Research on Simulation of Multi-node Motion States of the Bench Dragon Based on Archimedean Spiral and ode45 Algorithm

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Abstract. In this study presents a groundbreaking simulation analysis of the multi-node motion states of the Bench Dragon, a cherished traditional Chinese folk activity. By seamlessly integrating Archimedean spiral theory with the ode45 algorithm, we systematically investigated the motion equations governing the Bench Dragon's movements along diverse paths. Focusing on a specific scenario where the dragon dance team progresses clockwise along an isometric spiral with a constant pitch of 55 cm, while maintaining a uniform speed of 1 m/s at the dragon head's front handle, we meticulously determined the dragon head's positional changes on the spiral using parametric equations and velocity decomposition. Leveraging the cosine theorem and iterative approximation techniques, we further derived the precise positions and velocities of subsequent nodes, encompassing both the body and tail handles. Numerical solutions were obtained by employing MATLAB's ode45 function to solve the underlying differential equations. Our findings provide comprehensive spatial coordinates and instantaneous speeds for pivotal nodes at multiple time points, offering profound insights into the dynamic stability and coordination of traditional Bench Dragon dances. This research not only deepens our comprehension of complex kinematic behaviors in cultural performances but also embodies a methodological innovation by intertwining mathematical modeling with cultural heritage studies. The proposed framework holds promising applications across diverse fields, including computer animation, sports training, and the preservation of cultural heritage.

Keywords: Bench Dragon, Archimedean Spiral, Big Data, ode45.

1. Introduction

The Bench Dragon is a traditional folklore activity originating from the He Luo region, with a history of hundreds of years, and has been widely passed down in the annual celebrations of many provinces and cities in southern China. This Lantern Festival custom is especially prevalent in Fujian, where it is a key activity for welcoming the gods and praying for good fortune, demonstrating the deep cultural heritage of the region [1-3]. Villagers connect sections of benches through drilled holes, with each household responsible for one section, eventually forming a spectacular dragon that can be more than two hundred meters long. During the process of coiling the dragon, the dragon head leads the way forward, followed by the dragon body and the dragon tail, circling into a disk shape. The faster the dragon travels, the less space it needs, and the more impressive the viewing effect.

However, the long body of the Bench Dragon is very inconvenient to move, in order to make the Bench Dragon's dance perfect, it is necessary to carry out the research on the movement state of the Bench Dragon. In this paper, the following problems are solved: The dragon dance team coils clockwise along an isometric thread with a pitch of 55 cm, and the traveling speed of the handle in front of the dragon head is always 1 m/s. The dragon head is initially located at point A on the 16th turn of the thread. Initially, the dragon head is located at point A on the 16th turn of the thread. Give the positions and velocities of the front handle of the dragon head, the front handles of the dragon's body and the rear handles of the dragon's tail in sections 1, 51, 101, 151 and 201 behind the dragon head at 0 s, 60 s, 120 s, 180 s, 240 s and 300 s.

The position and velocity of each node of the dragon dance team moving along the spiral line are calculated according to the given parameters. First of all, the position change of the dragon head on the spiral line needs to be determined. From the pitch of the spiral line is 55 cm, the equation of the spiral line is calculated, and the velocity relationship is used to calculate the equation of motion of the dragon head. Since the length of each section of the bench is fixed, the cosine theorem and the position of the faucet can be used to derive the equation [4-6]. The position and velocity of each node is derived.

2. Modeling Theoretical Foundations

The dragon dance team is moving along an isometric spiral, in order to find out the equations of motion of the center of each handle of the dragon dance team, first of all, should be solved to change the isometric spiral trajectory equation. Let L is a ray from the O point, M is a point on the ray, along the L uniform linear motion, and L along the O to do uniform circular motion, then the trajectory of the M point that is isometric thread [6-10]. The schematic diagram is shown in Figure 1.

