Sunlight Spacing in Building Clusters Based on Solar Geometry Models

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Abstract. With rapid urbanization and increasing land scarcity, optimizing building layouts to meet residents' sunlight requirements has become crucial. This study proposes a method to analyze sunlight spacing in building clusters based on solar geometry models, aiming to improve urban planning and protect residents' sunlight rights. Focusing on Hangzhou as a case study, the apparent motion trajectory of the sun is integrated with a variable-step algorithm to calculate the minimum building spacing for different floor heights. For example, a 10-meter building height requires a minimum spacing of 13.3 meters. Furthermore, the study investigates the impact of building rotation angles on sunlight spacing, particularly in non-north-south-oriented layouts. The results demonstrate that rotation angles significantly affect sunlight distribution, highlighting the need to consider orientation in urban design. This research provides a practical framework for optimizing building layouts to balance land use efficiency and sunlight accessibility, offering valuable insights for urban planners and policymakers.

Keywords: Geometric Models, Sunlight Spacing, Building Layout, Rotation Angle.

1. Introduction

With the accelerating urbanization process in China, land resources are becoming increasingly scarce. In urban construction, ensuring reasonable spacing between buildings to meet the basic sunlight requirements of residents under limited land conditions has become a critical issue in urban planning. However, determining the sunlight spacing in building clusters is a complex engineering problem due to factors such as the sun's position changes across different seasons and times, as well as the height and orientation of buildings. Traditional building spacing designs often rely on empirical values and simplified calculations, lacking scientific theoretical support and precise mathematical models. This approach frequently results in excessive or insufficient building spacing, negatively impacting urban space utilization efficiency and residents' quality of life. In the face of constantly emerging high rise and irregular building clusters. These static rules expose three major defects: first, they cannot adapt to the dynamic changes of the sun's position, leading to severe distortions in shadow calculations; second, they only vaguely require "increased spacing" for buildings that are not oriented directly south, lacking a scientific basis for regulation; third, existing technical standards are outdated, and developers often exploit loopholes in the standards to reduce the spacing between buildings, resulting in numerous sunlight disputes that severely restrict urban sustainable development. Therefore, establishing a scientific research method based on solar geometry models to study the relationship between sunlight spacing in building clusters holds significant theoretical and practical value for optimizing urban spatial layouts, improving land use efficiency, and safeguarding residents' sunlight rights.

In this study, a sunlight model was first constructed to establish the relationship between the spacing of adjacent buildings and their heights, which was then solved using MATLAB. Building on this foundation, the influence of building orientation on sunlight spacing was considered by introducing rotation angles as a variable, further optimizing the model. Through practical calculations and analysis, it was found that traditional fixed search interval methods have certain limitations, particularly when determining sunlight spacing for taller buildings. Therefore, this study adopted a dynamic adjustment strategy for the search interval to improve the accuracy and rationality of the results

2. Mathematical Expression of Building Sunlight Model

2.1. Calculation of Solar Altitude Angle and Azimuth Angle

By integrating the solar geometry model, this study focuses on newly constructed residential buildings in Hangzhou that are arranged in parallel and oriented in a true north-south direction[1]. To characterize the shadow conditions of these buildings at different times, the solar altitude angle and solar azimuth angle are utilized. The specific representation is as follows:

2.1.1. Solar Altitude Angle α

The solar altitude angle refers to the angle between the line connecting the sun to the origin of the coordinate system and the horizontal plane [2]. Optimization calculation of heliostat field based on the topic of the 2023 Higher Education Society Cup National College Students Mathematical Contest in Modeling, the formula for calculating the solar altitude angle is [3]:

$$\sin\alpha = \cos\delta\cos\varphi\cos\omega + \sin\delta\sin\varphi \tag{1}$$

In the formula, α represents the solar altitude angle, with a range of [0, 90°], and δ denotes the solar declination, which is the latitude at which the sun is directly overhead. φ is the local latitude, and ω is the solar hour angle. The methods for calculating the solar declination and solar hour angle are as follows:

(1) The solar declination, which is the latitude at which the sun is directly overhead, varies between the Tropic of Cancer and the Tropic of Capricorn throughout the year. It can be expressed by the following formula[4]:

$$\sin\delta = \sin\frac{2\pi D}{365}\sin(\frac{2\pi}{365}23.45) \tag{2}$$

In the formula, *D* represents the number of days between the current date and the spring equinox.

