

Modeling And Analysis of Motion Path and Collision Detection for The Bench Dragon Using Spiral Equations and Differential Algorithms

Wanxin Nie ^{*, #}, Jingyu Zhao [#]

School of Intergrated Circuits, Shandong University, Shandong, China, 250101

* Corresponding Author Email: 202200400095@mail.sdu.edu.cn

[#]These authors contributed equally.

Abstract. This study establishes a model for the motion path and collision detection of the bench dragon by analyzing spiral and motion trajectories, describing the system's motion characteristics through mathematical models and differential equations, and verifying the model's reliability. Given the current lack of systematic research in this area, this paper pioneers a relevant model for detailed analysis. Initially, a differential equation model for θ and t was developed based on the spiral equation and polar coordinate arc length formula, with the improved Runge-Kutta algorithm used for iterative solving to determine connection point positions and finite difference methods applied for velocity calculations. The Separating Axis Theorem was then introduced for collision detection, and a function relating pitch to turning space was initialized, with traversal of possible pitches and step lengths revealing a minimum collision pitch of 0.452m. Finally, geometric methods derived the positions and velocities of connection points on the shortest path, demonstrating that the turning path length is fixed and unoptimizable. The model successfully generated motion paths, collision times, and minimum pitch under varying conditions, with results visualized in images and tables, effectively capturing the bench dragon's motion characteristics while ensuring reliability through numerical validation and error analysis. Future work may focus on algorithmic optimization to enhance computational efficiency.

Keywords: Bench Dragon, Collision Detection, Spiral Equation, Runge-Kutta Algorithm.

1. Introduction

In Chinese folk culture, the bench dragon, originating from Zhejiang-Fujian, is a unique traditional art. Composed of 223 connected benches, it forms a dynamic structure with a dragon head and tail. Its circular and spiral movement creates a fascinating visual effect. The performance demands high choreography and spatial coordination. To achieve smooth movement and minimal space use, this paper models the dragon's path as an equidistant spiral. Through mathematical modeling and analysis, it aims to determine the spiral's minimum pitch for optimized fluidity and calculate coordinates for seamless transitions.

This research is significant as it enhances understanding of the bench dragon's complexity and technical aspects, and provides a scientific basis for protecting and innovating this intangible cultural heritage. By simulating and analyzing the dragon's movement and collision conditions, it offers scientific support for actual performances, improving their safety and ornamental value. However, systematic research on the bench dragon's movement path is currently lacking, making its scientific modeling and analysis a challenging task..

This study focuses on the modeling of the bench dragon's movement path and collision detection. By establishing precise mathematical models, it delves into the movement characteristics of the bench dragon. Through the analysis of spiral equations, the research aims to reveal the geometric shape and dynamic laws of the bench dragon. Furthermore, the study also explores how to predict and avoid potential collisions during complex movements using mathematical methods, which is crucial for ensuring the safety and smoothness of the performance [1].

During the research process, various mathematical tools and theories are employed, including spiral equations, differential equations, and geometric methods, in order to comprehensively describe

and analyze the movement trajectory of the bench dragon. It is hoped that through in-depth research on these mathematical models, this study can provide more scientific guidance for bench dragon performances and offer new perspectives and methods for related research fields.

In conclusion, by conducting in-depth analysis on the modeling of the bench dragon's movement path and collision detection, this research not only helps us better understand and appreciate this traditional art form but also provides strong support for the preservation and cultural inheritance of intangible cultural heritage. Future research is expected to further refine and enhance these models and methodologies to improve computational efficiency and application effectiveness.

2. The basic fundamental analysis of the target to be constructed

By establishing a Cartesian coordinate system, the relevant equations for the equidistant Archimedean spiral can be derived, expressed as a function of time θ . The coordinates of the dragon head position are obtained using the Runge-Kutta method. Subsequently, we apply the forward difference method, central difference method, and backward difference method to compute the specific velocities at each point, iteratively gathering and storing the data for designated points [2].

Next, we utilize the Separating Axis Theorem to determine collision conditions. To ascertain the final collision time, a flag variable is introduced for collision traversal detection. By systematically varying the pitch from small to large values and utilizing the spiral equation, we can calculate the tangent equations for the entry and exit points, thereby clarifying the tangent equation for the inner circular arc [1].

Initially, a genetic algorithm was intended for determining the most suitable turning space structure. However, practical progress revealed that only one optimal scenario exists, leading us to conclude the use of geometric computation to determine the arc length within the turning space.

In the computational process, we establish the following assumptions, as shown in Fig 1:

- ① The connection points are assumed to perfectly trace the spiral path along the center of the spiral line during movement.
- ② The effects of velocity transmission due to friction at the connection points are neglected.
- ③ Variations in the pitch of the equidistant spiral caused by human factors in practical scenarios are disregarded.
- ④ Differences in height between long boards resulting from interleaving connection points are ignored.