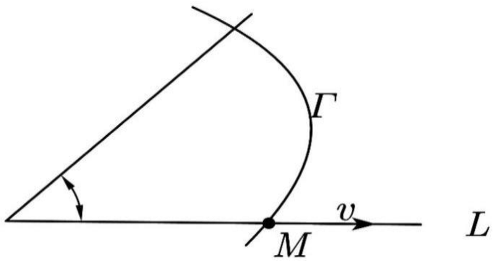

Figure 1. Isometric solenoid formation diagram

The polar equation of the isometric solenoid is given by Eq(1):

$$\rho = a + b\theta \tag{1}$$

Using the given conditions, the solenoidal equation is obtained as Eq(2):

$$\rho = 8.8 + \frac{0.55}{2\pi}\theta \tag{2}$$

Where $\theta \in [-32\pi, 0]$, when the front handle of the dragon's head starts traveling along the isometric spiral, the dragon's body and the dragon's tail are implicated by the rigid body to obtain the velocity, and then they are driven forward. In order to analyze the motion of the bench dragon as a whole, it is first necessary to analyze the motion of the front handle of the dragon's head, solve its equation of motion, and find the velocity of the dragon's head. The remaining nodes can be solved for their positions and velocities by using the cosine theorem with the velocity implicates.

The linear velocity of motion on an isometric solenoid can be derived from the Eq(3):

$$v = \frac{ds}{dt} \tag{3}$$

Where s represents the arc length of the helix and its expression is Eq(4):

$$s = \int_{-32\pi}^{\theta_0} \sqrt{r^2(\theta) + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \tag{4}$$

Ds is then the arc differential of the isometric solenoid, which can be expressed as Eq(5):

$$ds = \sqrt{dr^2 + (r d\theta)^2} \tag{5}$$

Substituting Eq(2) into Eq(5), and dividing dt on both sides of the equation simultaneously yields the relationship between angular velocity and linear velocity as Eq(6):

$$\omega = \frac{v}{\sqrt{(\frac{0.55}{2\pi})^2 + (0.88 + \frac{0.55}{2\pi}\theta)^2}} \tag{6}$$

Faucet of the front handle A_1 along the isometric screw movement, after a certain time t, turn over the angle θ, at this time we can write the parametric equation of the trajectory of A_1 for the Eq(7):

$$\begin{cases} x(\theta) = (8.8 + \dfrac{0.55}{2\pi}\theta)\cos\theta \\ y(\theta) = (8.8 + \dfrac{0.55}{2\pi}\theta)\sin\theta \end{cases} \tag{7}$$

The sum velocity of A_1 is a synthesis of linear and radial velocities, and the sum velocity can be decomposed into horizontal velocity v_x and vertical velocity v_y. Derivation of x, y from t in the equations of motion gives the horizontal and vertical velocities, and thus the sum velocity $v = \sqrt{(v_x)^2 + (v_y)^2}$. At the initial moment the dragon's body and the center of the dragon's tail handle are located on an equidistant solenoid, presenting a row as shown in Figure 2.

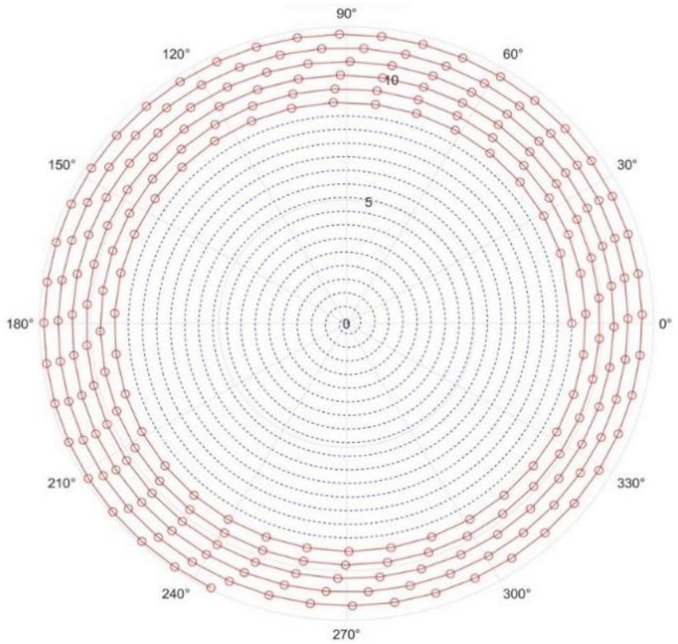

Figure 2. Initial State Bench Dragon Arrangement Diagram

3. Iterative Method

For each handle center A_i, according to the cosine theorem can be found out the corresponding coordinates, for the faucet after the handle center, also for the first piece of bench body of the front handle position, that is, for A_2, set A_2 of the polar diameter of r_2, the polar angle of α_2, at this time the faucet before the handle center of A_1 corresponds to the polar diameter of $r_1 = 8.8m$, the polar angle $\theta_1 = 0$, and the distance between the two points is l_1. According to the cosine theorem can be derived as Eq(8).

