A Study on The Assessment of Staircase Wear Problem Based on Hybrid Construction Of 1D-2D Models

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Abstract. In this study, the scientific method is used to obtain and preprocess the data, and after comparing the results of data clustering visualization by EM and KMeans method, the DBSACN model, which is divided into six clusters, is selected for the clustering and classification of stairs, and the corresponding models are established according to the frequency of staircase use, the preference of direction of use, the use of multiple people, the wear and tear and historical information, and the existence of the time of repair and alteration, etc., such as the one-dimensional linear model, two-dimensional discrete model, multi-step spatial expansion model, and so on. The corresponding models are established, such as one-dimensional linear model, two-dimensional discrete model, and multi-step spatial expansion model, etc. The friction bending model is analyzed by assuming parameters. The friction bending depth and other data are analyzed by assuming the parameters, and differential evolutionary fitting is carried out to obtain the relevant parameters and residual paradigms, and the MSE of the fitting analysis iteration is 0.10, which is verified by the sensitivity analysis, indicating that the model has strong robustness in different scenarios.

Keywords: DBSACN model, one-dimensional linear, two-dimensional discrete, staircase wear.

1. Introduction

In the field of construction, the wear and tear of stairs is widely concerned, and scholars at home and abroad have carried out research from different perspectives and achieved results. Some domestic studies focus on the actual wear phenomenon and protective measures, such as "impregnated paper laminated wood flooring"[1] to provide reference for the assessment of wear resistance of wooden stairs. Foreign scholars focus on scientific quantitative analysis and advanced technology, such as "Our Lord in the Attic: A Case Study"[2] on the 17th century staircase wear case study; "A Model for Analyzing Stair Wear Patterns in Archaeological Research"[3] to establish a dynamic "A Model for Analyzing Stair Wear Patterns in Archaeological Research"[4] builds a dynamic wear model to reveal the wear mechanism; and "Wear Modeling of Stairs: A New Approach for Predicting and Evaluating Stair Degradation"[5] utilizes a deep learning model to provide a new idea for research. However, there are limitations in the existing studies, which mostly focus on a single factor and lack of model generalization and accuracy. This paper aims to remedy these shortcomings by first performing data acquisition and preprocessing, and then using the DBSACN model to categorize the staircase in order to preserve the data characteristics. The model is constructed separately for different issues such as frequency of use, direction of use and multiple users, wear and tear, and historical information, etc. Finally, the model is evaluated by sensitivity analysis, and the results show that the model has strong accuracy and robustness in different scenarios. (Data sources: nationalgeographic.com, https://www.census.gov/, https://data.un.org/, http://hao.199it.com/)

2. Typical staircase model selection

After completing the data organization and preliminary analysis, this paper uses the DBSCAN model to cluster analyze the data of stair width and depth. The model parameters, including neighborhood radius (eps) and minimum number of points (minPts), are adjusted through several tests, and the stair width and depth data are divided into 6 classes, and then the ideal stair model is constructed with the average value. The visualization is shown in Fig1 below:

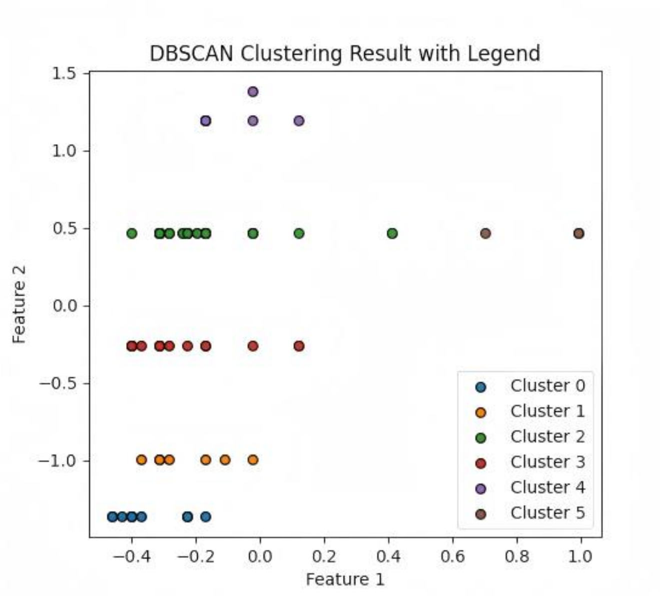

Fig. 1 Plot of clustering results for DBSCAN data

Different shapes of stairs are subjected to different pressures and wear patterns during use, and the shape classifications obtained based on cluster analysis can be used to analyze the relationship between wear and bending degree in a more targeted way, so as to establish more accurate mathematical models to predict the wear and bending of different shapes of stairs after long-term use.