(2) The solar hour angle represents the angular displacement between the local hour circle and the solar noon meridian. It is calculated based on the difference between local time *T* and solar noon (12:00 PM). The formula is expressed as[5]:

$$\omega = \frac{\pi}{12}(T-12) \tag{3}$$

2.1.2. Solar Azimuth Angle γ

The solar azimuth angle refers to the angle measured clockwise from true north along the horizon to the line connecting the origin of the coordinate system and the sun's projection on the ground. The formula is expressed as [6]:

$$\cos\gamma = \frac{\sin\delta - \sin\alpha\sin\varphi}{\cos\alpha\cos\varphi} \tag{4}$$

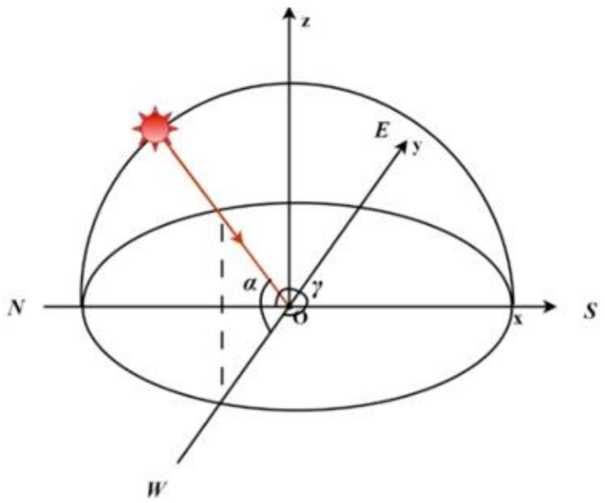

Figure 1. Coordinate Axes and Solar Azimuth Angle.

2.2. Construction of the Building Sunlight Model

Let H be the height of the building. By solving for the solar altitude angle and solar azimuth angle, the shadow conditions of the residential building at different times can be determined[7]. Since the parameters such as solar altitude for the two adjacent buildings in the horizontal coordinate system are similar, they are considered identical in this model. Based on this, the projection of the shadow in the direction of the building spacing on the ground is established as[8]:

$$\frac{H}{\tan\alpha}\times\left|\cos\gamma\right| \tag{5}$$

According to the data in Hangzhou City planning and management technical regulations, let X be the building spacing. To ensure that the shadow of the front building does not block the balcony of the first-floor resident in the rear building during sunlight hours, the relationship between the projection and the building spacing should satisfy[9]:

$$\frac{H}{\tan\alpha}\times\left|\cos\gamma\right|\le X+0.9,\Delta t\ge 2,t\in[8,16] \tag{6}$$

This equation ensures that the sunlight duration on the winter solstice day falls within the true solar time range and is no less than 2 hours. A building sunlight model is constructed to describe the relationship between the minimum spacing of adjacent residential buildings and the height of the residential buildings.

3. Analysis of Building Spacing Optimization

The initial building spacing is set to 0. By gradually increasing the value of the building spacing X with a step size of $\Delta X=0.1$ (meters), and within the true solar period from 8:00 to 16:00, the shadow length at each moment is calculated step by step until the requirement of a cumulative 2-hour sunlight duration is met, at which point the calculation is terminated early. The step size for adjusting the floor height is set to $\Delta H=1$ (meters), and the step size for sunlight duration is set to $\Delta hours=0.1$ (hours).

