Optimized design based on maximum output power from wave energy

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Abstract. A wave energy output device is a renewable energy technology that harnesses the vertical and horizontal motion of ocean waves to convert marine kinetic energy into mechanical or electrical power , offering significant development potential. This paper models the vertical and longitudinal motion of a wave energy device's float and oscillator using Newton's second law and solves the derived differential equations with an improved Lungkuta algorithm, obtaining displacement and velocity data over 40 cycles . Based on the motion model, it derives the damper's output power per cycle and builds a single-objective optimization model to maximize average output power. Using a small curved-edge trapezoidal approximation and a variable-step search, the optimal damping coefficient is found. With constant damping, the maximum average output power is 229.6819 W at a damping coefficient of 37,639; when the damping varies with velocity, the maximum average output power reaches 230.0163 W with an optimal coefficient of 81,478 and a power coefficient of 0.3377.

Keywords: Newton's second law, Longgkuta algorithm, Variable step size search method.

1. Introduction

As an important form of marine renewable energy, wave energy is widely distributed, abundantly available, and holds promising application prospects. The energy conversion efficiency of wave energy devices is one of the key challenges for large-scale utilization [1-3]. A typical wave energy device consists of a float, an oscillator, a central shaft, and a power take-off (PTO) system, which includes springs and dampers. Under the action of waves, the float moves and drives the oscillator, and their relative motion powers the damper, with the generated work serving as the energy output.

Researchers have conducted extensive studies on wave energy output devices, focusing on aspects such as device structure design, energy conversion mechanisms, optimization of damper and spring parameters, and dynamic response analysis under various sea conditions. Many scholars have proposed mathematical models based on the relative motion between the float and oscillator, using numerical simulations or experimental methods to evaluate the energy capture efficiency. Additionally, different types of power take-off (PTO) systems—such as hydraulic, pneumatic, and electromagnetic—have become research hotspots to improve energy conversion efficiency and device stability. In recent years, optimizing control strategies, enhancing resistance to harsh sea conditions, and reducing manufacturing costs have also emerged as key research directions.

In previous studies, some mathematical models were overly idealized, neglecting the effects of complex sea conditions and structural nonlinearities [4-5]. Additionally, limited optimization methods for damper and spring parameters made it difficult to balance efficiency and stability, while practical losses and reliability issues of the power take-off systems were insufficiently addressed. To overcome these shortcomings, this paper considers two cases for the damping coefficient of the float and oscillator during vertical oscillation: constant damping and velocity-dependent damping. Mathematical models of the float and oscillator motions are established, and a variable-step search method [6-10]is applied to optimize the output power. This study develops a mathematical model describing the motions of the floater and the oscillator in a wave energy conversion device, along with an optimization model for its power output. Two cases are considered: one with a constant damping coefficient and the other with a damping coefficient that varies with the relative velocity between the floater and the oscillator. For each case, the maximum output power is determined using a variable step-size search algorithm.

2. Modeling of the motion

2.1. Modeling of float and oscillator pendant motion

The improved Longgkuta algorithm is a widely used numerical method for solving ordinary differential equations, offering enhanced stability and accuracy in nonlinear, coupled, and multi-degree-of-freedom systems. Compared to the traditional version, improvements such as adaptive step-size control and error estimation reduce cumulative errors and improve efficiency. It has been effectively applied in engineering fields like mechanical vibration, spacecraft dynamics, biomedical modeling, and wave energy device analysis, providing a powerful tool for complex system simulation and optimization.

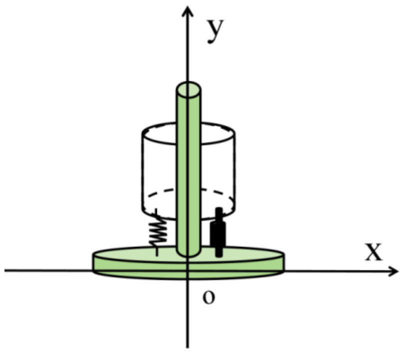

Figure 1. Cartesian coordinate system with the intersection of the central axis and the base plane as the origin.

Since this question only discusses the float and vibrator in the wave to do pendulum motion, in order to facilitate the calculation, the axis of the central axis for the y-axis, the wave is stationary when the axis of the axis of the bottom intersection as the center of the circle to establish a one-dimensional coordinate system. The one-dimensional coordinate system is set up as illustrated in Figure 1.