3. Frequency of use issues

3.1. One-dimensional mathematical modeling of the degree of wear bending

Ideally for the same step, this paper defines a one-dimensional transverse coordinate $x, x \in [0, w]$, where w denotes the transverse width of the tread surface (the horizontal plane of the step), and w is the depth of wear curvature of the step at the transverse position x at time $G(x,t)$ (or the t year) (with the deviation value being relative to the original smooth surface that has not experienced wear). The wear increment at absolute position x for each step is Δz (the single force bearing area and the friction incremental entropy are jointly determined). Let the contact pressure (of a person or object passing over) be a constant. C The unit wear bending deformation brought about by a person stepping on a step once is proportional to the bending coefficient δ [6]and is denoted as $\delta \bullet C$. From this we calculate the bending coefficient δ inherent in the construction of the respective staircase that is introduced.

Definition $D(X)$ is the tread distribution, i.e., the spatial distribution of the shape of the wear scratches. By the practical experience of life, many people walking side by side, when stepped on, the step of the tread distribution $D(X)$ more uniform, especially in the for two people walking, will show a bimodal distribution; if a single person forward, a single column stepped through, the step of the tread distribution $D(X)$ shows only a single linear distribution, closer to the physical and mathematical sense of the single peak distribution.

In order to simplify the problem sufficiently and obtain an absolutely idealized special model, we assume that the difference in wear in all directions of the steps is negligible or uniform, and focus only on the difference in the lateral distribution, i.e.,

$$\int_0^w D(x)dx = 1 \tag{1}$$

Assuming a total of N stepping uses in a year, the total wear increment on the position x in that year is

$$\Delta G(x) = N \bullet \delta \bullet C \bullet D(X) \bullet \Delta z \tag{2}$$

If the number of years of use of the stair treads at this point is t year, the total wear and tear is

$$G(x,t) = N_t \bullet \delta \bullet C \bullet D(x) \bullet \Delta z \tag{3}$$

If time is considered as a discrete number of years, noting $G(x,t)$ as the depth of wear at absolute position x at the end of year t, the

$$G(x,t+1) = G(x,t) + N \bullet D(x) \bullet \Delta z \tag{4}$$

If the repair is carried out in year t^* (the most recent year of a large-scale repair is sufficient), then $G(x,t^*)$ needs to be "partially reset" or "smoothed" to represent the trace of its redoing and repairing or sanding and reconstruction.

With this simple model, we analyze that the shape of the wear curvature is spatially influenced mainly by $D(X)$, and that the wear increases linearly with the year of time (if other factors remain constant).

If we extrapolate the theory of the formula to the real situation, assuming that the footprint distribution $D(X)$ is different due to "unequal foot traffic up and down the stairs" or "walking side preference", then we need to $D(X)$ split it into "footprint of people going up the stairs $D_\uparrow(X)$ " and "footprint of people going down the stairs $D_\downarrow(X)$ ". If the number of people going up the stairs is $N_\uparrow$ and the number of people going down the stairs is, $N_\downarrow$ then the

$$G(x,t) = \delta \bullet C \bullet (N_\uparrow t \bullet D_\uparrow(x) + N_\downarrow t \bullet D_\downarrow(x)) \bullet \Delta z \tag{5}$$

3.2. Two-dimensional discrete wear model for single-stage steps

A two-dimensional array can be constructed by discretizing the tread data into a regular grid (e.g., 100×100 points) and recording the depth of wear (or degree of depression) at each grid point.

Define the discretization of this step tread as i_x point in the transverse width direction and j_y points in the longitudinal depth direction. Discretize the tread data on this step into a regular grid to construct a grid of $i_x \times j_y$ points, denoted as $\{(\alpha,\beta) \mid \alpha = 1,...,i_x; \beta = 1,..., j_y\}$. At each grid point (α,β), measure the depth of wear curvature of the step tread "relative to the original smooth, un-stepped, wear-curved horizontal/reference surface", denoted as

$$z_{\alpha,\beta}(\text{work unit : m}) \tag{6}$$

As can be seen from the comparison of the following collection of 2D discrete wear models for the various types of stair shapes graphed to show a single step, in this paper, the most typical staircase No. 3 is selected as a template to continue the construction. The data clustering results based on the DBSCAN model are shown in Fig2-Fig13 below.

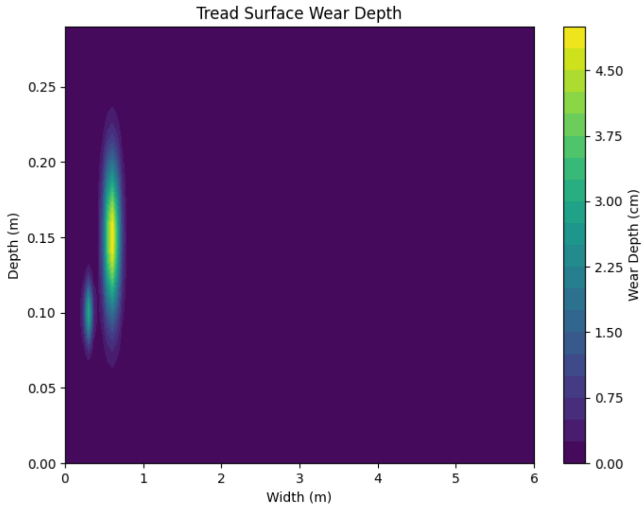

Fig. 2 Class 6 staircase model shape analysis diagram(a)

