Demand Forecasting and Cross Elasticity Analysis Based on Stochastic Processes

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Abstract. This study investigates a dynamic demand forecasting model based on stochastic processes, employing adaptive filtering and multilevel correlation analysis for regression forecasting of time-series data. The model utilizes cross-elasticity principles to construct a demand response structure and systematically analyzes elasticity fluctuations across multiple periods and states. By incorporating risk control and sensitivity analysis, the model evaluates decision robustness under market uncertainty. Results demonstrate that the model has strong adaptability in handling high-dimensional data, providing a theoretical foundation for complex time-series demand forecasting.

Keywords: Stochastic process, cross-elasticity analysis, demand response.

1. Introduction

In complex environments, demand forecasting and elasticity analysis are essential for optimizing resource allocation [1-5]. Demand for different variables is influenced by market dynamics, environmental conditions, and price fluctuations, exhibiting significant volatility and uncertainty. Traditional forecasting methods struggle to address these complex demand variations, making accurate prediction of uncertain demand a key research focus.

Cross-elasticity analysis is used to explore the substitutive and complementary relationships between variables. Substitutive relationships reveal how changes in the demand for one variable affect another, while complementary relationships reflect how different variable combinations impact overall outcomes. In a multi-variable context, accurate analysis of these relationships can effectively support decision-making optimization.

This study employs stochastic process modeling and cross-elasticity analysis to build a demand forecasting model that supports decision-making in complex conditions, enhancing the stability and effectiveness of allocation strategies.

2. Method

2.1. Stochastic Process Modeling

A core component of this study is analyzing the impact of uncertainty factors—such as sales volume, yield, cost, and price—on optimal allocation strategies. In actual allocation schemes, these uncertainty factors are often influenced by external factors such as market demand and environmental conditions, leading to significant volatility. To ensure that the allocation strategy remains profitable under uncertain market and environmental conditions, this study uses simulated annealing to solve the optimal allocation model and incorporates Monte Carlo simulation to manage uncertainty, thereby ensuring the strategy's robustness in dynamic environments.

(1) Simulated Annealing for Optimal Allocation Model Construction

1) Objective Function: In a scenario with a 50% price discount, the objective function is designed to maximize total revenue, as follows:

$$maxZ_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \times ((E(Y_{ij}) \times E(P_{ij}) \times S_j) + ((1-S_j) \times E(Y_{ij}) \times \frac{E(P_{ij})}{2}) - E(C_{ij})) \quad (1)$$

E(Yij): Expected yield of variable j at location i (unit yield per unit area).

E(Pj): Expected sales price of variable j (currency units).

E(Cij): Expected cost of variable jjj at location i (currency units per unit area).

Sj: Sales ratio of variable j, indicating the proportion of actual sales.

The objective function aims to maximize expected total revenue while accounting for the volatility of various parameters in the face of uncertainty. The Monte Carlo simulation method generates simulated values for the yield expectation E(Yij) price expectation E(Pj), and cost expectation E(Cij), which are then incorporated into the objective function for calculation.

2) Constraints

(a) Land Adaptability Constraint: Each crop can only be planted on suitable land to meet adaptability requirements for growth conditions. If crop j is unsuitable for land i, the planting area xij should be set to zero, ensuring no planting occurs on unsuitable land. The mathematical expression of this constraint is:

$$x_{ij} = 0 \tag{2}$$

This constraint ensures that each crop grows in an appropriate environment, enhancing yield and quality.

(b) Repetitive Planting Restriction: In the same land plot, the same crop cannot be planted in consecutive planting seasons to prevent soil nutrient depletion and pest build-up from repeated planting. For crop j on land iii, if it is planted in season t, it should not be replanted in the following season t+1. This restriction is expressed as:

$$\mathrm{x}_{\mathrm{ij}}^{\mathrm{t}} \cdot \mathrm{x}_{\mathrm{ij}}^{\mathrm{t+1}} = 0, \forall \mathrm{i}, \mathrm{j}, \mathrm{t} \tag{3}$$

(c) Bean Crop Planting Requirement: To improve soil quality and increase soil fertility, each land plot is required to plant at least one bean crop within three years. For land i, the total planting area of bean crops must meet the following condition:

$$\sum_{\mathrm{j}\in\mathrm{b}} \mathrm{x}_{\mathrm{ij}} \geq \mathrm{A}_{\mathrm{i}} \tag{4}$$

Where b is the index of bean crops. This constraint ensures that the crop planting plan for each land plot provides ecological benefits, particularly by enhancing nitrogen content through bean crop planting.