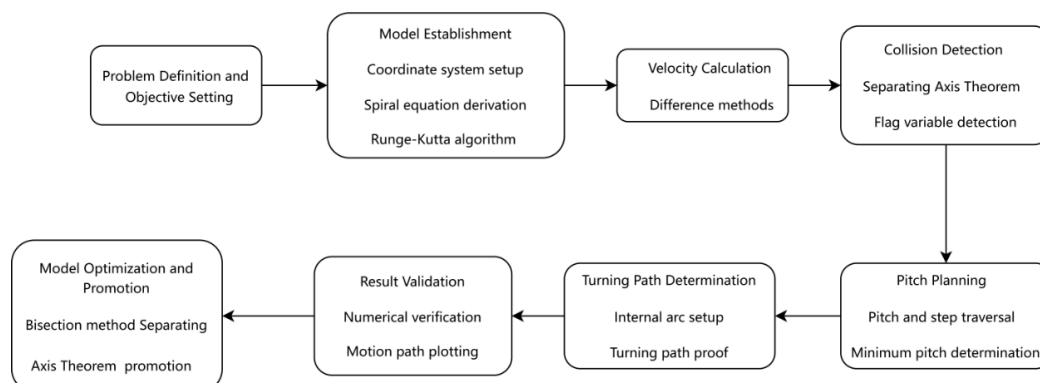


Fig 1 Modeling Flowchart

First, a series of key parameters are defined in the code, including the pitch, the distance between the head and the body of the dragon, and the distance between two holes of the head and other seat handles. Next, the evolution of the position of the first handle of the head over a duration of 300 seconds is analyzed. Using the differential equations obtained during the modeling process for the angle (θ) and time (t) , the Runge-Kutta method is applied to iteratively solve for the angle corresponding to the position of the head at each time point, thus deriving the spatial position at each

moment and ultimately obtaining the trajectory of the center of the connection point at the front of the head [3].

Furthermore, we need to determine the position of a hole located behind the head seat (also positioned along the spiral) at each time point, as well as the positions of the holes on the dragon's body and tail seats. To achieve this, we define X and Y arrays that store the coordinate values of the front handle holes of each seat at various time points, along with the corresponding angles for these holes at each moment.

During this process, logical conditions are implemented to filter for points that meet the criteria: if the computed angle is less than the angle corresponding to the previous hole, or if the newly computed hole does not lie on the same spiral, a new angle must be recalculated until the conditions are satisfied—that is, the point must lie on the same spiral as the previous hole and have an angle greater than that of the previous hole.

A flag variable is introduced within the loop to indicate whether a collision has occurred. If flag = 0, it signifies that no collision has occurred in the current state, allowing the system to continue progressing [4]. Subsequently, we need to determine the positions of each hole at each time point to assess whether any contact has taken place between them. To achieve this, a subfunction is called to perform collision detection.

This subfunction implements the separation axis theorem, utilizing the normals of the two plates for coordinate transformation and overlap assessment. If a collision is detected, the value of flag is reset to 1, thereby interrupting the loop to identify the moment of collision.

We start by clearly defining the initial position of the first handle of the head under the current pitch conditions. This position is assumed to be an initial point, and the values for pitch and step length are initialized. Next, we will iterate through each possible combination of pitch and step length, employing the methods outlined in the previous research, along with the flag variable and the separation axis theorem, to determine the minimum angle θ corresponding to the collision of the front handle of the head [1][4].

After completing the iteration, we will plot the relationship curve between the updated pitch and the minimum θ obtained, and compare it with the theoretical curve representing the collision at the boundary of the turning space of the front handle [5]. The intersection point of the two curves will indicate the pitch corresponding to the minimum required pitch, ensuring both accuracy in calculations and efficiency in execution.

3. Results

In this study, the minimum pitch value for the head to collide at the critical boundary of the turning space is determined to be $r=0.452m$. To present this result more intuitively, a line graph is utilized. The x-coordinate of the intersection point of the two lines corresponds to the desired minimum pitch value [6]. In the Fig 2, the red line represents the relationship between the angle and pitch at the intersection of the circumference and the spiral line, while the blue line indicates the minimum angle the connection point of the head can achieve under any current pitch condition.

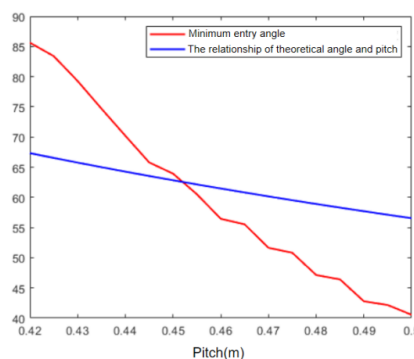


Fig 2 Pitch

In order to obtain the coordinates of each bench segment at different moments, it is necessary to determine the overall trajectory of the bench dragon. The determination of this trajectory is a highly significant and complex issue.

