

Research on the Estimation of Frequency of Speed Signal Based on Fast Fourier Transform

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Abstract. In modern aviation flight, airspeed measurement is crucial. Traditional airspeed measurement methods such as Pitot tubes are limited in extreme environments, making laser velocimetry the preferred choice due to its high accuracy and strong adaptability. However, it is necessary to address the noise interference and dynamic changes of the Doppler frequency shift signal. In this study, a phased frequency estimation model is proposed to systematically address the challenges of noise analysis and signal frequency estimation according to the characteristics of different flight phases. For signals with known parameters, noise components separation and statistical characteristics analysis reveal specific frequency components of noise, low-frequency energy concentration, and Gaussian white noise characteristics. When the amplitude and phase of the signal are known but the frequency is not, if maximum likelihood estimation fails, the FFT is used for calculation, and its effectiveness under known parameters is verified using the Welch method. If all signal parameters are unknown, the autocorrelation analysis method is employed to achieve phase-free frequency estimation, combined with the Hanning window function to improve the FFT, forming an adaptive parameter-free frequency estimation method. This study establishes a systematic frequency estimation method system, and the models at each stage demonstrate strong adaptability in complex noise environments, providing a general solution for dynamic signal processing. The results provide new algorithmic support for airspeed measurement technology, promoting the improvement of laser velocimetry accuracy and reliability.

Keywords: Fast Fourier Transform (FFT), White Noise, Frequency Estimation, Frequency Domain Analysis.

1. Introduction

With the rapid development of modern aviation technology, airspeed measurement, as an important parameter for aeronautics and navigation, its precision and reliability are crucial for assessing aircraft performance [1]. Traditional airspeed measurement methods such as pressure tube Pitot tube, hot-wire anemometer, and Laser Doppler Velocimetry, although performing well in many situations, may be limited in extreme temperatures, pressures, or complex environments. Laser velocimetry [2] is a feasible method for airspeed measurement. Its principle is to emit laser at a fixed frequency, and then obtain the signal light by using the Mie scattering effect on aerosol particles in the air for Doppler frequency shift [3]. By utilizing the coherent interference principle, a signal containing information about the Doppler frequency shift is obtained, and the frequency of this signal is estimated. In recent years, the method based on aircraft signal frequency estimation [4] has gradually become a research hotspot due to its potential application in various complex environments without the need for additional sensors.

Zhang et al. proposed a method for airspeed measurement of unmanned aerial vehicles based on radar signal frequency estimation. This method can effectively estimate the instantaneous airspeed and climb rate of the aircraft and performs well in complex environments by analyzing modulation information in radar echo signals. Through the introduction of adaptive filtering technology, Li et al. proposed an improved signal frequency estimation algorithm, which significantly enhanced the robustness of the method in complex environments. The research results show that this algorithm can maintain high estimation accuracy even in high noise and multi-environment interference scenarios. Smith and Thompson demonstrated that the airspeed measurement method based on frequency

estimation techniques can meet the high-precision measurement requirements under extreme conditions in the hypersonic environment by simulating radar signals of hypersonic aircraft.

Based on signal frequency estimation, the airspeed measurement method does not require additional sensors, is highly adaptive, minimally affected by the environment, and has strong versatility. It can measure airspeed in various environments, simplify complex problems, and is widely used. However, due to high computational complexity, the impact of signal noise on airspeed measurement accuracy, and performance in low-frequency and complex environments, there is currently no systematic frequency estimation method providing an algorithmic basis for airspeed measurement.

This study involves examples of spacecraft in spaceflight, with a sampling interval of $T_s = 2 \times 10^{-9} s$, receiving multiple Doppler frequency shift signals. The actual data received comes from <https://www.nmmcm.org.cn/>.

Given the complex situations of flight signals, this study establishes a systematic frequency estimation method system and discusses the following cases:

(1) In stage 1 of the flight, it is known that the non-noise portion of the received signal has an amplitude of 4, a frequency of 30×10^6 Hz, and a phase of 45° . The noise characteristics of the received data in flight stage 1 are to be analyzed.

(2) In flight phase 2, it is known that the amplitude of the actual received signal is 2, and the phase is 0° . Design a method to estimate the non-noise part frequency of the received signal in flight phase 2.