When the sea level is at rest, respectively, the float and vibrator force analysis and the establishment of force equilibrium equations, the vibrator's center position coordinates and the center of the bottom of the central axis, respectively, the vibrator and the float position. The force analysis of the float and the oscillator is illustrated in Figure 2 and Figure 3.

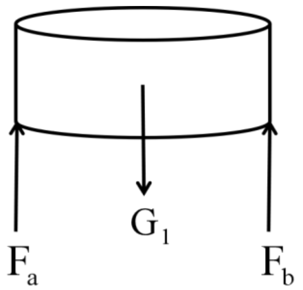

Figure 2. Analysis of the forces on the pendulum.

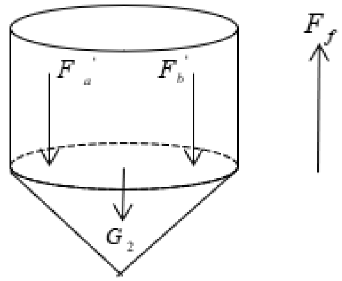

Figure 3. Analysis of the forces on the float.

The equilibrium equations of force are:

$$\begin{cases} F_a + F_b - G_1 = 0 \\ F_f - F_a' - F_b' - G_2 = 0 \end{cases} \quad (1)$$

Where G_1 and G_2 are the gravitational forces of the oscillator and the float itself, and are the external forces of the spring and damper, respectively, and F_f is the buoyant effect of seawater on the float.

At this time both the oscillator and the float are at rest, the relative speed of motion is 0, while the spring is in compression and is subjected to the force of the spring and the damper, respectively:

$$\begin{cases} F_a = k(x_1 - x_2) \\ F_b = -C(v_1 - v_2) = 0 \end{cases} \tag{2}$$

Where k is the spring stiffness, C is the damping coefficient of the damper, x_1 and x_2 are the distances of the oscillator and the float from the origin on the y-axis of the coordinates, v_1 and v_2 are the velocities of the two, respectively. Then at rest, the distance between the vibrator and the float in the vertical height is 0.2980425m, which is the initial position of the vibrator.

Force analysis of the float, the float in a pendulum motion, the external forces are: its own gravity, wave excitation force, hydrostatic restoring force, and spring elasticity and damper damping force.

Wave incentive force $F_{w,z}(t)$ meets:

$$F_{w,z}(t) = f\cos\omega t \tag{3}$$

The spring elasticity $F_a^{'}$ is proportional to the relative displacement of the oscillator and the float and can be expressed as:

$$F_a^{'}(t) = -k\left(x_1(t) - x_2(t) - l\right) \tag{4}$$

The damper damping force $F_b^{'}$ is proportional to the relative velocity of the oscillator and the spring and can be expressed as:

$$F_b^{'}(t) = C\left(x_1^{'}(t) - x_2^{'}(t)\right) \tag{5}$$

The hydrostatic restoring force is caused by the change in the magnitude of the buoyancy force on the float as it makes a pendulous motion, and the expression for its magnitude can be divided into the following two cases:

Assuming that the height of the cone below the float is h_1, the height of the cylinder above it is h_2, and the radius of the base of the cone is R, the depth of the float submerged in seawater when the sea level is stationary is h, and the depth of its submergence in seawater when the float is doing a pendulum motion is $h + x_1(t)$.

When $h + x_1(t) \le h_1$, the magnitude of the hydrostatic restoring force can be expressed as follows:

$$F_r(t) = \frac{1}{3}\rho g\pi R^2 h_1\left(\frac{h + x_1(t)}{h_1}\right)^3 \tag{6}$$

When $h + x_1(t) > h_1$, the magnitude of the hydrostatic restoring force can be expressed as follows:

$$F_r(t) = \rho g\pi R^2\left(h + x_1(t) - \frac{2}{3}h_1\right) \tag{7}$$

From Newton's second law, the equation of motion for the pendulous motion of the float is:

$$(m_1 + m_1^{'})x_1^{''}(t) = G_a + F_{w,z}(t) - F_a(t) - F_b(t) - F_r(t) \tag{8}$$

