{10}(t)$	Recovery Period (ewes)	14	18

Where $T_{Lactation}$ denotes the lactation duration, selectable within 35–45 days. The population state of the Hu sheep breeding dynamic system can be described by a state vector composed of these functions:

$$\boldsymbol{x}(t) = [x_1, x_2, \cdots, x_{10}]^T \tag{1}$$

This study simulates the large-scale Hu sheep breeding system by developing kinetic evolution equations for the state functions $\boldsymbol{x}(t)$, enabling quantitative investigations into breeding configuration optimization.

2.3. Dynamic Programming Framework for Multi-Parameter Breeding Optimization

After lambs complete the fattening period, the farm will sell them. Let the number of lambs sold be denoted by the function $x_e(t)$, The annual lamb output is calculated as $X_e = \sum_{t=1}^{365} x_e(t)$. For the cost calculation of pen usage, this study assumes that the daily cost per self-owned standard pen is 1 unit. When self-owned pens are insufficient, temporary pens can be rented at a daily cost of 3 units per pen, based on empirical data.

The Key system parameters include: Number of rams x_0, Number of ewes y_0, Number of proprietary pens P_0, Lactation duration (35–45 days) $T_{Lactation}$. This study focuses on cost control under the constraint of achieving annual production targets. Taking an annual production target $X_{Plan} = 1500$ lambs/year as an example, we formulate the following optimization problem to determine the optimal configuration:

$$(Objective\ Function)\ \ min \quad cost\ (x_0, y_0, T_{Lactation}, P_0) \tag{2}$$

$$s.t.(Constraints)\begin{cases} 9 \le x_0 \le 16,\ 270 \le r_0 \le 800,\ 35 \le T_{Lactation} \le 45,\ 130 \le P_0 \le 160 \\ X_e \ge 1500 \end{cases} \tag{3}$$

The parameter ranges are selected to be sufficiently broad to encompass potential optimal solutions. Let $P_n(t)$ represent the daily number of pens used. The daily pen usage cost is defined as:

$$cost(t) = \begin{cases} 3P_n(t) - 2P_0 & (P_n(t) > P_0) \\ P_n(t) & (P_n(t) \leq P_0) \end{cases} \tag{4}$$

The annual pen usage cost is then computed as $cost_{year} = \sum_{t=1}^{365} cost(t)$.

Following the pen usage specifications described in Section 2.1, $P_n(t)$ is calculated as:

$$P_n(t) = \left\lceil \frac{x_1(t)}{4} \right\rceil + \left\lceil \frac{x_2(t)}{14} \right\rceil + \left\lceil \frac{x_4(t)}{14} \right\rceil + \left\lceil \frac{x_5(t)}{8} \right\rceil + \left\lceil \frac{x_6(t)}{8} \right\rceil + \left\lceil \frac{x_7(t)}{6} \right\rceil + \left\lceil \frac{x_9(t)}{14} \right\rceil + \left\lceil \frac{x_{10}(t)}{14} \right\rceil \tag{5}$$

To accurately estimate production costs, a simulation system must be developed to: 1. Predict the population state evolution $\boldsymbol{x}(t)$ under parameter combinations $(x_0, y_0, T_{Lactation}, P_0)$; 2. Forecast daily pen usage $P_n(t)$; 3. Calculate daily spatial costs. Subsequently, intelligent optimization algorithms are employed to identify the optimal parameter combination $(x_0, y_0, T_{Lactation}, P_0)$, This dual approach integrates dynamic system modeling with computational optimization to address the complex trade-offs between production efficiency and cost control in large-scale Hu sheep farming operations.

3. Model Construction of Stochastic Difference Equations

3.1. Fundamental Framework Design

To establish a state transition model for the Hu sheep production cycle, this study employs difference equations to characterize inter-state conversion relationships. Using days as the basic temporal unit for numerical simulation, we define the first-order difference of state functions to represent daily population increments:

$$\Delta x_i(t) = x_i(t) - x_i(t-1) \tag{6}$$

The increment can also be expressed as:

$$\Delta x_i(t) = x_i^{in}(t) - x_i^{out}(t) \tag{7}$$

Where $x_i^{in}(t)$ denotes the number of sheep entering state i from $t-1$ to t, and $x_i^{out}(t)$ represents those exiting state i. Assuming a deterministic duration T_i for state i, the outflow satisfies:

$$x_i^{out}(t) = x_i^{in}(t - T_i) \tag{8}$$

For sequential state transitions (e.g., gestation period following mating), the inflow-outflow relationship becomes:

$$x_i^{in}(t) = x_{i'}^{out}(t) = x_{i'}^{in}(t - T_{i'}) \tag{9}$$

Combining Equations (6)-(9), we derive the difference equation system:

$$\begin{cases} x_i(t) = x_i(t-1) + x_{i'}^{in}(t - T_{i'}) - x_i^{in}(t - T_i) \\ x_i^{in}(t) = x_{i'}^{in}(t - T_{i'}) \end{cases} \tag{10}$$