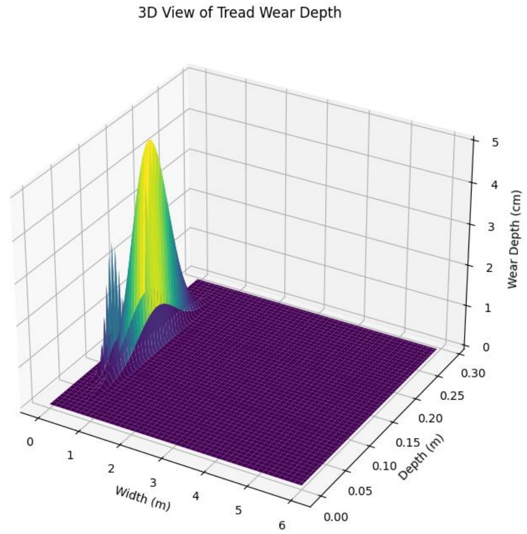

Fig. 3 Class 6 staircase model shape analysis diagram(b)

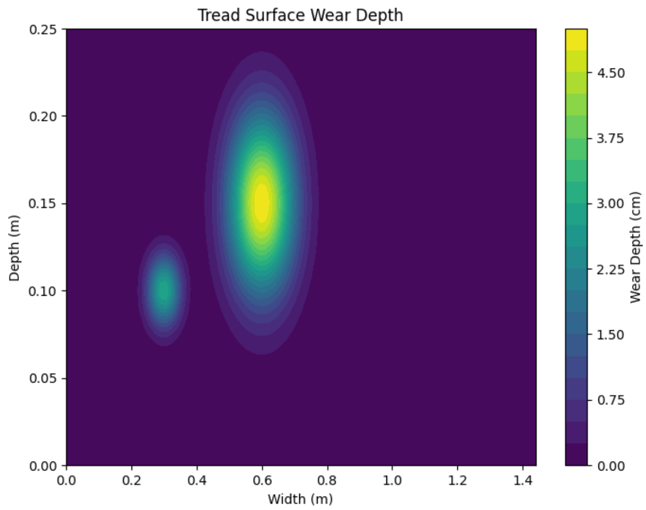

Fig. 4 Class 6 staircase model shape analysis diagram(c)

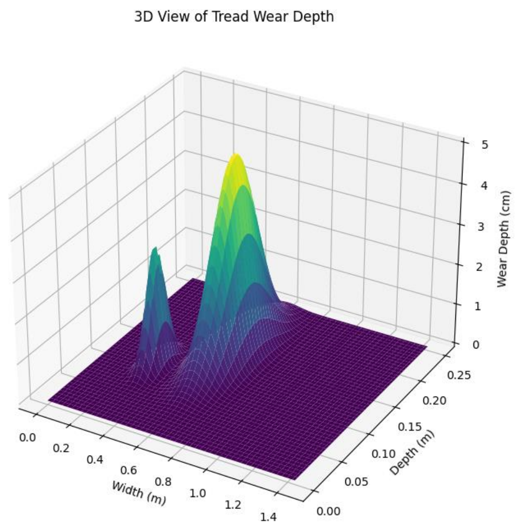

Fig. 5 Class 6 staircase model shape analysis diagram(d)

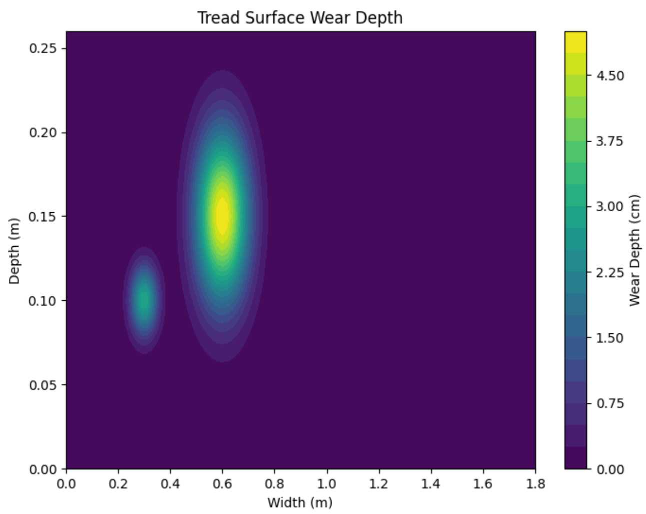

Fig. 6 Class 6 staircase model shape analysis diagram(e)