(3) Estimate the frequency of the received signal during flight phase 3 based on the database when the amplitude and phase of the non-noise part of the received signal are unknown.

2. Analyze the characteristics of the noise $z(t)$ received during flight phase 1

2.1. Model Establishment

A key step in airspeed measurement is the estimation of frequency information from the time series signal, where the signal satisfies the following expression:

$$x(t) = A \sin(2\pi f_0 t + \varphi) + z(t) \quad (1)$$

Where A represents the amplitude of the signal, f_0 is the frequency of the signal, φ is the phase of the signal, and $z(t)$ represents the noise information. The variables A , f_0 , φ are all known quantities.

2.2. Spectrum analysis method

The received signal $x(t)$ can be considered as the sum of the noise part $z(t)$ and the non-noise part $A \sin(2\pi f_0 t + \varphi)$. Therefore, the expression for the noise $z(t)$ can be obtained:

$$z(t) = x(t) - A \sin(2\pi f_0 t + \varphi) \quad (2)$$

Given the amplitude $A = 4$ of the non-noise part, the frequency at a specific value (unit: Hz), and the phase, by substituting these values into equation (2), we can obtain a discrete data-set of noise $z(t)$ changing with time t . The image of the separated received signal changing with time is shown in Figure 1. It is evident that the variations in the non-noise signal exhibit strong regularity, indicating that the randomness of the original signal mainly stems from the superimposed effects of the noise component.

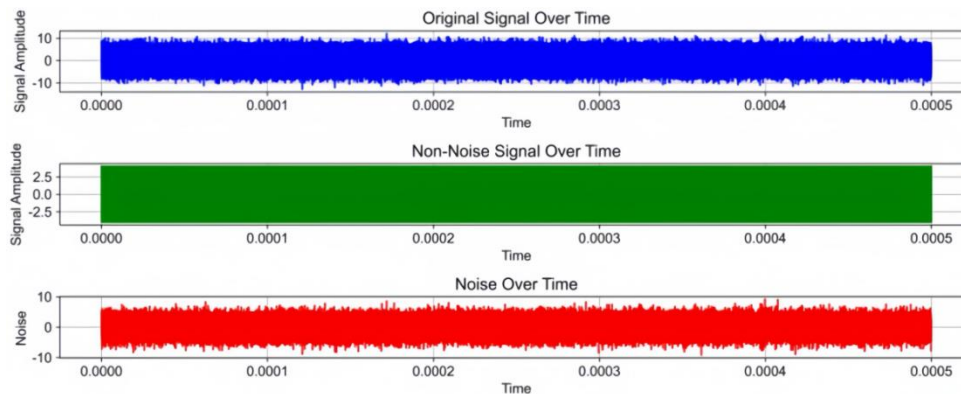


Figure 1. Graph of signal variation over time

Next, plot the frequency distribution histogram of the noise $z(t)$ with time t as the independent variable, as shown in Figure 2.

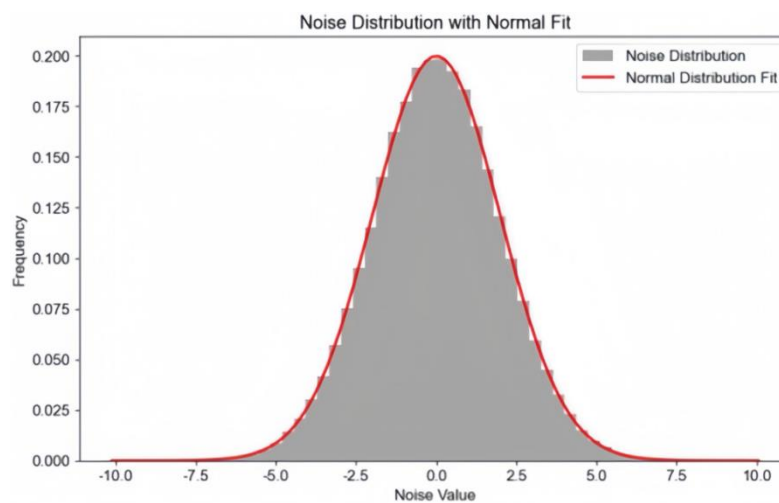


Figure 2. Histogram of the frequency distribution of noise $z(t)$

Obviously, noise basically follows a normal distribution with a mean of zero and no autocorrelation, in which case $p = 1.0$, meaning that the null hypothesis of "noise follows a Gaussian white noise process with a mean of 0" cannot be rejected.