(d) Land Area Limitation: The total planting area for each plot must not exceed the actual available area Ai of that plot. This is to prevent the allocation of planting areas beyond the physical limitations of the land, ensuring feasibility. The mathematical expression is:

$$\sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{x}_{\mathrm{ij}} \leq \mathrm{A}_{\mathrm{i}}, \forall \mathrm{I} \tag{5}$$

This constraint prevents the allocation of planting areas beyond the land's available area, ensuring operational feasibility and rationality.

(e) Minimum Planting Area Threshold: The planting area of a single crop must not be less than 10% of the plot area, to avoid excessively fragmented planting that would complicate management. For crop j on land i, the planting area must meet the following condition:

$$\mathrm{x}_{\mathrm{ij}} \geq 0.1 \times \mathrm{A}_{\mathrm{i}}, \mathrm{x}_{\mathrm{ij}} \geq 0 \tag{6}$$

(2) Monte Carlo Simulation Model Construction

In practical allocation, variables such as sales volume, price, cost, and yield are influenced by various uncertainties. Therefore, this study applies Monte Carlo simulation to model these variables and evaluate the robustness of the allocation strategy in a dynamic environment [6-10]. By defining reasonable distribution forms and performing multiple random samples, future fluctuations of each variable can be simulated, thereby improving the stability of the optimization results. The process is illustrated in Fig. 1.

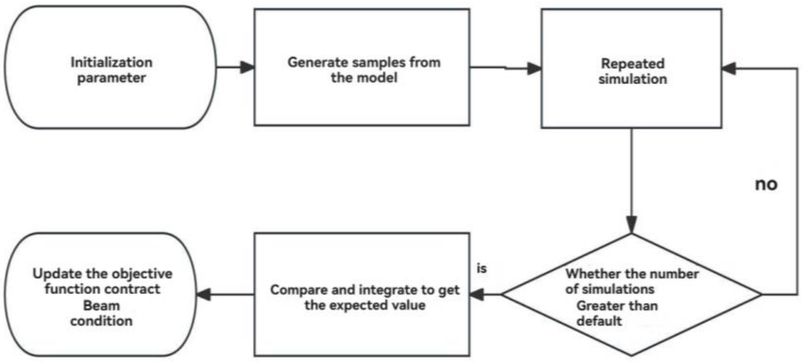

Figure 1. Monte Carlo Simulation Flowchart

The following defines the uncertain variables and determines their random distributions.

1) Sales Volume Variation Distribution: To reflect future market demand fluctuations for different variables, we assume that the growth rate of sales volume follows a normal distribution. For example, the annual average growth rate of certain crops is set as μy with a standard deviation σy, thus constructing a probability distribution model for future sales volume. The volatility of sales volume, Vy, is defined as:

$$V_y \sim N(\mu_y, \sigma_y^2) \tag{7}$$

Where the mean and standard deviation of sales volume fluctuations are set based on market demand forecasts from past sales data to capture possible annual growth or decline trends.

2) Yield Fluctuation Distribution: Crop yield per unit area is influenced by environmental factors such as climate, with a ±10% fluctuation each year. This environmental impact aligns with the characteristics of a normal distribution. Therefore, we use a normal distribution to simulate changes in yield per unit area:

$$Y_{ij} \sim N(\mu, \sigma^2) \tag{8}$$

3) Cost Variation and Distribution: Due to market price increases and inflation, planting costs are expected to grow by an average of 5% per year. To simulate cost variations, we use a uniform distribution within a certain range to generate data, simulating cost fluctuations over different years. The uniform distribution implies that all possible cost changes occur with equal probability within a specified range:

$$C_{ij} \sim U(0.05, 0.10) \tag{9}$$

The annual growth rate of costs is set between 4% and 6%, reflecting the impact of market conditions on crop planting costs.