To more accurately simulate real-world conditions, an additional calculation condition is introduced: the starting point is set at the bottom windowsill (0.9 meters above the indoor ground level at the exterior wall position). Therefore, during the solving process, the starting point for the shadow projection calculation is adjusted to the building height minus 0.9 meters. Finally, MATLAB was used to process the calculation tasks of different heights, and the results were drawn as shown in Figure 2 below:

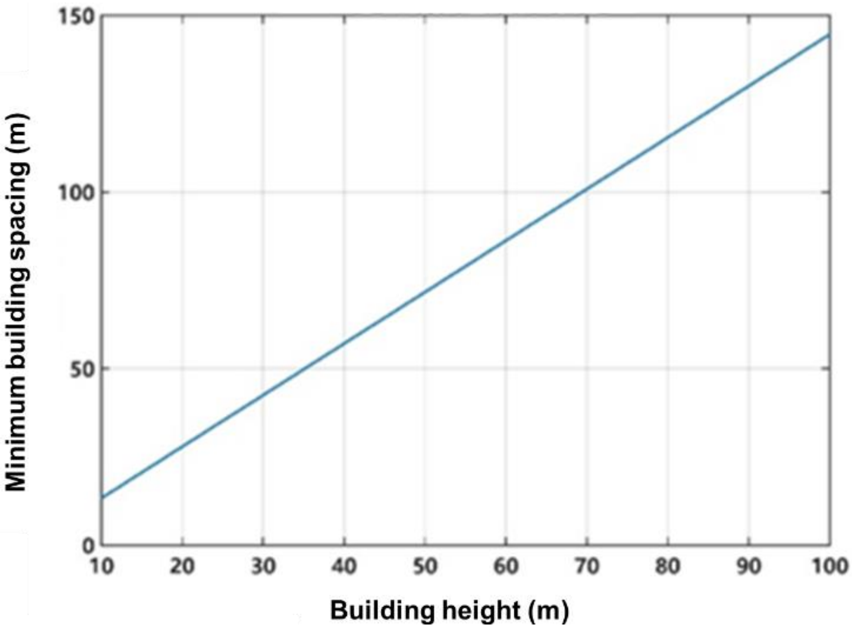

Figure 2. Minimum Building Spacing Corresponding to Different Building Heights.

The results for the first 9 floor heights and their corresponding minimum building spacing are presented in Table 1 below:

Table 1. First 9 Floor Heights and Their Corresponding Minimum Building Spacing.

Floor Height	Minimum Building Spacing	Floor Height	Minimum Building Spacing	Floor Height	Minimum Building Spacing
10	13.3	11	14.8	12	16.2
13	17.7	14	19.2	15	20.6
16	22.1	17	23.5	18	25.0

Residential buildings are classified into five height categories. Low-rise buildings comprise one to three floors, primarily including single-family detached houses, duplexes, and townhouses. Mid-rise buildings range from four to six floors and typically feature shared staircases as vertical circulation [10]. Medium-high-rise buildings span seven to nine floors and offer superior lighting and ventilation conditions. High-rise buildings extend beyond ten floors and are generally equipped with comprehensive amenities including elevators, fitness facilities, and swimming pools. Super high-rise buildings exceed thirty floors, demand sophisticated engineering solutions and substantial investment, and are predominantly located in central urban districts of major cities [11].

According to the standard floor height of 2.80 meters and the height of window sills in the Code for Residential Design, residential buildings are divided into the following Table 2:

Table 2. Classification of Residential Building Heights.

	Low-Rise	Mid-Rise	Medium-High-Rise	High-Rise	Super High-Rise
Floor (Unit: m)	<3	4-6	7-9	10-30	>30
Height (Unit: m)	<9.3	12.1-17.7	20.5-26.1	28.8-84.9	>84.9

For the height of representative residential buildings, the shadow length variation curve from 8:00-16:00 on the winter solstice is drawn as shown in Figure 3 below:

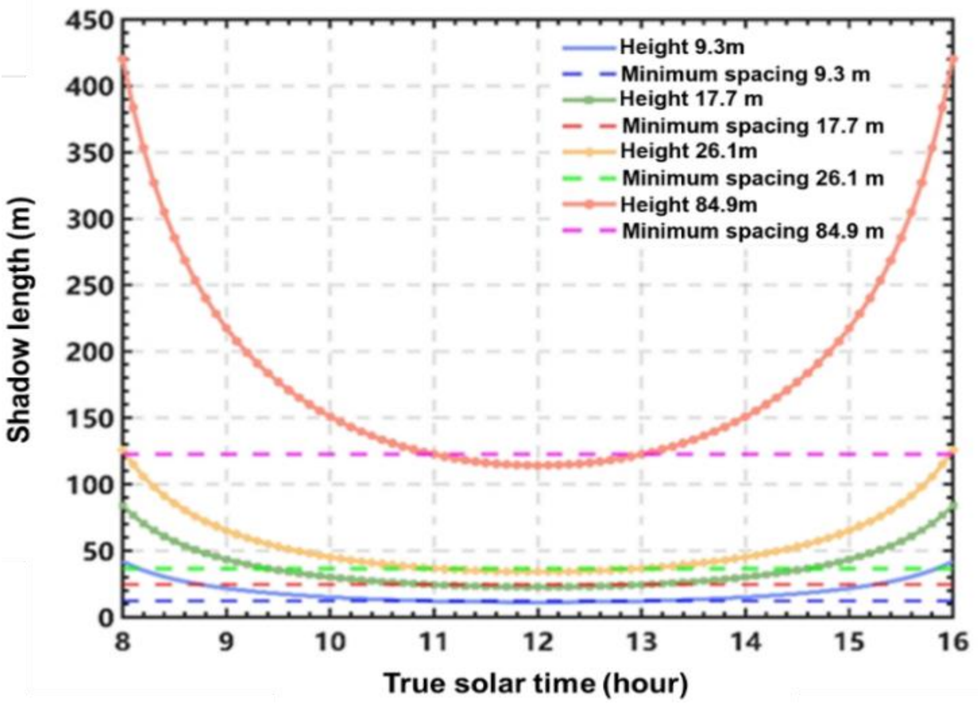

Figure 3. Shadow Length Variation Corresponding to Different Building Heights.

The analysis reveals a positive correlation between building height and shadow length. Consequently, taller structures require proportionally greater minimum spacing to ensure unobstructed sunlight exposure, while shorter buildings need comparatively smaller spacing. This height-spacing relationship necessitates careful consideration of building dimensions during residential planning to satisfy sunlight requirements.

4. Analysis of Building Spacing Under Rotation Angles

Building on the previous section, to describe the difference between the two parallel buildings and their original positions, a rotation angle θ is introduced to represent the angle by which the current position is rotated clockwise relative to the original position, as shown in Figure 4:

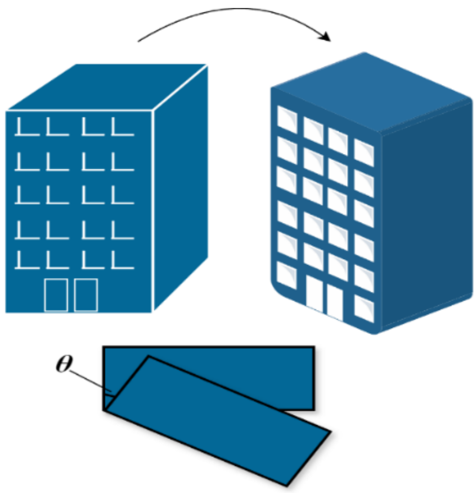

Figure 4. The building rotates clockwise.

At this point, the relationship between the minimum spacing of adjacent residential buildings and the height of the residential buildings satisfies[12]:

$$\frac{H}{\tan\alpha}\times\left|\cos\left(\gamma-\theta\right)\right|\leq X+0.9,\Delta t\geq 2,t\in\left[8,16\right] \tag{7}$$

4.1. Step-by-Step Incremental Spacing Method

Similar to the previous analysis, the building spacing X increases incrementally until satisfying sunlight requirements: ≥2 hours cumulative sunlight during the winter solstice (8:00-16:00 true solar time) [13]. The rotation angle affects shadow projection through the $\cos\theta$ term in the shadow length formula, altering shadow extension.