Finally, utilizing the fast Fourier transform will convert it, enabling the transformation of time domain signals to frequency domain signals, as shown in Figure 3.

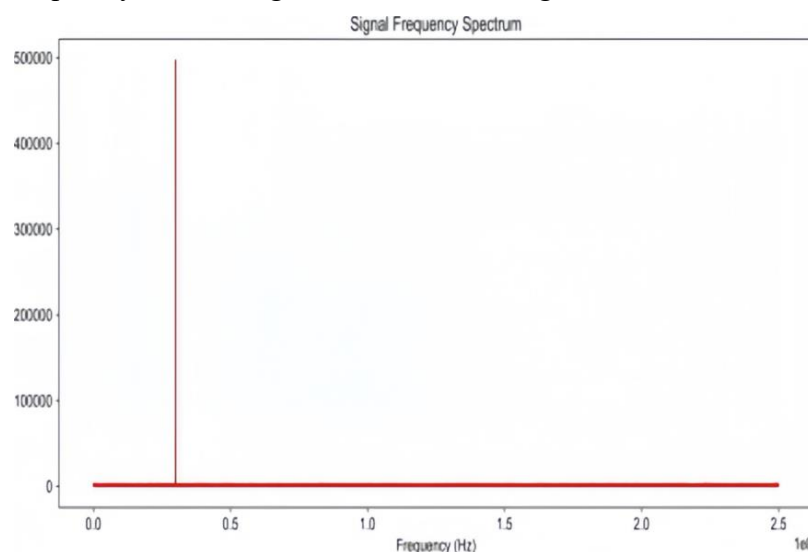


Figure 3. Frequency spectrum under fast Fourier transform (FFT) ($A \cdot \varphi$, f_0 known)

Further analysis reveals the characteristics of the noise $z(t)$: the noise mainly exhibits specific frequency components, with power concentrated at specific frequencies, low-frequency energy aggregation, energy attenuating at high frequencies, and the noise follows a Gaussian white noise process with a mean of 0.

3. Frequency estimation of non-noise components of the received signal in Flight Phase 2 based on Fast Fourier Transform (FFT)

3.1. Attempt to solve using Maximum Likelihood Estimation (MLE)

The maximum likelihood estimation [5] is a common method in frequency estimation. The goal is to estimate the frequency. First, write out the likelihood function, which is the conditional probability density function of the frequency given the observed data. Assuming the noise is Gaussian white noise, the likelihood function is:

$$L(f_0) = \prod_{i=1}^N \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x(t_i) - A \sin(2\pi f_0 t_i + \varphi))^2}{2\sigma^2}\right) \quad (3)$$

In order to maximize the likelihood function, it is necessary to take the logarithm and obtain the log-likelihood function:

$$\ln L(f_0) = -\frac{N}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^N (x(t_i) - A \sin(2\pi f_0 t_i + \varphi))^2 \quad (4)$$

The goal of maximum likelihood estimation is to find the values that maximize the log-likelihood function.

$$f_0 = \arg \max_{f_0} \ln L(f_0) \quad (5)$$

Given that the amplitude of the non-noise part at this time is $A=2$, phase $\varphi = 0^\circ$, these values are substituted into the model above to calculate the estimation of the non-noise part frequency under maximum likelihood estimation (MLE) using Python software, as shown in Figure 4.