By restructuring the state vector as $\boldsymbol{x}'(t) = [x_3, x_{10}, x_1, x_2, x_4, x_5, x_6, kx_6, x_8, x_7]^T$, (where k represents lambing rate), Equation (10) can be vectorized:

$$\begin{cases} \boldsymbol{x}(t) = \boldsymbol{x}(t-1) + \boldsymbol{x}'^{in}(t - \boldsymbol{T}') - \boldsymbol{x}^{in}(t - \boldsymbol{T}) \\ \boldsymbol{x}^{in}(t) = \boldsymbol{x}'^{in}(t - \boldsymbol{T}') \end{cases} \tag{11}$$

This formulation enables iterative numerical simulation of the dynamic system.

3.2. Stochastic Process Integration

To address real-world randomness in the production cycle, we integrate three stochastic factors into the framework:

(1) Conception Probability Model

Based on the model assumptions in Section 3.1, let $x_5^{out}(t)$=$x_5^{in}(t-T_5)$ denote the number of ewes completing the pregnancy confirmation period at time t. Each ewe has a conception probability $P_{conception}=85\%$. The probability $P(x)$ that exactly x ewes ($x \in [0, x_5^{out}]$) successfully conceive follows a binomial distribution:

$$P(x)=C(x_5^{out},x)P_{conception}^{x}\left(1-P_{conception}\right)^{x_5^{out}-x} \tag{12}$$

Where $C(x_5^{out},x)$ represents the binomial coefficient. In computational implementation, we employ stochastic sampling to generate a random integer $x_6^{in}\in[0,\ x_5^{out}]$ conforming to Equation (12). This integer quantifies the number of successfully conceived ewes at time t. This stochastic process is formalized as $Model_{conception}$, yielding the conception count:

$$x_6^{in}(t)=Model_{conception}\left(x_5^{in}(t-T_5)\right) \tag{13}$$

Ewes failing to conceive re-enter the production cycle for subsequent mating arrangements.

(2) Gestation Duration Variability Model

Due to the fluctuation of actual parturition dates between 127 and 150 days after entering the gestation period, the number of ewes giving birth at time t correlates with those entering gestation between $t-150$ and $t-127$. For ewes entering gestation at $t-d(d\in[127,150])$, the probability P of parturition at time t is defined as:

$$P=\frac{d-126}{24} \tag{14}$$

In computational implementation, stochastic sampling is employed to generate random integers conforming to this probability distribution. These integers represent the number of ewes that entered gestation at $t-d$ and give birth at t. This stochastic process is formalized as $Model_{childbirth}$. The total number of parturient ewes at t is calculated as:

$$x_7^{in}(t)=\sum_{d=127}^{150}Model_{childbirth}\left(x_6'(t-d)\right) \tag{15}$$

Where $x_6'(t-d)$ denotes the number of ewes that have been in gestation for d days but have not yet delivered. Post-parturition, these ewes and their lambs enter the lactation period.

(3) Lambing Number Variability Model

The number of lambs per delivery exhibits stochastic fluctuations, with neonatal mortality considered. Empirical data indicate a mean of 2.2 lambs per delivery and a 3% mortality rate. Simplifying assumptions include: 20% probability of singleton births, 40% probability of twins and 40% probability of triplets.

This configuration ensures the mean lambs per delivery equals 2.2. Given $x_7^{in}(t)$ parturient ewes from $Model_{childbirth}$, the total lambs $x(x\in[x_7^{in},3x_7^{in}])$ follows the probability distribution:

$$P(x)=\sum\frac{x_7^{in}!}{a!\,b!\,c!}\cdot 0.2^{a}\cdot 0.4^{b}\cdot 0.4^{c} \tag{16}$$

with constraints:

$$a+b+c=x_7^{in} \tag{17}$$

$$a+2b+3c=x \tag{18}$$

Combining Equations (15)– (18), the probability mass function becomes:

$$P(x) = \sum_{a=max\left(0,2x_7^{in}-x\right)}^{\frac{3x_7^{in}-x}{2}} \frac{x_7^{in}!}{a!\left(3x_7^{in}-x-2a\right)!\left(x+a-2x_7^{in}\right)!} \cdot 0.2^{a} \cdot 0.4^{x_7^{in}-a} \quad (19)$$