Based on the aforementioned issues, the coordinates of the entry point have been defined, and the line segment AB is established as the diameter of the turning space circle [7]. By drawing tangents l1 and l2 to the entry spiral and exit spiral from AB, these tangent equations also serve as the tangent equations for the internal turning arc, as shown in Fig 3.

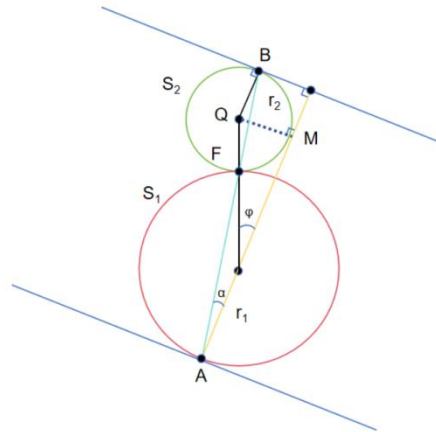


Fig 3 Internal arc diagram of the turning space

It can be observed that α is a constant value. At this point we can obtain the expression S_1+S_2 as follows.

$$S_1 + S_2 = (r_1 + r_2)(\pi - 2\alpha) \tag{1}$$

We also need to determine the value of $r_1 + r_2$, which we can find using the following formula:

$$\begin{aligned} r_1 + r_2 &= \frac{2R_T \sin \alpha}{\sin \varphi} \\ &= \frac{2R_T}{2\cos \alpha} \end{aligned} \tag{2}$$

It follows that S_1+S_2 is a definite value.

In the drawing of the solution diagram, the first curve is the disc inlet curve, the second curve is the arc C1 segment, the third curve is the arc C2 segment, and the fourth curve is the disc outlet spiral symmetric with the center of the disc inlet spiral.

In the time interval of -100s to 0s, that is, before the tap handle just reaches the tangent point between the disc inlet curve and the C 1 section of the arc, the differential equation can be solved by using the differential equation [3][8]. When solving the position of the tap handle on the disc outlet spiral, the differential equation can also be established by using this principle, but because the disc outlet spiral and the disc inlet spiral are center symmetric, the equation of the disc outlet spiral becomes $r = k(\theta + \pi)$ [5][7]. Then the differential equation for the position of the faucet handle established on the disk-exit spiral becomes as follows:

$$\frac{d\theta}{dt} = \frac{1}{k\sqrt{1+(\theta+\pi)^2}} \tag{3}$$

The initial value at time zero can be directly used to find the function of t in the arc C1 and C2. Correspondingly, $r=r_1$ at C1 and $r=r_2$ at C2.

$$\theta = -\frac{1}{r} + \theta_0 \tag{4}$$

Next, we need to determine the position of the dragon body. The difficulty here is that when the dragon head handle and the dragon body handle are not in the same curve segment, it will cause trouble for the establishment and solution of the model. Therefore, we need to establish the solution

model for the position of the dragon body handle for different curve segments. First of all, when the time is less than or equal to 0s, the dragon body handle and the dragon body handle are located in the disc into the spiral, at this time, the simultaneous circle equation and the spiral equation can be used to solve the position of the dragon body handle [6][8].

In the second case, the joint point of the dragon body is located in the disc entry spiral, and the position coordinates of the dragon body handle are known, then the nonlinear equation of the Angle θ corresponding to the dragon body handle can be obtained, where d is the distance between the dragon body handle and the dragon body handle.

$$(k\theta \cos \theta - x_1) + (k\theta \sin \theta - y_1) = d^2 \quad (5)$$

When the handle of the dragon head is located in the arc of C2 section, the first case is that the connection point of the dragon body is also located in the arc of C2 section. In this case, the isoscel triangle can be established as before to solve the position of the handle of the dragon body. In the second case, the dragon handle is located in the circular arc in section C2, where, according to the coordinates of the center of the circular arc in section C1 and the position of the dragon handle, d_1 can be obtained, that is, the distance between the center of the circle and the connecting point in front of the dragon handle [6]. Then, the cosine theorem can be established to solve it, and then the position of the dragon handle can be determined.

$$\cos \theta = \frac{d_1^2 + r_1^2 - d^2}{2d_1 r_1} \quad (6)$$

Finally, when the leading end handle 2 is located on the outgoing spiral curve, the equations of the circle and the outgoing spiral curve are simultaneously solved [9][10]. When the body part handle is on the arc of section C2, the position of the body - part handle is obtained by solving the nonlinear equation of the distance constraint.

As shown in Fig 4, the following is a demonstration diagram of the running path's variation with time in the coordinate system.