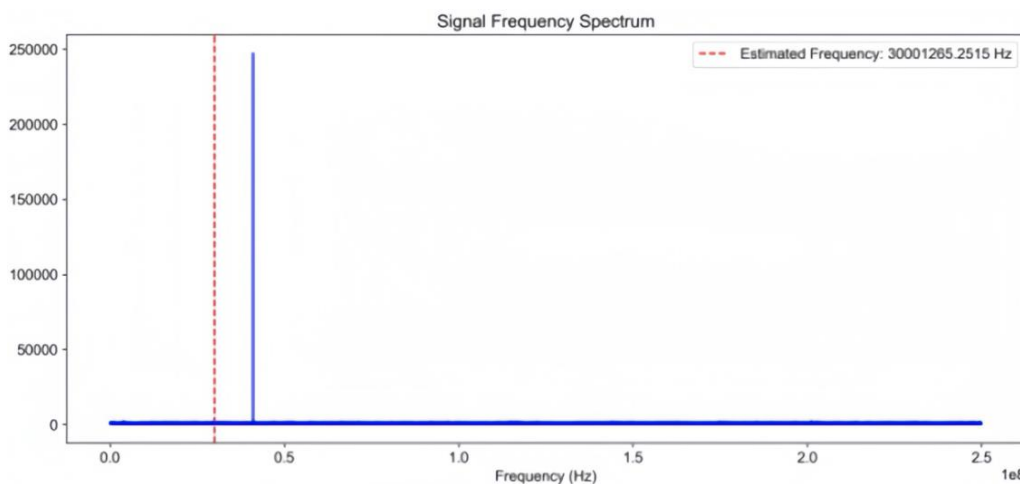


Figure 4. Frequency spectrum under maximum likelihood estimation (A , φ known)

Although the non-noise signal frequency under the maximum likelihood estimation is estimated to be 30001265.2515Hz, there is a significant difference between the maximum likelihood estimation peak and the actual peak (red dashed line) at this time, so the maximum likelihood estimation is not applicable for solving the flight stage 2. Next, the more suitable Fast Fourier Transform is used for calculation, and the results are verified using the Welch method.

3.2. Solving using Fast Fourier Transform (FFT)

Fast Fourier Transform (FFT) [6] is an efficient frequency analysis method that allows the transformation of time-domain signals to the frequency domain, making it easier to identify the main frequency components. The FFT method is suitable for stationary signals and can quickly find the main frequency components of a signal $x(t)$ during flight phase 2.

- (1) Perform FFT transformation on signal $x(t)$ to obtain frequency domain signal $X(f)$.
 - (2) Calculate the spectrum amplitude and find the frequency distribution with the maximum amplitude.
 - (3) Use the frequency estimated by FFT as the initial value of the maximum likelihood estimation.
- The formula for FFT estimation is:

$$X(f_0) = \sum_{i=1}^N x(t_i) \exp(-j2\pi f t_i) \quad (6)$$

When the amplitude $A=2$ and phase $\varphi = 0^\circ$ are plugged into the model mentioned above, calculations were performed using Python software. The estimated non-noise component frequency based on Fast Fourier Transform (FFT) was found to be 40999836Hz, as shown in Figure 5. Thus, Fast Fourier Transform is an effective method for estimating the non-noise component frequency under the known conditions of amplitude and phase.

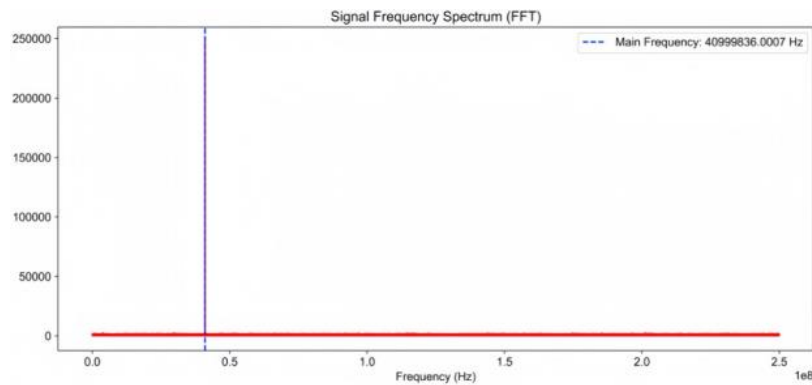


Figure 5. Frequency spectrum under fast Fourier transform (FFT) (A , φ known)

3.3. Verification Analysis of FFT Results using Welch Method

Given the lack of actual frequency data in Flight Phase 2, the Welch method was used to perform preliminary validation of the Fast Fourier Transform (FFT) estimates. A brief explanation of the Welch method is provided next:

In signal processing, the Welch method divides the signal into multiple overlapping segments, applies a window function to each segment (such as a Hamming window or Hanning window), calculates the Fourier transform of each segment, and then averages the power spectra of all segments to obtain a smoothed estimate of power spectral density. This method provides a smooth and stable spectral estimate, suitable for analyzing non-stationary signals. By substituting the amplitude $A=2$ and phase into the Welch model, and performing calculations using Python software, the estimated frequency of the non-noise part under the Welch method is 41015625Hz, as shown in Figure 6.