In computational implementation, Monte Carlo sampling generates random integers $x_8^{in} \in [x_7^{in}, 3x_7^{in}]$ conforming to Equation (19). This stochastic process is formalized as $Model_{newborn}$, yielding:

$$x_8^{in}(t) = Model_{newborn}\left(x_7^{in}(t)\right) \quad (20)$$

This study develops a stochastic differential equation model integrating three key factors—reproductive cycles, environmental stressors, and husbandry management—to enhance livestock population dynamics prediction. The framework demonstrates three advantages: 1) A data-driven probability distribution mechanism dynamically characterizes stochastic variables; 2) Monte Carlo simulations enable multi-factor interaction analysis; 3) Modular architecture permits flexible incorporation of new stochastic elements. Its innovation lies in reconciling computational efficiency (through difference equations) with biological realism (via probability density function-embedded stochastic terms). This hybrid approach, combining Bayesian inference and numerical simulation, provides a paradigm-shifting tool for analyzing complex biological systems with inherent uncertainties, offering both theoretical robustness and practical applicability.

4. Optimization Methodology and Experimental Results

4.1. Genetic Optimization Algorithm

For the optimization problem defined by Equations (2) and (3), which contains nonlinear constraints, heuristic optimization algorithms must be employed for solving. This study employs a single-objective genetic algorithm to address the optimization problem [12]. The workflow of the genetic algorithm utilized in this study is illustrated in Fig. 2. The genetic algorithm initiates with population initialization, followed by fitness evaluation incorporating an adaptive penalty mechanism to guide solution quality. Selected individuals then undergo crossover and mutation operations before updating the population, forming an iterative cycle that continuously refines solutions. This process repeats until meeting predefined termination criteria, at which point the optimization results are output. The streamlined workflow ensures efficient convergence through dynamically balanced exploration and exploitation across successive generations.

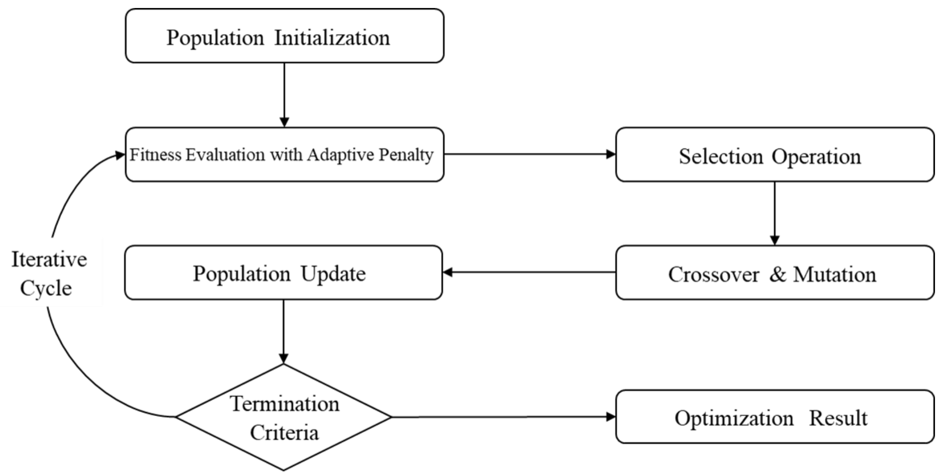

Figure 2. Flowchart illustrating the genetic algorithm implementation process

4.2. Experimental Settings and Optimization Results

Following the optimization problem defined in Equations (2) and (3), this study configures the parameter lower bounds as lb= [9,270,35,130], upper bounds as ub= [16,800,45,160], (Note: Likely a typo; upper bounds should be ub), population size of 16, maximum iterations of 200 generations, adaptive penalty coefficient $\lambda = 100.0$ to enforce the annual lamb production constraint, random crossover rate $\alpha = 0.8$, and mutation rate $p_m = 0.05$.

For individual fitness function evaluation, numerical simulations span 20 years, with annual spatial utilization costs computed using Equations (4) and (5). The optimization trajectory of the objective function (spatial utilization cost) throughout the evolutionary process is depicted in Fig. 3:

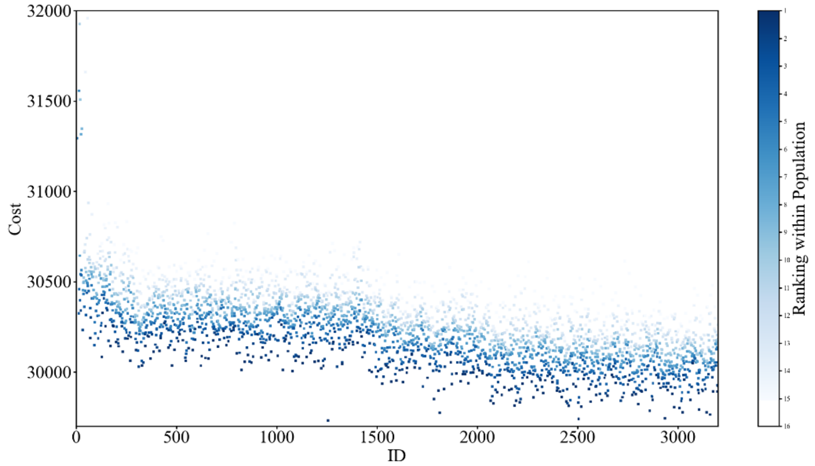

Figure 3. exhibits the cost function's monotonic decline

As the optimization progresses, the spatial utilization cost function continuously decreases, demonstrating the effectiveness of the optimization methodology employed in this study. The solution with the minimal annualized cost derived from the optimization experiments is presented in Table 2.

Table.2. Optimal Solution Obtained by Genetic Algorithm and Its Simulation Results

$(x_0, y_0, T_{Lactation}, P_0)$	$cost_{mean}$	$\overline{X_e}$
(9,423,35,136)	29852	1504

The optimization results indicate that reducing the lactation period duration contributes to spatial cost reduction. A 10,000-day numerical simulation of the optimal solution obtained from experiments was conducted, and the resulting variation curves of pen occupancy by Hu sheep across production cycles are illustrated in Fig. 4:

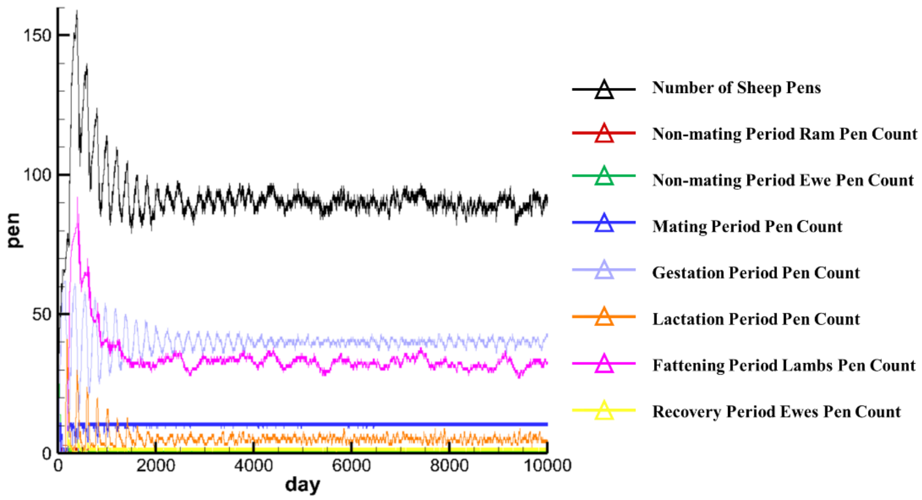

Figure 4. Numerical Calculation-Based Variation Curve of Sheep Pen Utilization Number

From the above production schedule curves, it is evident that the number of pens utilized across various states fluctuates around a stable baseline value. As shown in the figure, the stochastic process-based difference equation model developed in this study accurately simulates the impact of stochastic factors on Hu sheep production planning and predicts the range of production schedule fluctuations induced by these factors, thereby providing data-driven decision support for farming operations. To ensure operational continuity, farms must allocate pen quantities tailored to each production cycle and maintain reserve pens to address potential fluctuations.

5. Conclusions

This study develops a dynamic modeling framework integrating biological stochasticity and engineering optimization, employing stochastic differential equations to characterize key variables in Hu sheep production cycles—including conception rates, gestation duration variability, and lambing number fluctuations. By coupling an enhanced genetic algorithm with multi-parameter collaborative optimization, the proposed dynamic compensation mechanism effectively coordinates resource competition across production phases, demonstrating the decision-making advantages of integrating stochastic modeling and intelligent optimization.

Three key innovations emerge: (1) an interdisciplinary analytical framework bridging biological processes and engineering management, overcoming limitations of traditional steady-state models; (2) a modular stochastic differential equation system providing a universal modeling tool for multifactor interaction analysis in livestock systems; (3) a dynamic response optimization strategy that balances production volatility and economic efficiency.

The findings hold triple application value: the framework is extensible to intensive breeding optimization for multiparous livestock (e.g., cattle and swine); integration with IoT-enabled data streams could establish digital twin systems for dynamic scheduling; and the methodology offers quantitative support for governmental elastic production planning and risk mitigation strategies. Future research will focus on multi-objective optimization under environmental constraints, advancing theoretical innovation and practical transformation toward sustainable livestock industry development.

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