$$r_2^2 + r_1^2 - 2r_1 r_2 \cos(\alpha_2 - \alpha_1) = l_1^2 \tag{8}$$

Where $l_1 = 2.86m$, also note that A_2 in the isometric solenoid, so $r_2 = 8.8 + \frac{0.55}{2\pi} cos(\alpha_2)$, the association can be solved for the polar coordinates of A_2, for the part of the body of the dragon, but also according to the cosine theorem to solve for the A_i of the polar diameter of the r_i, the polar angle of α_i, the distance between the two points as l_i=1.65m. According to the cosine theorem, we can derive the Eq(9):

$$r_{i+1}^2 + r_i^2 - 2r_{i+1} r_i \cos(\alpha_{i+1} - \alpha_i) = l_i^2 \tag{9}$$

Since they are on the equidistant solenoid, so $r_i = a + bcos(\alpha_i), r_{i+1} = 8.8 + \frac{0.55}{2\pi} cos(\alpha_{i+1})$, and the polar coordinates corresponding to the body of the dragon can be solved by using an iterative method.

When the faucet front handle movement, at this time the $\theta_1 = \omega t$, set the movement of A_i after the polar angle of θ_i, for the faucet before and after the handle A_1, A_2, according to the cosine theorem has the Eq(10):

$$r_2^2 + r_1^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1) = l_1^2 \tag{10}$$

From Eq(6), we can obtain Eq(11), which is a first-order differential equation to find the relationship between θ_1 and t. The relationship between θ_2 and t is then calculated using Eq(10).

$$\frac{d\theta_1}{dt} = \frac{1}{\sqrt{(\frac{0.55}{2\pi})^2 + (8.8 + \frac{0.55}{2\pi}\theta_1)^2}} \tag{11}$$

Similarly, for the handle centers A_i, A_{i+1} at the front and back of the dragon's body, let the corresponding polar angles be θ_i, θ_{i+1} when it is in motion, and by the cosine theorem, we can get the Eq. The polar coordinates of A_i in the state of motion can be obtained using iterative operations. Using Eq(11), Eq(7) can find the equation of motion and then the velocity.

$$r_{i+1}^2 + r_i^2 - 2r_i r_{i+1} \cos(\theta_{i+1} - \theta_i) = l_1^2 \tag{12}$$

The solution schematic is shown in Figure 3.

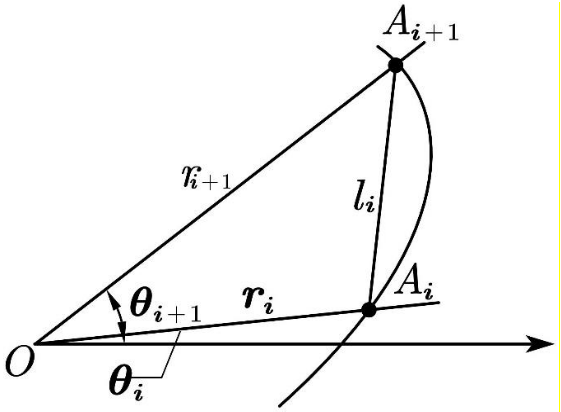

Figure 3. Cosine Theorem State of Motion Schematic

4. The step of solving problems

The model is solved by using the built-in ode45 function of matlab, numerical differentiation method, and iterative approximation method. ode45 function is essentially the Lung-kuta method, which has a high computational accuracy, and Eq(11) is a first-order differential equation, which can be numerically solved by ode45 function to find out the numerical solution of θ_1. In order to find out the corresponding partial velocity, take a smaller time step, calculate the corresponding horizontal displacement and vertical displacement, carry out the difference, can find out the corresponding partial velocity. In Eq(10), Eq(11), it is difficult to write the explicit expression of θ_i, so the approximation iteration method is used for numerical solution.