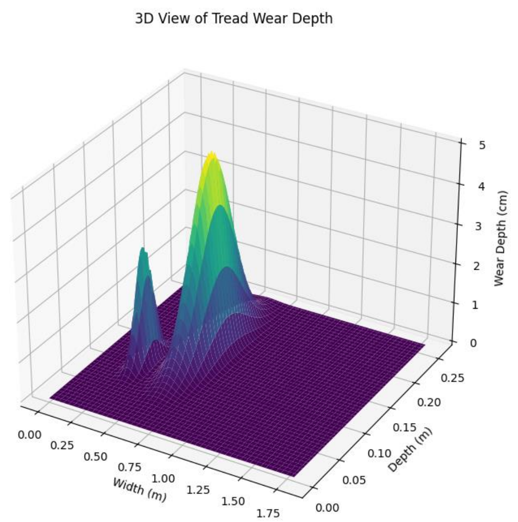

Fig. 7 Class 6 staircase model shape analysis diagram(f)

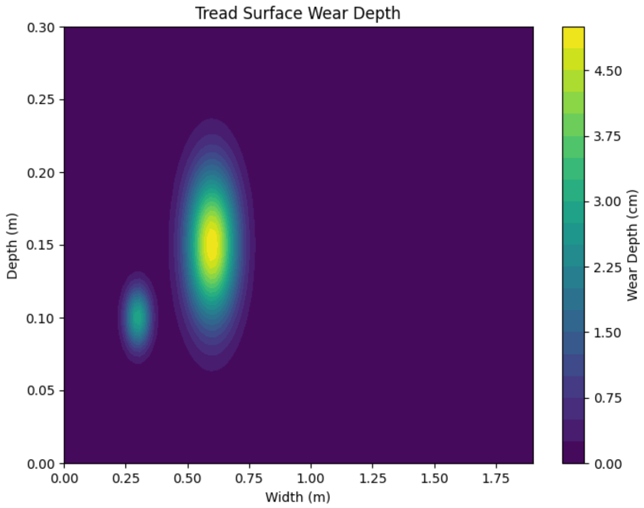

Fig. 8 Class 6 staircase model shape analysis diagram(g)

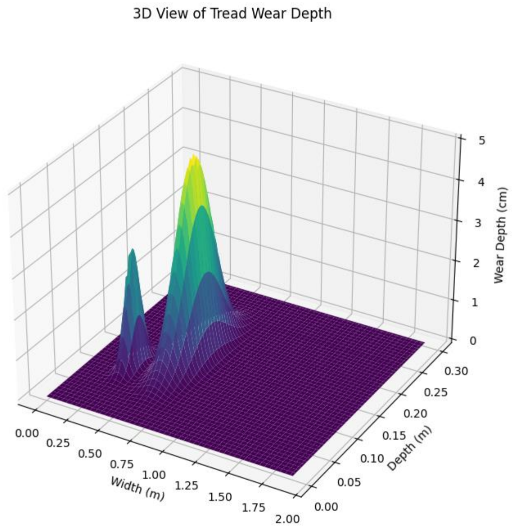

Fig. 9 Class 6 staircase model shape analysis diagram(h)

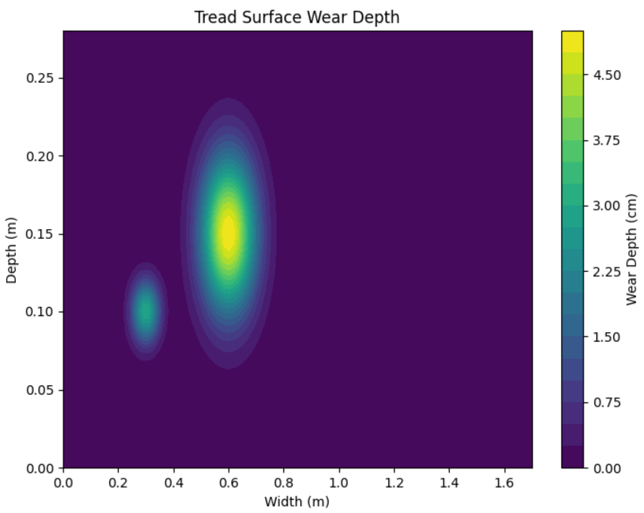

Fig. 10 Class 6 staircase model shape analysis diagram(i)

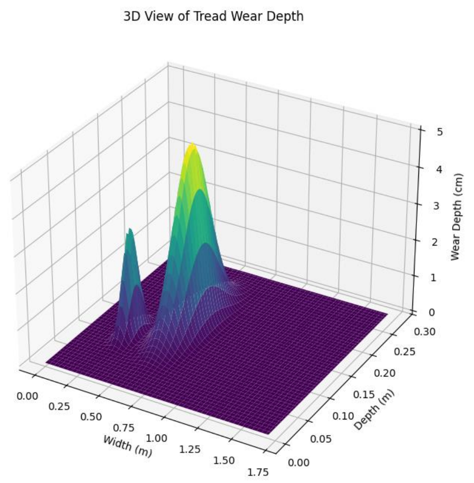

Fig. 11 Class 6 staircase model shape analysis diagram(j)