4) Price Variation and Distribution: Grain crop prices (such as wheat and corn) are relatively stable, so the price can be considered a constant or simulated using a normal distribution with minimal fluctuation. For vegetable crops, prices show an upward trend with an estimated 5% growth per year. To capture the uncertainty of this growth trend, we use a random distribution, assuming an annual price increase of 5% with some fluctuation:

$$P_{ij} \sim N(0.05, \sigma^2) \tag{10}$$

For mushroom crops, prices decline by 1%-5% per year, with a fixed 5% decrease for specific varieties like morels. For this downward trend, a random distribution is used for mushroom crop prices, while a fixed annual decrease of 5% is applied to morel prices.

2.2. Cross-Elasticity Coefficient Model

(1) Model Construction

The demand cross-elasticity coefficient is used to analyze the substitutive or complementary relationships between different variables. By calculating the demand cross-elasticity coefficient Exy, we can determine whether a substitutive or complementary relationship exists between variables x and y. The calculation formula is:

$$E_{xy} = \frac{\%\Delta Q_x}{\%\Delta P_y} \tag{11}$$

Where %Qx represents the percentage change in sales volume of crop xxx, and % Py represents the percentage change in the price of crop y.

The magnitude of Exy indicates the sensitivity of demand and price between different crops. By analyzing these coefficients, we can understand the substitutive and complementary relationships between crops, which helps optimize planting strategies.

For Exy>0, a positive cross-elasticity coefficient indicates that the two crops are substitutes. When the price of crop y rises, the demand for crop x will increase, and vice versa. A larger coefficient implies a stronger substitutive relationship, suggesting that high-substitutive crops can be considered in the planting strategy to accommodate demand changes caused by price fluctuations.

For Exy<0, a negative cross-elasticity coefficient indicates that the two crops are complements. When the price of crop y rises, the demand for crop xxx will decrease, and vice versa. A larger negative value implies a stronger complementary relationship. Complementary crops can be planted together to meet the demand for product combinations, thereby increasing revenue.

For Exy=0, the two crops have no related demand, meaning the price change of one crop has minimal impact on the demand for the other, indicating that they are neither substitutes nor complements in the market.

(2) Correlation Analysis

To analyze the relationships among sales volume, sales price, and planting cost, this study uses the Spearman rank correlation coefficient to measure the associations among variables. This approach enables us to identify whether there is a significant positive, negative, or no correlation between variables. If the coefficient is 1, it indicates a perfect positive correlation, where an increase in one variable is accompanied by a proportional increase in another. If the coefficient is -1, it indicates a perfect negative correlation, where an increase in one variable corresponds to a proportional decrease in another. A coefficient close to 0 suggests no significant correlation; values between 0 and ±1 indicate partial positive or negative correlations, with values closer to ±1 indicating stronger associations.

To determine the statistical significance of the correlation coefficients, we also calculate the p-value. If p≤0.05, the correlation is considered significant, indicating a non-random association between variables; if p>0.05, the correlation is not significant, possibly resulting from random factors. This analysis allows for a more precise understanding of the relationships among sales volume, sales price, and planting cost, offering insights that are crucial for optimizing planting strategies.

(3) Construction of the Substitute-Based Optimized Planting Model

Aiming to enhance production safety and stability, we develop a substitute-based optimized planting model to optimize rural crop planting strategies under different production conditions. The model considers land type, crop suitability, planting costs, yield per unit area, sales price, planting restrictions (e.g., crop rotation, legume constraints), and the demand cross-elasticity coefficient among crops.

$$Z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \times ((E(Y_{ij}) \times E(P_{ij}) \times S_j) + ((1 - S_j) \times E(Y_j) \times \frac{E(P_{ij})}{2}) - E(C_i)) \tag{12}$$

$$Z_a = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ib} \times ((E(Y_{ib}) \times E(P_{ib}) \times S_a) + ((1 - S_a) \times E(Y_{ib}) \times \frac{E(P_{bib})}{2}) - E(C_{ib})) \tag{13}$$

$$Z_b = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ib} \times ((E(Y_{ib}) \times E(P_{ib}) \times S_b) + ((1 - S_b) \times E(Y_{ib}) \times \frac{E(P_{ib})}{2}) - E(C_b)) \tag{14}$$