The model uses representative building heights from the established residential classification. Since Hangzhou's new residential areas feature parallel but non-north-south arrangements, we examine rotation angles $\theta = [15, 30, 45, 60, 75]$ to model differently oriented buildings.

Finally, MATLAB was used to process the calculation tasks of different heights and rotation angles. The results are shown in Table 3 below:

Table 3. Minimum Building Spacing Corresponding to 4 Building Heights.

Minimum Building Spacing(Unit: meters)	Rotation Angle				
	15°	30°	45°	60°	75°
9.3	12.7	14.2	17.4	24.6	47.4
17.7	25.4	28.4	34.7	49.1	94.8
26.1	38.1	42.5	52	73.6	142.1
84.9	126.9	141.6	173.4	245.2	473.6

For buildings of different heights, the shadow length variation curves over time under five rotation angles are shown in Figure 5 below:

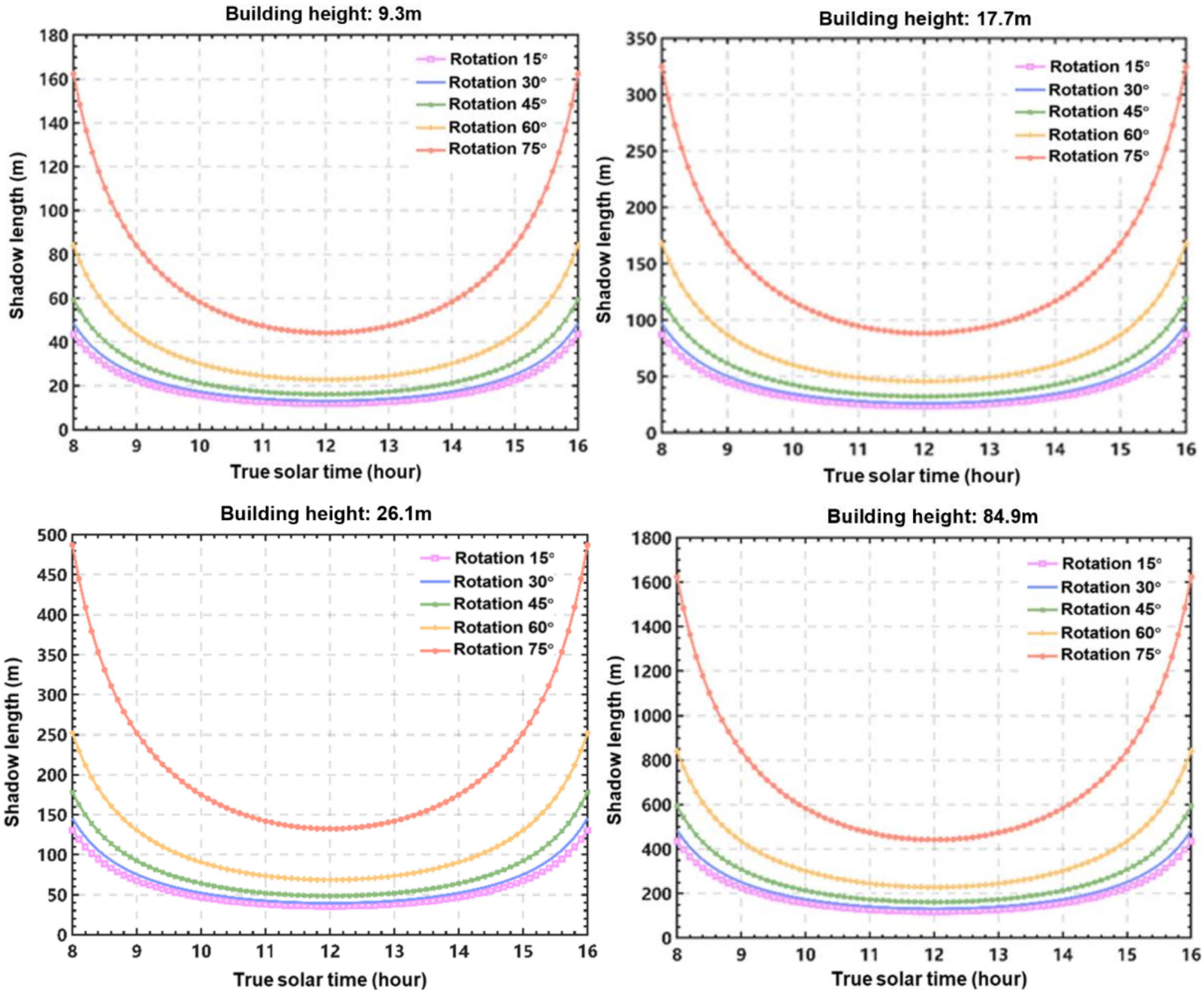

Figure 5. Shadow Length Variation for 4 Building Heights Under 5 Rotation Angles.

As can be seen from Figure 6, for the same building height under different rotation angles, when the rotation angle $\theta \in (0,90)$ increases, the variation range of the shadow length over time also increases. Initially, the curves almost overlap, then gradually diverge, and finally form a trend of significantly exceeding.

4.2. Optimization Using the fminbnd Function

To verify and optimize the calculation method, the MATLAB built-in function fminbnd is used for recalculation. The search intervals for fminbnd are set to[0,50], [0,65], [0,80], and [0,95], ensuring that the solution, i.e., the minimum building spacing, is found within a reasonable range. The curves of minimum building spacing and rotation Angle under different building heights are shown in Figure 6 below:

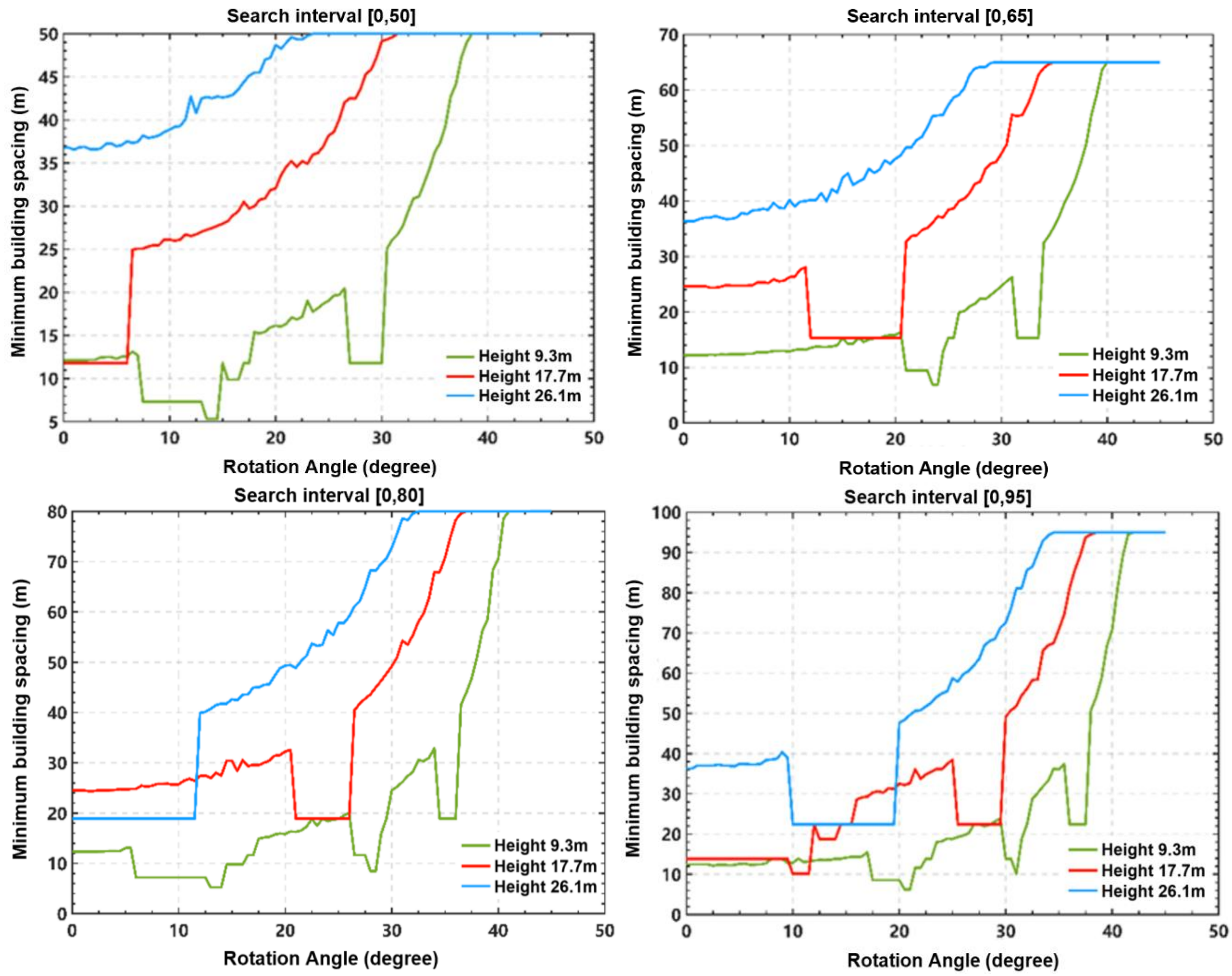

Figure 6. Minimum Building Spacing Versus Rotation Angle for Different Building Heights

According to the figure above, the minimum building spacing calculated by the fminbnd function is not entirely consistent with the step-by-step incremental spacing method for different search intervals, and there are some anomalous intervals. To find a solution that meets the requirements and is universally applicable, this study adopts a strategy of controlling the same building height and then plots the relationship curves between the minimum building spacing and the rotation angle under different search interval conditions. The results are shown in Figure 7 below:

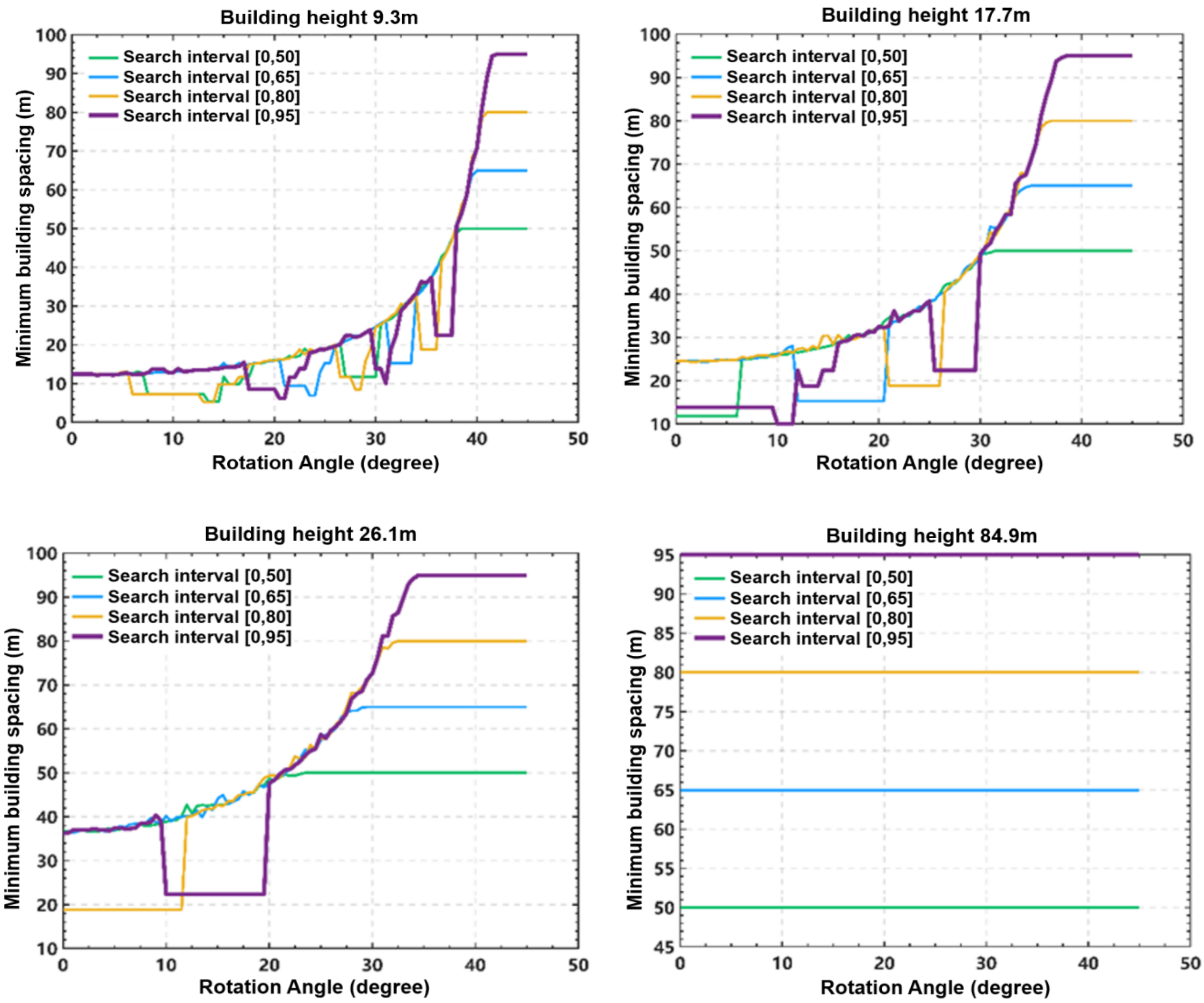

Figure 7. Minimum Building Spacing Versus Rotation Angle for the Same Building Height.

The four curves of different colors represent the four search intervals of the fminbnd function. The new curve formed by the overlap of the four curves is the relationship curve between the minimum building spacing and the rotation angle for the given building height, satisfying the condition of a cumulative daily sunlight duration of no less than 2 hours. However, when the building height is 84.9 meters, the solution obtained by this method has significant errors. Therefore, for taller residential buildings, this method is not applicable, and only the step-by-step incremental spacing method can be used to obtain the results.

4.3. Comparative Analysis

Two methods are available for calculating minimum building spacing at given rotation angles: the stepwise increment method and MATLAB's fminbnd optimization. The stepwise method increases spacing in fixed increments until meeting sunlight requirements, with accuracy dependent on step size - smaller steps improve precision but increase computation time. The fminbnd function, employing golden-section search and parabolic interpolation, efficiently finds local minima within specified intervals, typically converging faster than stepwise methods. However, its performance depends on objective function characteristics and may produce errors with improperly defined search intervals.

Verification shows both methods work effectively for moderate-height buildings. The fminbnd function efficiently generates complete spacing-angle curves, while the stepwise method provides precise numerical spacing values. However, for taller structures (e.g. 84.9m), only the stepwise method remains viable as fminbnd fails to converge properly.

5. Conclusions

This study's model offers valuable applications for urban planning, particularly in optimizing land use efficiency. Establishing precise height-to-spacing relationships (e.g., 10m height requires 13.3m minimum spacing), enables planners to balance land utilization with sunlight access. The analysis of rotation angles proves especially crucial for non-north-south layouts. Through comparative evaluation of the stepwise increment and fminbnd methods, we demonstrate their respective advantages: while both works for moderate heights, only the stepwise method remains viable for taller structures (≥84.9m). Future improvements should focus on algorithm optimization for high-rise applications and the incorporation of additional variables like terrain and climate conditions.

This study develops a solar geometry-based method to analyze building cluster spacing, offering a scientific approach to urban planning through mathematical modeling and computational analysis. The research optimizes spatial layouts, enhances land use efficiency, and protects sunlight access—critical for sustainable urban development under limited land resources. By quantifying sunlight requirements, the model enables efficient residential area design that balances solar exposure with land utilization, supporting both livability and sustainability. The findings provide valuable theoretical and practical insights for urban planners and architects.

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