By comparing the estimated non-noise frequency component at 40999836Hz obtained through the Fast Fourier Transform (FFT) in the previous section, it can be seen that the estimated non-noise frequency components calculated by the two methods are very close. In other words, the results obtained by the Welch method can largely provide strong evidence for the Fast Fourier Transform.

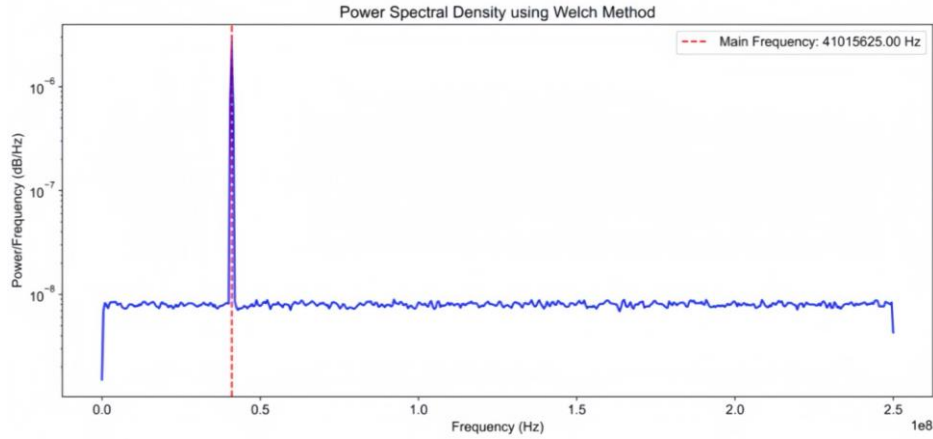


Figure 6. Frequency spectrum under Welch method (A , φ unknown)

3.4. Error Analysis for Flight Phase 2

Given the absence of true values, the frequency estimate of the non-noise portion obtained by the Welch method is used as a substitute for the true value in the error analysis. Let the true frequency be $f_0 = 41015625\text{Hz}$. Thus, the frequency estimate of the non-noise portion based on the Fast Fourier Transform is taken as the measurement result, denoted as $f_0^* = 40999836\text{Hz}$. Substituting into the error analysis model [7], we obtain: Absolute error $\varepsilon = 15789$, Relative error $r = 0.384951\%$.

Therefore, when the amplitude and phase of the non-noise portion of the received signal are known, the frequency estimation using the Fast Fourier Transform (FFT) can have an error controlled within a certain range (0.04%), showing excellent precision.

4. Based on the improved Fast Fourier Transform for frequency estimation of the non-noise part of the received signal in flight phase 3

4.1. Autocorrelation Methods

Using autocorrelation method to estimate the periodicity of the signal, and hence infer the frequency. The autocorrelation function of a signal is defined as:

$$R(\tau) = \int x(t)x(t+\tau)dt \quad (7)$$

Determine the frequency of the signal by assessing the stage characteristics of the function $R(\tau)$. Estimate the period of the signal $T = \tau_0$ by finding the lag time of the first significant peak τ_0 in the autocorrelation function, and then calculate the signal's frequency f_0 .

The autocorrelation signal plot shows the variation of signal autocorrelation with delay time, as shown in Figure 7. The first significant peak in the plot appears at around 35.714MHz delay, indicating that the signal's main frequency estimate is 35.714 MHz. This estimate accurately reflects the periodic characteristics of the signal, confirming the effectiveness of autocorrelation analysis.