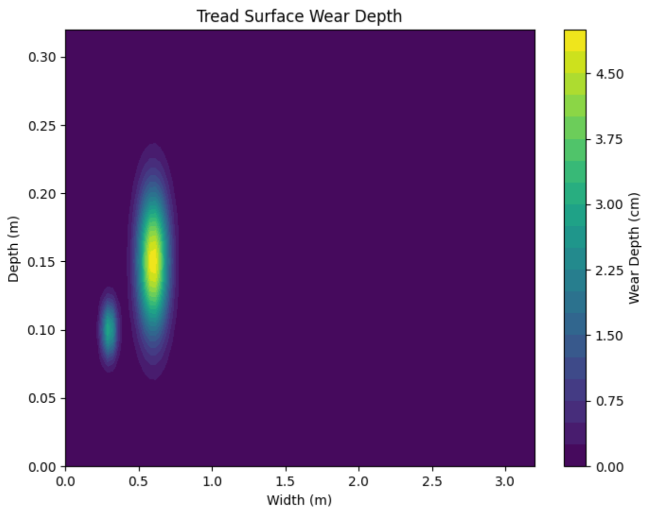

Fig. 12 Class 6 staircase model shape analysis diagram(k)

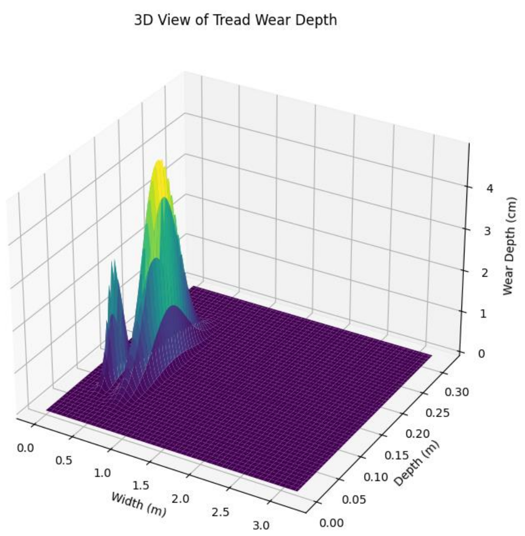

Fig. 13 Class 6 staircase model shape analysis diagram(l)

In the preliminary calculations, we define $i_x = 50, j_y = 50$ to simplify the subsequent modeling process.

3.3. Basic Assumptions for Wear Bending Accumulation

This paper defines the distribution of forces, points of action and their benefits on the mesh for each "step" as an abstract function $\Delta z_{\alpha,\beta}^{(N)}$, so that the benefit of the N' nd step on the stair step to the mesh point (α,β) is the total wear increment. The subsequent introduction of material bending stresses, i.e., the wear coefficient δ, makes the contribution of the benefit of a single step to the degree of wear bending at grid point (α,β)

$$\delta \bullet \Delta z_{\alpha,\beta}^{(N)} \tag{7}$$

In practice, the estimation of the degree of wear damage is unavoidably related to the use of fatigue years, natural disasters, changes in humidity in bad conditions, etc. Therefore, this paper defines the existence of a random error term $\gamma_{\alpha,\beta}$, which is used to characterize the sum of the deviations of the trace errors that may be brought about by the above types of factors, and thus the final wear bending depth $\gamma_{\alpha,\beta}$ is defined as follows:

$$z_{\alpha,\beta} = \sum_{N'=1}^{N} [\delta \bullet \Delta Z_{\alpha,\beta}^{(N)}] + \gamma_{\alpha,\beta} \tag{8}$$

$$\alpha = 1,...,i_{x,\beta}; \beta = 1,..., j_{\alpha,y} \tag{9}$$

Analysis of the established mathematical model yielded $\gamma_{\alpha,\beta} = 0.005$ [7], obtaining a frictional bending depth of up to 0.0855m and an intermediate value of roughly 0. 0262m.As shown in Fig14-16.

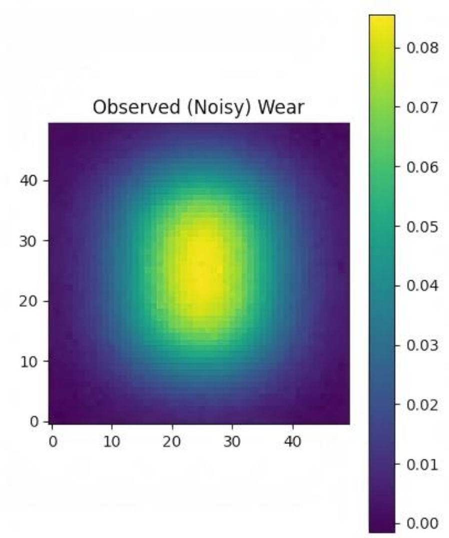

Fig. 14 Plot of outlier noise deviation results(a)