Where m_1 is the oscillator mass and $m_1^{'}$ is the additional inertial mass

2.2. Modeling of damper output power optimization

The power expression when the object is moving flat is:

$$P = Fv \tag{9}$$

The expression for damper resistance is:

$$F_b(t) = -C\left(x_1^{'}(t) - x_2^{'}(t)\right) \tag{10}$$

The expression for the output power of the damper is:

$$P(t) = F_b(t)\left(x_1^{'}(t) - x_2^{'}(t)\right) \tag{11}$$

Let the period of the pendulum motion of the float and the oscillator be T ,then the work done by the resistance of the damper to the outside world during one cycle of motion can be expressed as:

$$W = \int_0^T F_b(t)\left(x_1^{'}(t) - x_2^{'}(t)\right)dt \tag{12}$$

The average power over a cycle can be expressed as:

$$\overline{P} = \frac{W}{T} \tag{13}$$

In order to find the best damping coefficient that makes the PTO system output power maximum, it is necessary to establish a mathematical optimization model with the output power as the optimization objective.

The oscillator can only move inside the float, which is shaped as a cylinder, with its geometric center as the object of study, and the length of the spring can not be zero, so there is:

$$\frac{H}{2} < x_1(t) \le h_2 - \frac{H}{2} \tag{14}$$

where H is the height of the vibrator cylinder

Since the float cannot fly off the surface of the water, there is:

$$x_2(t) < h \tag{15}$$

In summary, the output power optimization model is:

Optimization Objective:

$$\arg\max \overline{P} = \frac{W}{T} \tag{16}$$

Constraints

$$s.t.\begin{cases} (m_1 + m_1^{'})\ x_1^{''}(t) = F_a(t) + F_b(t) - G_1 \\ F_b(t) = -C\left(x_1^{'}(t) - x_2^{'}(t)\right) \\ \frac{H}{2} < x_1(t) \le h_2 - \frac{H}{2} \\ x_2(t) < h \end{cases} \tag{17}$$

When the damping coefficient is constant, there is a damping coefficient $C \in [0,10^5]$,when the damping coefficient varies, there is a damping coefficient $C = \alpha\left(x_1^{'}(t) - x_2^{'}(t)\right)^{\beta}$, $\alpha \in [0,10^5]$, $\beta \in [0,1]$.

3. Results

3.1. Solution of the pendant motion model

Determination of initial conditions: the incident wave slope is $\omega = 1.4005\,s^{-1}$, and there are: the initial moment, the float and the oscillator of the equilibrium in still water. Set the initial conditions so that the initial displacement and initial velocity of the object are zero.

Processing of differential equations: Since the equations solved in this problem are second-order binary differential equations, it is necessary to transform the equations into first-order multivariate equations, and at the same time move the first-order derivative term to the left-hand side of the equation, and the residual term to the right-hand side of the equation, and then solve the problem using the Lungkuta algorithm.The transformed multivariate first order equations are:

$$\begin{cases} x_f^{'}(t) = u(t) \\ x_z^{'}(t) = \omega(t) \\ u^{'}(t) = (f\cos\omega t + C_h(\omega(t) - u(t)) + K_h(x_z(t) - x_f(t)) - f_{zh}u - \rho g A x_f(t))/(m_f + m^{'}) \\ \omega^{'}(t) = -(C_h(\omega(t) - u(t)) + K_h(x_z(t) - x_f(t))/m_z \end{cases} \tag{18}$$

Iteration of the algorithm: set the step size $h = 0.0001$, traversing the length of 40 wave cycles, in accordance with the recursive formula to find out the above four first-order differential equations corresponding to $k_1 . k_2 . k_3 . k_4$, and then find out the corresponding next discrete value $y(n+1)$.

The application of the algorithm and model solving: it is easy to know that the displacement of the float is $x_f(t)$, the displacement of the vibrator is $x_z(t)$, the derivative of the displacement of the float $u(t)$ is the velocity of the float, and the velocity of the vibrator is the velocity of the vibrator, and the value corresponding to the interval of 0.02s is taken.

3.2. Display and analysis of model solution results

The following figure shows the change of displacement and velocity of the pendulum when the linear damping coefficient is fixed for the first 40 cycles. Observation can be found that the motion of the float and oscillator reaches stability after a period of time, and after the model test, the period of the motion of the float and oscillator after stabilization and the period of the wave excitation.

The states of motion of the float and oscillator in geodetic coordinates are shown in Table 1:

Table 1. Velocity and displacement of the float and oscillator when the damping coefficient is constant.