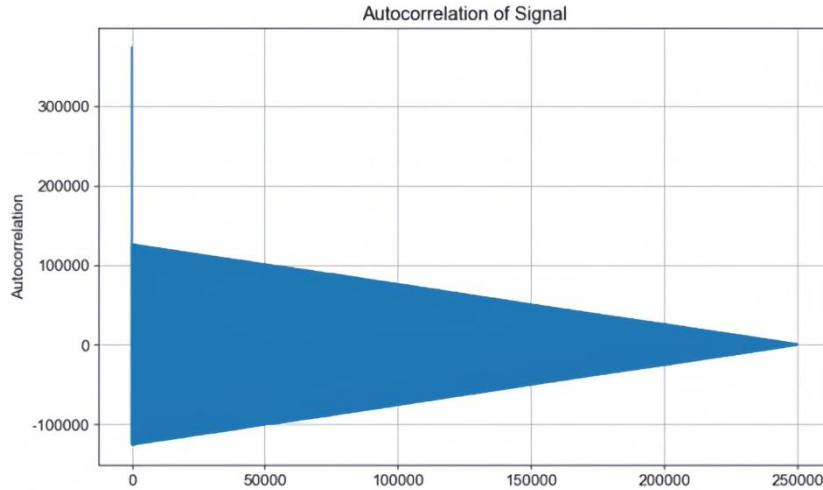


Figure 7. Autocorrelation Signal (Peak Detection) Graph

The autocorrelation analysis method is particularly suitable for phase signals with unknown phase and amplitude. After detecting the phase, the estimated frequency can be used as a signal to adjust the signal processing algorithm to ensure accurate frequency estimation even without phase information. Through frequency domain analysis, the main peak frequency of the "Flight Period 3" signal was found to be approximately 33333333Hz, as shown in Figure 8. This frequency is the highest energy frequency point in the spectrum plot, indicating it is the main frequency component of the signal.

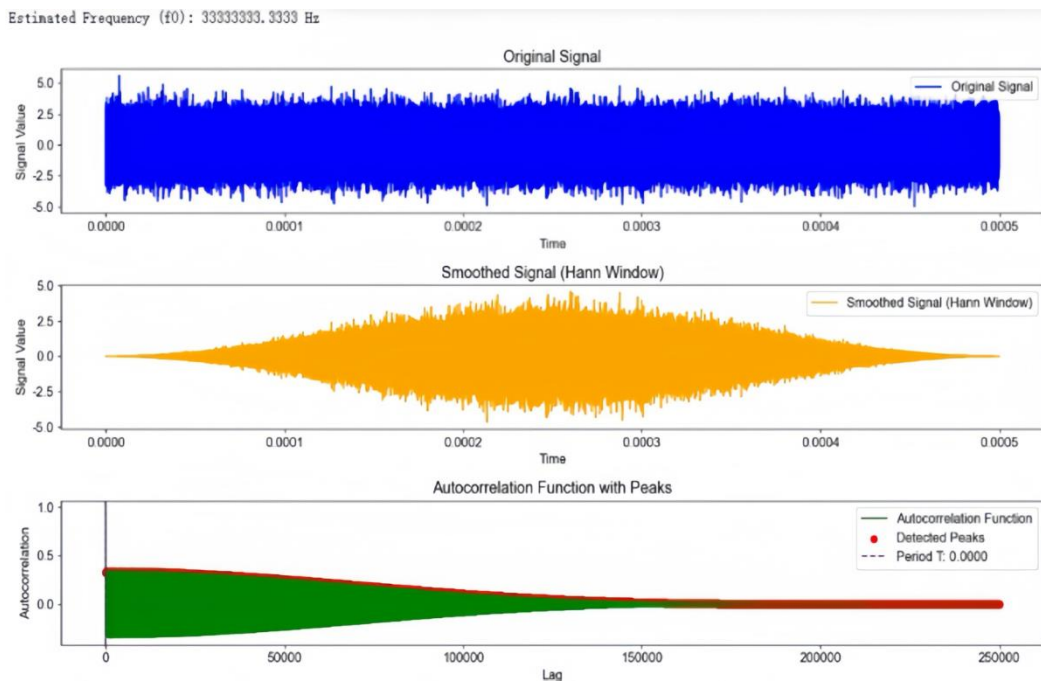


Figure 8. Original signal, smoothed signal, and autocorrelation analysis results

4.2. Improved Fast Fourier Transform

Introduction to the Hanning window function: The Hanning window is a widely used window function in the fields of signal processing and spectrum analysis. The mathematical expression of the Hanning window is as follows:

$$w(n) = 0.5 - 0.5 \cos\left(\frac{2\pi \cdot n}{N-1}\right) \quad (8)$$

Among them: $w(n)$ is the value of the window function at position, and n is the index of the sample points in the window, starting from 0. N is the length of the window (the number of sample points).

This formula describes the values of the Hanning window function at discrete sample points. The characteristic of the window is that it has a smaller amplitude at the two ends, and a relatively higher amplitude in the central region. This helps reduce spectral leakage generated by truncating signals, improving the accuracy of spectral analysis.