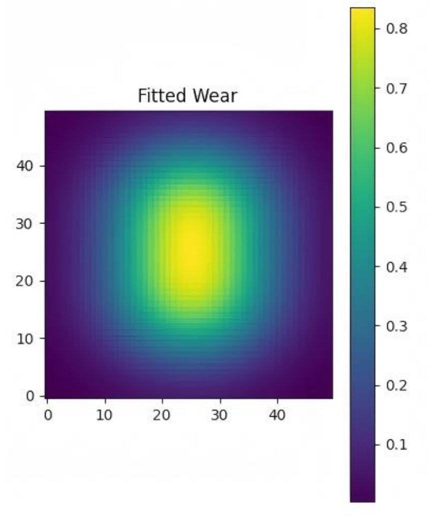

Fig. 15 Plot of outlier noise deviation results(b)

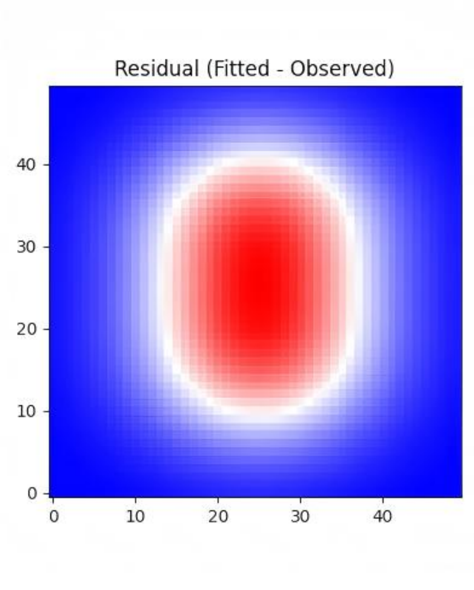

Fig. 16 Plot of outlier noise deviation results(c)

4. Direction of use and multi-user issues

4.1. One-dimensional parametric definition of stepping bending modes

Define E typical mode, each mode is denoted as $\theta_1, \theta_2, ..., \theta_E$. For each of the fixed all modes that have been set up, bring into the model function calculation and estimate its unit tread bending distribution function at grid point (α, β) $d_{\alpha,\beta}^{(e)}$ If defined at the construction material hardness coefficients δ are constant and each time a e th mode occurs, the incremental bending of the abrasion for the network grid (α, β) point can be written as

$$\Delta z_{\alpha,\beta}^{(e)} = d_{\alpha,\beta}^{(e)} \tag{10}$$

4.2. Fixed Mode A: Single Walking

For a single person, the tread distribution $d_{\alpha,\beta}^{(e)}$ of the steps exhibits only one single linear distribution, which is closer to the physico-mathematical sense of a single peaked distribution. Modeling is done to bias the staircase towards a certain position (in this case the "leading edge" or "trailing edge" of the single-peaked distribution) in order to classify it into a single mode of computationally defined functions.

$$d_{\alpha,\beta}^{(1)} = C_1 \bullet \exp(-\frac{(\alpha - x_e)^2}{2\varphi_x^2} - \frac{(\beta - y_e \pm \Delta\tau)^2}{2\varphi_y^2}) \tag{11}$$

Definition of x_e for the step tread transverse width in the middle of the position; y_e for the step tread longitudinal depth in the middle of the position; Δh for the offset, $\Delta h > 0$ indicates that the upper floor off the leading edge, $\Delta h < 0$ indicates that the lower floor off the trailing edge; φ_x, φ_y for the distribution of pedal force range.

4.3. Fixed Mode B: Two-Person Parallel

Two people walking side by side, their stepping force points and force benefits are mostly concentrated in the distribution of the left and right sides of the staircase steps, there is a distribution of symmetrical tendency, and presents a bimodal distribution.

$$\begin{aligned} d_{\alpha,\beta}^{(2)} &= C_1 \bullet [\exp(-\frac{(\alpha - x_1)^2}{2\varphi_x^2} - \frac{(\beta - y_e)^2}{2\varphi_y^2}) \\ &+ \exp(-\frac{(\alpha - x_2)^2}{2\varphi_x^2} - \frac{(\beta - y_e)^2}{2\varphi_y^2})] \end{aligned} \tag{12}$$

The x_1, x_2 in the defining equation are the center positions of the left and right force benefits, $x_1 < x_e, x_2 > x_e$ [8].

The resulting figurative analysis shows that the distribution of wear bending at random steps of a staircase consists of a linear summation of these 2 modes:

$$d_{\alpha,\beta} = \sum_{e=1}^{2} A_e \bullet d_{\alpha,\beta}^{(e)} \tag{13}$$

Definition A_e represents the relative proportion of stepping mode e in the total number of stepping, which satisfies $\sum_{e=1}^{2} A_e = 1$, then the above equation can be combined and simplified to

$$\tau_{\alpha,\beta} = \sum_{e=1}^{E} [\delta \bullet A_e \bullet d_{\alpha,\beta}^{(e)}] + \omega_{\alpha,\beta} \tag{14}$$