Time(s)	Float displacement(m)	Float velocity(m/s)	Oscillator displacement(m)	Oscillator velocity(m/s)
10	-0.190711329	-0.641008552	-0.21167884	-0.69395345
20	-0.590684336	-0.240951445	-0.634248453	-0.272775651
40	0.285374415	0.31297134	0.296499146	0.332912423
60	-0.314505563	-0.479455012	-0.331435947	-0.515727968
100	-0.083614704	-0.04211498	-0.084068054	-0.643001946

The data presented in the table above indicate that the float and oscillator experience irregular vibrations when subjected to wave forces.

The pendulum displacement and velocity for the first 40 cycles of linear damping coefficient variation are shown in Table 2:

Table 2. The velocity and displacement of the float and oscillator when the damping coefficient varies with velocity.

Time(s)	Float displacement(m)	Float velocity(m/s)	Oscillator displacement(m)	Oscillator velocity(m/s)
10	-0.205711329	-0.654816818	-0.234529161	-0.699955345
20	-0.611107559	-0.254789324	-0.664821131	-0.277275651
40	0.268767727	0.295301668	0.280154837	0.312912423
60	-0.327164218	-0.491517963	-0.346592337	-0.525727968
100	-0.088408175	-0.609830583	-0.095578533	-0.643001946

3.3. Solution of the optimization model of damper output power

The system is stationary at sea level at the initial moment, and the initial values of the equations are set so that the displacement and velocity are zero at the initial moment, $\alpha \in [0,10^5]$,when the frequency of the incident wave is 2.2143 s^{-1}.

In order to facilitate the calculation, the power integral function in this optimization model is processed by the trapezoidal method, and the area of each curved-edge trapezoid can be used to approximate instead of the function value when the chosen step size is small enough. For the optimal damping coefficient solution when the damping coefficient is constant, the Lungkuta method can be continued. In order to make the trapezoidal area as close as possible to the true value of the function, it is necessary to reduce the step size as much as possible, but if the value of the step size is too small, it will make the traversal time is too long, which is not conducive to quickly find the optimal solution, so this paper adopts the following treatment: first set a larger step size for traversal to get a rough result, and then reduce the step size within the range of the preliminary traversal to get the accurate result.

The relationship between the damping coefficient and the average power is shown in Figure 4:

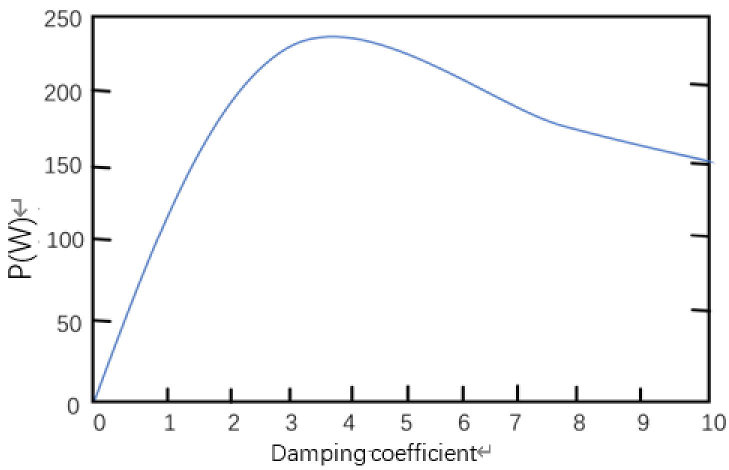

Figure 4. The relationship between damping coefficient and average power.

By traversing the above method, the maximum output power when the damping coefficient is constant $P_{\max} = 229.6819\,w$, the optimal damping coefficient $C = 37639$, the maximum output power when the damping coefficient varies with the speed of the oscillator and the float $P_{\max} = 230.0163\,w$, the optimal damping coefficient $C = 81478$, and the corresponding power exponent $\beta = 0.3377$.

4. Conclusions and outlooks

This paper investigates wave energy output devices by modeling the float and oscillator motions using Newton’s second law within a Cartesian system. A modified Runge-Kutta algorithm solves the differential equations, yielding displacement and velocity data. The damper’s output power formula is derived, and an optimization model is built to maximize power. Using small trapezoid approximation and variable-step search, the study identifies the maximum average output power and optimal damping coefficients under both constant and velocity-dependent damping conditions.

Since this study only examines the heaving motion of the wave energy device, while in real conditions the float and oscillator may also experience other forms of motion under wave action, future research could incorporate surge or pitch motions to make the mathematical model more representative of actual operating conditions.

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