The steps for solving the problem are as follows:

(1) Solve the numerical solution of Eq(11) using the ode45 function.

(2) Substituting to Eq(6) yields the A_1 equation of motion Eq(6).

(3) Solve for the velocity of A_1 using numerical differentiation.

(4) Determine the coordinates of A_i in the initial state using the cosine theorem.

(5) When A_1 starts moving, the iterative approximation method solves the equations of motion for A_i.

(6) Solve for the velocity of A_i using numerical differentiation.

5. Results

The locations of the front handle of the dragon's head, the front handle of the dragon's body in the 1st, 51st, 101st, 151st, and 201st sections behind the dragon's head, and the rear handle of the dragon's tail are obtained, as shown in Table 1.

Table 1. Position results

~	0 s	60 s	120 s	180 s	240 s	300 s
Mixer x (m)	8.800000	5.792952	-4.093304	-2.952754	2.580986	4.424471
Mixer y (m)	0.000000	-5.777240	-6.298877	6.099874	-5.363021	2.312063
1 x (m)	8.363824	7.452994	-1.455371	-5.229322	4.812182	2.467572
1 y (m)	2.826544	-3.448322	-7.403824	4.368736	-3.573786	4.397764
51 x (m)	-9.518732	-8.683770	-5.535449	2.901531	5.971822	-6.300521
51 y (m)	1.341137	2.548499	6.384491	7.244718	-3.840182	0.475134
101 x (m)	2.913983	5.679922	5.353640	1.887079	-4.929043	-6.232643
101 y (m)	-9.918311	-8.006396	-7.563402	-8.474108	-6.370657	3.943839
151 x (m)	10.861726	6.689036	2.398545	1.017048	2.979344	7.045591
151 y (m)	1.828753	8.128921	9.724912	9.423364	8.394622	4.385043
201 x (m)	4.555102	-6.612554	-10.625842	-9.292599	-7.466524	-7.463963
201 y (m)	10.725118	9.030696	1.369854	-4.235739	-6.169190	-5.255709
End x (m)	-5.305444	7.357792	10.975038	7.392348	3.255067	1.794175
End y (m)	-10.676584	-8.803563	0.833397	7.483891	9.464390	9.299317

The speeds of the front handle of the dragon's head, the front handle of the dragon's body in the 1st, 51st, 101st, 151st and 201st sections behind the dragon's head and the rear handle of the dragon's tail are as in Table 2 Speed results.

Table 2. Speed results

~	0 s	60 s	120 s	180 s	240 s	300 s
Mixer (m/s)	1.000000	1.001143	1.001668	1.002436	1.003664	0.992351
1 (m/s)	0.999971	1.001104	1.001613	1.002353	1.003522	0.992063
51 (m/s)	0.999742	1.000805	1.001206	1.001765	1.002601	0.990431
101 (m/s)	0.999575	1.000596	1.000936	1.001404	1.002094	0.989674
151 (m/s)	0.999448	1.000441	1.000745	1.001160	1.001772	0.989236
201 (m/s)	0.999349	1.000322	1.000601	1.000984	1.001550	0.988952
End (m/s)	0.999312	1.000278	1.000549	1.000921	1.001472	0.988856

6. Conclusion

This study presents a rigorous analysis of the multi-node motion states of the Bench Dragon, a quintessential Chinese folk activity, through the innovative application of Archimedean spiral theory and the ode45 algorithm. By developing and solving motion equations, we obtained precise positions and velocities of critical nodes at different time instants, offering unprecedented insights into the dynamic stability and coordination of the Bench Dragon dance. The methodology employed is proven feasible and effective, delivering high fidelity in simulating the intricate movements of the dragon. Looking to the future, this work establishes a robust foundation for advancing research on traditional cultural activities, potentially guiding improvements in dance choreography, enhancing performance aesthetics, and facilitating the digital preservation of cultural heritage. Furthermore, the integration of mathematical modeling with cultural heritage research showcases a transformative approach that can inspire similar investigations across diverse disciplines.

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