Assuming that the distribution function of the force benefits of the two pedaling modes is written as a "pedaling mode matrix" (of the form $((i_x \times j_y) \times E)$, denoted as $d^{(e)}$); and that $\{\tau_{\alpha,\ \beta}\}$ forms a random vector τ; and that $\omega_{\alpha,\beta}$ forms a random vector ω, and that $E = (E_1, ..., E_E)^T$; then

$$\begin{aligned} &\tau = \delta \bullet \sum_{e=1}^{E} A_e d^{(e)} + \omega \\ &\Leftrightarrow \tau = \delta \bullet [d^{(1)} d^{(2)} \cdots d^{(E)}] A + \omega \end{aligned} \tag{15}$$

From this, a linear modeling framework can be constructed for the stepping patterns. The existing data are listed and analyzed, and the initial construction of the model matrix is fitted to understand the qualitative measurements of τ. The inversion yields the required parameters δ and the stepping weights A_e for each mode, which are then extrapolated to find information such as "whether or not multiple people are using the staircase", "preference of staircase direction of use " and so on.

4.4. Spatial extension model construction for multilevel steps

Define the number of steps of a particular staircase as F (numbered $f = 1,2,...,F$) and do a 2D discrete data collection of $\{\tau^{(f)}_{\alpha,\beta}\}$ for each step analogous to the 1D parameterization of the tread bending pattern to construct a 3D number set:

$$\tau_{\alpha,\beta,f} = 1,...,F \tag{16}$$

Define the stepping pattern spatial model as $d^{(e)}_{\alpha,\beta,f}$, one by one corresponds to the typical stepping pattern of different stepping behaviors falling on the f nd step; perhaps, the number of times of use $A_{e,f}$ will also be subject to changes in the spatial location of the floor f . Therefore, define $A_{e,f}$ to indicate the number of times the standard stepping pattern e occurs on the f th step and the weight.

In order to simplify the calculation and modeling, define the steps of the same staircase construction material is the same, that is, has the same building material hardness coefficient δ ; but because of the design and construction or repair of the same staircase at all levels of the step material is different, notation for δ_f to differentiate the brought into the calculation. And set the random generation of various types of factors brought about by the deviation error and the Gaussian distribution characteristics of the parameter $\gamma_{\alpha,\beta}$.

As a result, the overall wear model of multi-level steps can be written as:

$$\tau_{\alpha,\beta,f} = \sum_{e=1}^{E}[\delta_f A_{e,f} d^{(e)}_{\alpha,\beta.f}] + \gamma_{\alpha,\beta.f} \tag{17}$$

$$\alpha = 1,...,i_x ; \beta = 1,..., j_y ; f = 1,... \tag{18}$$

5. Modeling of the problem of determining the timing of stair wear and construction repairs

5.1. Construction of year-of-construction segmentation model

Define that the building stairs are used in time interval $[0,t]$ and a number of time points $\{t_1, t_2,...\}$ where partial repair or overall renovation occurs, leading to a "partial recovery" of the wear depth, bringing noise points and outliers that affect the accuracy of the model.

For this scenario, we choose to treat the model segmented, given a new constraint or definitional domain: define the period of use of a building staircase $[0,t]$ segmented into k intervals $[t_{k-1}, t_k]$ (of which $t_0 = 0, t_k = t$). Based on each fixed time interval, define different tread frequencies μ_k per year and different wear force benefit distributions $\{d_{e,k}\}$ in a fixed pattern, so that the total tread coefficient for the k time periods can be written as

$$A_{e,f,k} = \mu_k \bullet (t_k - t_{k-1}) \bullet d_{e,k} \tag{19}$$

Subsequently, we analyzed the time series and the spatial occupancy in combination to arrive at a total wear bending increment for stair step f in time period k of

$$\sum_{e=1}^{E}[\delta_e A_{e,f,k} d^{(e)}_{\alpha,\beta,f}] \tag{20}$$

When a repair or renovation operation is carried out at a specific point in time $t^*_k \in [t_{k-1}, t_k]$, this action is defined to reset the wear state at that moment to an initial value*, which does not necessarily return to the value at the moment of perfectly smooth and unworn curvature, and from which a new accumulation of wear will be sustained in the subsequent time. In order to quantify this process, a building repair and renovation coefficient $\zeta_{f,k}(\alpha,\beta)$ is introduced for grid point (α,β) on step f, which takes a value between 0 and 1 to characterize the degree of retention of the depth of wear of the step ($\zeta = 1$ for no repair or renovation operation, $\zeta = 0$ for a complete restoration to the original thickness).

The resulting segmentation function is:

$$\tau_{\alpha,\beta,f}^{(k)} = d_{f,k}(\alpha,\beta)\tau_{\alpha,\beta,f}^{(k-1)} + \sum_{e=1}^{E}[\delta_e A_{e,f,k} d_{\alpha,\beta,f}^{(e)}] \quad (21)$$

From this, the analysis shows that at moment $k = K$, which is time t, the specific total wear bending depth visible and measurable at this time is calculated to be

$$\tau_{\alpha,\beta,f} = \tau_{\alpha,\beta,f}^{(K)} \quad (22)$$

Based on the repair or refurbishment operation, it is equivalent to a "partial reset" or "partial reduction" of a specific existing wear value in this time series and spatial location. This segmented model provides a more realistic simulation of the wear and tear of a long-term building in service and the complex process of multiple renovations, providing a more informative analytical tool for building maintenance and life expectancy prediction.