Objective Function and Constraints:

$$maxZ_3 = Z_2 + Z_a - Z_b \tag{15}$$

$$S.t.\ \begin{cases} x_{ij} = 0, if\ crop\ j\ is\ not\ suitable\ for\ plot\ i \\ x_{ij}^{t} \cdot x_{ij}^{t+1} = 0, \forall i, j, t \\ \sum_{i=1}^{3} x'_{ib} \geq A_i \\ \sum_{j=1}^{n} x_{ij} \leq A_i, \forall i \\ x_{ij} \geq 0.1 \times A_i, x_{ij} \geq 0 \\ E_{ab} > 1.6, \\ elasticity\ between\ a\ and\ b\ greater\ than\ 1.6 \end{cases} \tag{16}$$

3. Result

3.1. Results of the Stochastic Process Model

During the Monte Carlo simulation, we first initialize parameters and set the simulation count to N=10. For each simulation run, we sample uncertainty factors such as sales volume, cost, yield per unit area, and sales price. By integrating the results of 10 simulations, we calculate the expected value for each crop combination, updating the objective function and determining the optimal planting strategy, thus completing the risk assessment.

The results from the simulated annealing solution show distinct planting plans across different land types and years. In 2024, 2026, 2027, 2029, and 2030, dry fields, terraces, and hillside areas are primarily allocated to high-yield grain crops (such as rice and corn). Irrigated fields are planted with vegetable crops in 2024 and 2029 and with single-season rice in 2026, 2027, and 2030. In conventional greenhouses, vegetables are grown in the first season and edible fungi in the second season, while smart greenhouses support mixed planting of various vegetables and edible fungi. In 2025 and 2028, legumes are the main crops, with leguminous vegetables planted in the first season of irrigated fields. The statistics show a diverse crop mix across years, strictly following crop rotation principles to ensure yield stability. The planting plan includes grains, vegetables, and some economic crops, with a balanced allocation of legumes.

To further evaluate the risk associated with different crop planting strategies, a risk assessment was conducted post-modeling to analyze the robustness of the planting plan under uncertain conditions. First, we used chi-square tests on annual yield and revenue data to identify significant differences between observed and expected values for each crop. The chi-square test results show a chi-square statistic of 6,291,804.83 with a degree of freedom of 54 and a corresponding p-value of 0. Since the p-value is 0, this indicates a highly significant difference between observed and expected values, suggesting notable deviations in planting results across different years. Examining the chi-square contributions by year, 2026 and 2027 show high contributions of 1,817,912.09 and 1,600,098.15, respectively, indicating the greatest deviation from expected values and thus higher instability in these years. Additionally, 2030 also exhibits a high chi-square contribution of 1,703,824.35, implying a considerable difference from expected outcomes. In contrast, the chi-square contributions for 2024, 2025, 2028, and 2029 are relatively low, indicating that the planting strategies for these years are closer to the expected values and thus more stable.

Furthermore, we used a risk assessment model to evaluate the volatility of different land types in terms of planting cost, yield, and price. We calculated risk metrics for each land type, including mean and volatility (standard deviation), resulting in a comprehensive risk assessment. The results indicate that 2026, 2027, and 2030 show higher-risk crops in dry fields and hillside areas, with higher risk weights compared to other land types, suggesting that planting strategies in these years may require further optimization to reduce risk. Fig. 2 and Fig. 3 present the risk assessment results for dry fields and hillside areas, providing guidance for future planting decisions to enhance adaptability and stability under varying conditions across years and land types.