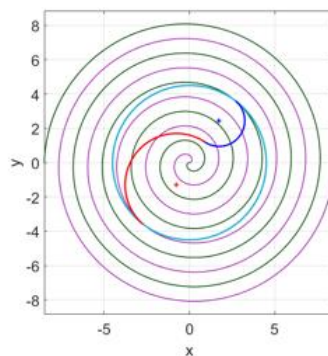


Fig 4 Motion Trajectory

By substituting the data and conducting the program calculation, we can obtain the following results

The angle corresponding to the left - most boundary of the blue arc S1, $\theta_{\max 1} = 4.0055$

The angle corresponding to the right - most boundary of the blue arc S1, $\theta_{\min 1} = 0.9841$

The angle corresponding to the left - most boundary of the red arc S2, $\theta_{\min 2} = -2.1575$

The angle corresponding to the upper - most boundary of the red arc S2, $\theta_{\max 2} = 0.8639$

A schematic diagram of the fixed turning path and the verification of the velocity obtained from the numerical calculation are presented [7]. The speed obtained from the modeling calculation is the standard value of 1 m/s.

4. Conclusions

Through experimental simulations and numerical analysis based on spiral equations and differential algorithms, the following key results were obtained: The minimum pitch required to avoid collisions during the Bench Dragon's motion was determined to be 0.452 m, achieved by iterating through pitch values and applying the Separating Axis Theorem. The Runge-Kutta method successfully calculated the positions and velocities of connection points, with the standard velocity validated as 1 m/s. Additionally, the angles corresponding to the boundaries of arcs S1 and S2 were identified, with $\theta_{\max 1} = 4.0055$, $\theta_{\min 1} = 0.9841$, $\theta_{\min 2} = -2.1575$, and $\theta_{\max 2} = 0.8639$, providing precise spatial constraints for the turning path.

The study demonstrated that the turning path length is a fixed value, unoptimizable due to geometric constraints, as confirmed by the sum of arc lengths (S1 + S2) being a constant. Motion trajectories and collision conditions were visualized, showing consistent alignment with theoretical predictions.

In this paper, this research addressed the problem of modeling the motion path and collision detection for the Bench Dragon under various conditions. By analyzing spiral equations and motion paths, this research established a comprehensive model that describes the system's motion characteristics using mathematical models and differential equations. Through the application of the Runge-Kutta algorithm and the Separating Axis Theorem, this research successfully determined the positions and velocities of connection points, identified collision conditions, and calculated the minimum pitch required for collision avoidance [10]. The results, including motion paths, collision times, and minimum pitch values, were validated through numerical verification and error analysis, confirming the reliability of the model.

Looking ahead, the methods and algorithms developed in this study can be further optimized to improve computational efficiency, particularly in solving for the minimum pitch.

References

- [1] Liu N, Mao X J. Collision Detection Algorithm Based on the Separating Axis Theorem[J]. Digital Technology and Applications, 2012(8): 70.
- [2] Zeng J H. Discussion on Vertical Profile Fitting of Existing Railways Based on Linear Regression and Genetic Algorithm[J]. Journal of Shijiazhuang Tiedao University (Natural Science Edition), 2021, 34(4): 1-8.
- [3] Gao H, Wu Y, Liu Z X, et al. Multi-Strategy Improved Runge-Kutta Optimization Algorithm[J]. Journal of Science of Teachers' College and University, 2024, 44(7): 1-10.
- [4] Liu Y C. Design of Yard Anti-Collision System Based on the Separating Axis Theorem[J]. Science Communication, 2015, 7(19): 131-132.
- [5] Liu C J. Principle and Calculation of Equidistant Spirals[J]. Mathematics in Practice and Theory, 2018, 48(11): 165-174.
- [6] Zhang L, Li H. Optimization of Spiral Paths in Robotics Using Geometric Methods[J]. International Journal of Advanced Robotic Systems, 2020, 17(2): 1-10.
- [7] Liu Q, Huang G, Zhang X, et al. Speed Planning and Interpolation Algorithm of Archimedes Spiral Based on Tangential Vector[J]. International Journal of Precision Engineering and Manufacturing, 2024, 25(11): 2235-2248.
- [8] Draelos M. Time-Optimal Spiral Trajectories With Closed-Form Solutions[J]. IEEE Robotics and Automation Letters, 2023, 8(4): 2213-2220.
- [9] Al-shimmary A F, Hussain A K, Radhi S K. Numerical Solution of Volterra Integro-Differential Equation Using 6th Order Runge-Kutta Method[J]. Journal of Physics: Conference Series, 2021, 1818(1): 012183.
- [10] Gan B, Dong Q. An improved optimal algorithm for collision detection of hybrid hierarchical bounding box[J]. Evolutionary Intelligence, 2021.