5.2. Modeling of pedaling frequency time speculation

Once measurements of specific total wear bending depths $\{\tau_{\alpha,\beta,f}\}$ have been accomplished, we often expect to extrapolate backward to the frequency of pedaling, or foot traffic.

Defining a segmented time model, which can also be based on historical information to initially speculate on the distribution of foot traffic μ_k versus the pattern distribution $\{d_{e,k}\}$, and substituting material coefficients δ_f and repair coefficients $\zeta_{f,k}(\alpha,\beta)$ [9]into the model to obtain the theoretically predicted depth of wear. $\tau_{\alpha,\beta,f}$ This involves the application of the Least Squares, or MLE, method, which fits the theoretically predicted values to the actual measured values. Parameters such as $\{\mu_k, d_{e,k}, \delta_f, d_{f,k}\}$ [10] are adjusted to minimize the difference between observed wear and model predictions. Ultimately, based on the optimal solution $\overline{\mu_k}$, etc., to determine the frequency of stair stepping, the analysis can be used to obtain the average number of users per year and the time period of concentrated traffic.

In this paper, the first set 9, the establishment of the model analysis can be obtained, differential evolutionary fitting parameters for $[5.00000004e-03, 1.00000001e+02, 1.00000000e+02]$ the final residuals of the paradigm of 247.7547852399316. After fitting analysis iteration of the MSE = 0. 099102.The results of the data fitting analysis are shown below in Fig17-19.

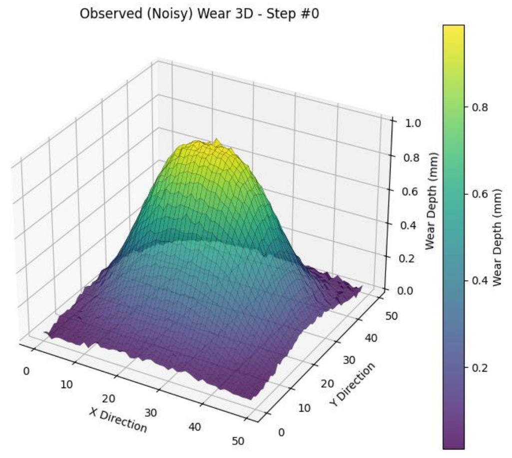

Fig. 17 3D results output graph(a)

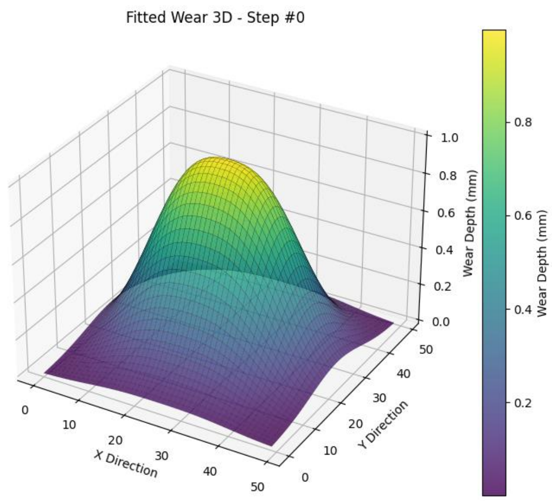

Fig. 18 3D results output graph(b)

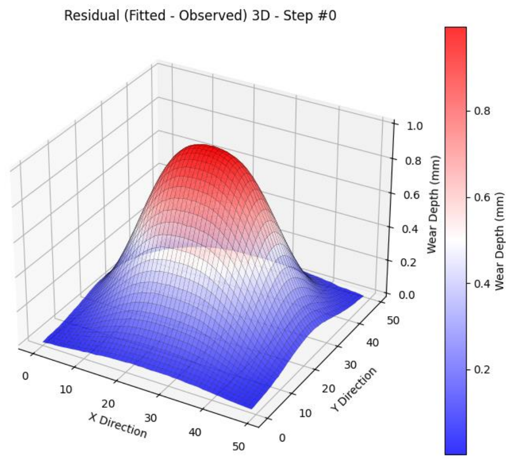

Fig. 19 3D results output graph(c)

6. Conclusion

The construction of this model is centered on staircase-related research, and the data acquisition and pre-processing were completed before the model was constructed, and the DBSACN model was selected for the classification of staircase types and shapes after comparison. The model was built to address the issues of staircase usage frequency, direction preference, multiple users, wear and tear, historical information, and construction time. Finally, the model is evaluated by sensitivity analysis to verify its accuracy and robustness in different scenarios.

7. References

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