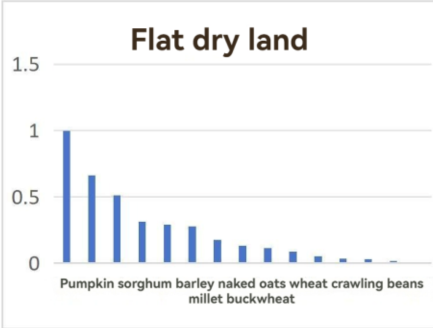

Figure 2. Risk Assessment Chart for Crops on Flatland

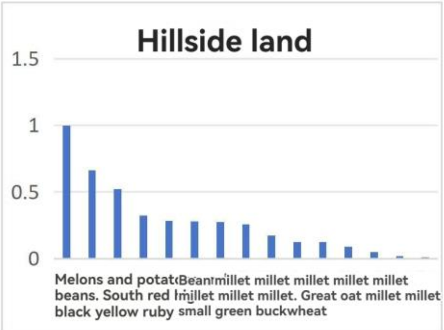

Figure 3. Risk Assessment Chart for Crops on Hillsides

3.2. Solution Results of the Cross-Elasticity Demand Model

To ensure scientific rigor in the model solution and the rationality of optimization results, we first conducted a correlation analysis among crops to assess their substitutive and complementary relationships, as well as the interdependence among variables such as sales volume, sales price, and planting costs. By calculating cross-elasticity coefficients, we determined the substitutive and complementary relationships between different crops, providing a scientific basis for subsequent adjustments in planting strategies. The resulting elasticity matrix for grain, edible fungi, and vegetable crops is partially displayed below due to space constraints.

	Soya beans	Black bean	Ormosia bean	Mung bean	Crawling bean
Soya beans	0.701	0.693	0.700	0.697	0.699
Black bean	0.213	0.211	0.213	0.212	0.212
Ormosia bean	0.538	0.532	0.537	0.534	0.536
Mung bean	0.076	0.075	0.076	0.075	0.076
Crawling bean	−0.206	−0.203	−0.205	−0.204	−0.205

Figure 4. Demand Elasticity Matrix for Grain (Legumes)

	Wheat	Corn	millet	sorghum	Broomcorn millet	buckwheat	Pumpkin	Sweet potato	Naked oats	barley
Wheat	0.164	0.161	0.161	0.163	0.159	0.156	0.159	0.152	0.159	0.158
Corn	1.640	1.601	1.604	1.629	1.587	1.560	1.583	1.515	1.585	1.580
millet	1.584	1.546	1.549	1.573	1.533	1.507	1.528	1.463	1.530	1.525
sorghum	−0.349	−0.341	−0.341	−0.347	−0.338	−0.332	−0.337	−0.322	−0.337	−0.336
Broomcorn millet	[illegible]	0.125	0.125	0.127	0.124	0.122	0.124	0.118	0.124	0.123
buckwheat	−0.166	−0.162	−0.162	−0.165	−0.160	−0.158	−0.160	−0.153	−0.160	−0.160
Pumpkin	0.758	0.740	0.742	0.753	0.734	0.721	0.732	0.701	0.733	0.730
Sweet potato	−0.320	−0.313	−0.313	−0.318	−0.310	−0.305	−0.309	−0.296	−0.309	−0.308
Naked oats	0.054	0.053	0.053	0.054	0.052	0.051	0.052	0.050	0.052	0.052
barley	0.191	0.186	0.187	0.190	0.185	0.181	0.184	0.176	0.184	0.184

Figure 5. Demand Elasticity Matrix for Grain (Legumes)

Additionally, we applied Spearman’s rank correlation coefficient to analyze the interdependence between yield per unit area, planting costs, and sales prices. The analysis revealed a significant positive correlation between yield per unit area and planting costs, while the correlation between sales price and yield per unit area was relatively weak. These findings support the weight assignment and constraint settings of each variable in the model, enhancing the optimization process's alignment with market demands and cost control requirements.

	Yield per mu	Planting cost	unit price of sales
Yield per mu	1	0. 8335	-0. 156
Planting cost	0. 8335	1	0. 011
Unit sales price	-0. 1597	0. 0109	1

Figure 6. Spearman Correlation Coefficient Matrix

	Yield per mu	Plant into this	Sales ticket price
Yield per mu	1	0	0. 1004
Planting cost	0	1	0. 9116
Unit sales price	0. 1004	0. 9116	1

Figure 7. Spearman Correlation Coefficient Matrix

In the 2027 planting plan, Monte Carlo simulations were combined to analyze future yield and production cost data over seven years. Results indicate a downward trend in the yield and production costs for crops such as lettuce, sweet potatoes, barley, millet, and sorghum. The cross-elasticity matrix further confirms that millet and sorghum are substitutive for each other. The optimized planting plan adjusted accordingly, achieving a revenue of 7,099,051 yuan.