Improved Fast Fourier Transform [8, 9]: For the signal $x(t)$ of flight stage 3, the following steps can be performed:

1) Apply a window function $w(n)$ to signal $x(t)$ first, then perform FFT transformation to obtain the frequency domain signal $x(f)$.

2) Calculate the spectrum amplitude $|x(f)|$ and find the frequency distribution with the maximum amplitude.

3) The frequency obtained by FFT estimation serves as the initial value for maximum likelihood estimation. The formula for FFT estimation is:

$$X(f_0) = \sum_{i=1}^N |x(f)| \exp(-j2\pi f t_i) \quad (9)$$

Taking the amplitude and phase of the non-noise part, substituting them into the aforementioned model, and calculating using Python software [10], the estimated frequency of the non-noise part based on the Fast Fourier Transform (FFT) was 34999860Hz, as shown in Figure 9.

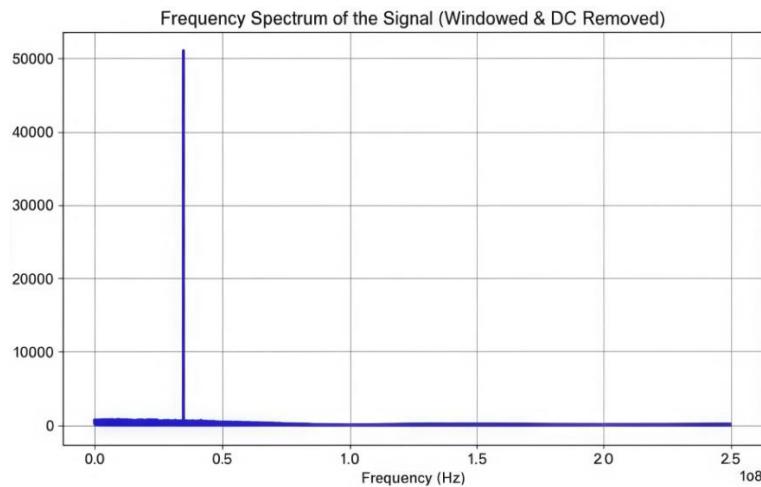


Figure 9. Frequency spectrum under Fast Fourier Transform (FFT) (A , φ unknown)

Thus, even under the conditions of unknown amplitude and phase, the improved fast Fourier transform remains an effective method for estimating the non-noise part frequencies.

4.3. Error Analysis for Flight Phase 3

Similarly, due to the absence of true values, the frequency estimate of the non-noise portion obtained using the autocorrelation method is used as a substitute for the true value in the error analysis. The frequency estimate of the non-noise portion based on the Fast Fourier Transform is taken as the measurement result. Thus, $f_0 = 33333333\text{Hz}$, $f_0^* = 34999860\text{Hz}$. Substituting into the error analysis model, we obtain: Absolute error $\varepsilon = 1666527$, Relative error $r = 0.04999581 < 5\%$.

Therefore, when the amplitude and phase of the non-noise portion of the received signal are unknown, the frequency estimation using the Fast Fourier Transform (FFT) can have an error controlled within a certain range (5%). Although the precision is slightly worse, it is still reliable.

5. Conclusion

This study conducted three tasks after noise separation: plotted $z(t)$'s frequency distribution histogram confirming Gaussian white noise (mean 0); analyzed four noise characteristics via FFT; and estimated noise-free signal frequency using FFT, Welch, and MLE to verify noise properties. MLE showed significant deviation in Phase 2 flight frequency estimation compared to actual peaks (red dashed line), prompting switch to FFT with Welch verification. For known amplitude/phase but unknown frequency, FFT effectively estimated non-noise frequency at 40999836Hz. For unknown parameters, autocorrelation analysis and Hanning window-enhanced FFT yielded 34999860Hz estimation. Error analysis revealed FFT's frequency estimation error for stationary signals remains within $\pm 5\%$, demonstrating its stability. The method boasts broad applicability, excellent interference resistance, and high measurement accuracy, providing a reliable technical means for stable continuous signal frequency analysis under normal conditions. Subsequent research will focus on addressing the challenges of non-stationary signal processing caused by meteorological disturbances, cloud scattering, and terrain occlusion in special circumstances, particularly optimizing algorithms for discontinuous signals with significant time interval spacing and brief duration characteristics.

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