Under current market conditions, although price volatility is challenging to predict, controlling planting costs and increasing the proportion of high-yield crops can help maintain revenue stability to some extent. Through rational optimization of land allocation and yield planning, crop revenues are effectively integrated. Adjustments in planting strategies allow adaptation to market dynamics and variations in crop demand, ultimately enhancing planting revenue. Optimizing the planting plan while ensuring production safety can improve economic efficiency.

4. Sensitivity Analysis

In this multi-parameter sensitivity analysis and stability evaluation, we focused on two critical factors in the agricultural yield model—sales price and planting cost. By adjusting these two factors, we assessed the impact of different parameter combinations on the system's total revenue and evaluated the system's stability. Fig. 8 shows a heat map of sensitivity, providing a visual representation of the impact of parameter changes on revenue.

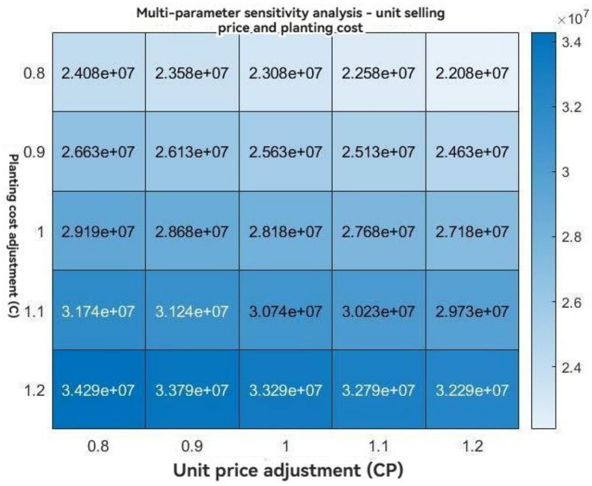

Figure 8. Sensitivity Heatmap

4.1. Impact of Sales Price and Planting Cost on Total Revenue

Sales price significantly influences total revenue in a positive direction. When the sales price increases from 0.8 to 1.2, total revenue gradually rises. Particularly in the range of 0.9 to 1.1, revenue changes linearly with minimal fluctuation, indicating good system stability in this range. Minor price adjustments in this range do not cause drastic changes in system revenue, and as prices exceed 1.1, revenue growth accelerates, further demonstrating the effect of price increases on revenue.

Conversely, an increase in planting costs compresses profit margins, leading to a gradual decline in total revenue. Specifically, as planting costs rise from 0.8 to 1.2, total revenue exhibits a linear downward trend, indicating system stability within this cost range. Although increased planting costs negatively impact revenue, the system remains relatively stable. However, when costs reach a higher level, the decline in total revenue accelerates, highlighting the adverse effect of high costs on revenue. Therefore, cost control within the range of C=0.9 to 1.0 is critical, as the system demonstrates a stable downward trend within this range, aiding revenue stability.

4.2. Optimal Point Analysis

When the sales price is 1.2 and planting cost is 0.8, total revenue reaches its maximum, indicating that this parameter combination allows the system to maximize revenue. Revenue changes around this optimal point are relatively stable, showing strong system stability, even with minor parameter fluctuations. This result offers clear guidance for decision-makers to optimize prices and control costs within the market conditions to maximize revenue and ensure system stability.

4.3. Stability Evaluation Summary

Increasing the sales price significantly boosts total revenue, and the system shows good stability in response to price adjustments. In contrast, rising planting costs compress revenue, with the rate of revenue decline accelerating as costs increase. However, total revenue remains relatively stable within a specific cost range. The optimal parameter combination achieves minimal revenue fluctuation and a strong capacity for withstanding volatility. Thus, in practical decision-making, appropriately raising prices and controlling costs can maximize revenue and maintain system stability under varying market conditions.

5. Conclusion

This study developed an optimization model based on stochastic processes and cross-elasticity analysis to address demand fluctuations and uncertainty. By analyzing substitutive and complementary relationships among crops, we optimized planting strategies, demonstrating that rational substitution and complementary configurations contribute to system stability. Sensitivity analysis revealed that raising sales prices significantly increased revenue, while cost control was critical for maintaining system stability. The optimal parameter combination maximized revenue and enhanced the system's resilience.

This study provides an effective method for planting optimization under complex conditions, improving revenue and system robustness, and offering valuable support for future planting decisions.

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