Multi-objective Optimization Strategy Model Based on Simulated Annealing

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Abstract. This paper proposes a multi-objective optimization model based on simulated annealing for sensitivity analysis of decision variables and time-series forecasting. The model is constructed with dynamic optimization conditions, using constraint solving and progressive algorithms to determine the optimal paths for variables. Integrating risk assessment and cross-elasticity simulation, the study presents a multi-objective approach to address complex market environments. Experimental results indicate that the model effectively balances demand and supply fluctuations across various simulation scenarios. To further refine the strategy, this paper analyzes the algorithm's adaptability under multiple constraints and provides robust improvement recommendations suitable for complex scenarios.

Keywords: Simulated annealing, constraint solving, multi-objective optimization.

1. Introduction

Multi-objective optimization faces challenges of high-dimensional data, dynamic changes, and multiple constraints in complex environments [1-5]. As data scale and problem complexity increase, traditional optimization algorithms gradually struggle to meet robustness and adaptability requirements in dynamic settings. Simulated annealing, with its global search capability and ability to escape local optima, has proven to be an effective tool for handling such problems, particularly for risk analysis under demand fluctuations and multiple constraints. Current research mainly focuses on single-objective or simple multi-objective scenarios, but optimizing multiple objectives under dynamic demand in complex market environments remains challenging. To address this, this paper constructs a multi-objective optimization strategy model that combines simulated annealing, aiming to handle demand fluctuations and risks under uncertainty. This approach provides robust optimization for complex systems, offering theoretical support and methodological innovation for dynamic decision-making.

2. Method

Based on the research objectives, we developed an optimization model for different sales scenarios to maximize strategic benefits. The model is constructed within a multi-variable linear programming framework, fully considering the effects of land types, variable adaptability, constraints, and market demand. The core of the model includes adaptability analysis, optimization of boundary constraints (e.g., planting limits), and cross-variable interaction analysis. By introducing simulated annealing for solution, the model achieves a global optimum in high-dimensional space, resulting in robust optimization under dynamic demand conditions.

2.1. Decision Variables and Objective Functions

The decision variable xij represents the area (in mu) of crop j planted on land i. To maximize total revenue, the objective function is defined as follows.

In the context of premium sales, the objective function is:

$$\max \quad Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \times \left(Y_{ij} \times P_{ij} \times S_j - C_{ij}\right) \tag{1}$$

Where Z1 represents total revenue under premium sales conditions.

In the scenario where sales are at 50% of the 2023 price, the objective function becomes:

$$Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \times \left(\left(Y_{ij} \times P_{ij} \times S_j\right) + \left(\left(1 - S_j\right) \times Y_{ij} \times \frac{P_{ij}}{2}\right) - C_{ij} \right) \tag{2}$$

Where Z2 represents total revenue under the 50% price reduction condition. Additional parameter descriptions are as follows:

(1) Z1: Total revenue (including the unsold portion).

(2) Z2: Total revenue (including the unsold portion at a 50% discount from 2023 sales prices).

(3) m: Number of land plots.

(4) n: Number of crops.

(5) xij: Area (in mu) of crop j planted on land i; the decision variable representing the allocation of planting area.

(6) Yij: Yield per mu (in kg/mu) for crop j on land i; reflects production efficiency on specific plots.

(7) Pij: Selling price (in yuan) of crop j on land i; represents the market sales price of the crop.

(8) Cij: Planting cost (in yuan/mu) for crop j on land i; includes production expenses such as fertilizers, labor, and irrigation.

(9) S j: Sales ratio of crop j, representing the proportion of total production that can be sold.

2.2. Constraints

(1) Land Adaptability Constraint

Each crop can only be planted on suitable land to meet adaptability requirements for growth conditions. If crop j is unsuitable for land i, the planting area xij should be set to zero, ensuring no planting occurs on unsuitable land. The mathematical expression of this constraint is:

$$x_{ij} = 0 \tag{3}$$

This constraint ensures that each crop grows in an appropriate environment, enhancing yield and quality.

(2) Repetitive Planting Restriction

In the same land plot, the same crop cannot be planted in consecutive planting seasons to prevent soil nutrient depletion and pest build-up from repeated planting. For crop j on land iii, if it is planted in season t, it should not be replanted in the following season t+1. This restriction is expressed as:

$$x_{ij}^{t} \cdot x_{ij}^{t+1} = 0, \forall i, j, t \tag{4}$$

This constraint ensures the implementation of crop rotation, maintaining soil health and reducing pest risks.

(3) Bean Crop Planting Requirement

To improve soil quality and increase soil fertility, each land plot is required to plant at least one bean crop within three years. For land i, the total planting area of bean crops must meet the following condition:

$$\sum_{j\in b} x_{ij} \geq A_i \tag{5}$$

Where b is the index of bean crops. This constraint ensures that the crop planting plan for each land plot provides ecological benefits, particularly by enhancing nitrogen content through bean crop planting.

(4) Land Area Limitation

The total planting area for each plot must not exceed the actual available area Ai of that plot. This is to prevent the allocation of planting areas beyond the physical limitations of the land, ensuring feasibility. The mathematical expression is:

$$\sum_{j=1}^{n} x_{ij} \leq A_i, \forall i \tag{6}$$

This constraint prevents the allocation of planting areas beyond the land's available area, ensuring operational feasibility and rationality.

(5) Minimum Planting Area Threshold

The planting area of a single crop must not be less than 10% of the plot area, to avoid excessively fragmented planting that would complicate management. For crop j on land i, the planting area must meet the following condition:

$$x_{ij} \geq 0.1 \times A_i, x_{ij} \geq 0 \tag{7}$$

Through the establishment of these constraints, the model effectively controls crop adaptability, rotation requirements, and land use efficiency, ensuring optimal allocation of planting areas while balancing ecological and economic benefits.

2.3. Linear Programming Model Formulation

In summary, the linear programming model for the optimal crop planting plan for the rural area from 2024 to 2030 is as follows:

$$maxZ_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \times (Y_{ij} \times P_{ij} \times S_j - C_{ij}) \tag{8}$$

$$Z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \times ((Y_{ij} \times P_{ij} \times S_j) + ((1 - S_j) \times Y_{ij} \times \frac{P_{ij}}{2}) - C_{ij}) \tag{9}$$

$$s.t.\begin{cases} x_{ij} = 0, \\ if\ crop\ j\ is\ unsuitable\ for\ planting\ on\ plot\ i \\ x_{ij}^{t} \cdot x_{ij}^{t+1} = 0,\ \forall i, j, t \\ \sum_{i=1}^{3} x_{ib}^{t} \geq A_i, \\ \sum_{j=1}^{n} x_{ij} \leq A_i,\ \forall i \\ x_{ij} \geq 0.1 \times A_i,\ x_{ij} \geq 0 \end{cases} \tag{10}$$

2.4. Simulated Annealing Solution

To optimize the planting strategy, this study applies the simulated annealing algorithm to solve the crop planting scheme. Simulated annealing is a random optimization algorithm based on the physical annealing process, which gradually lowers the "temperature" to balance global search and local optimization, thereby gradually approaching the global optimal solution. In the specific solution process, the algorithm generates an initial configuration of solutions based on initial conditions and adjusts the planting area allocation in each iteration to maximize the objective function value [6-10].

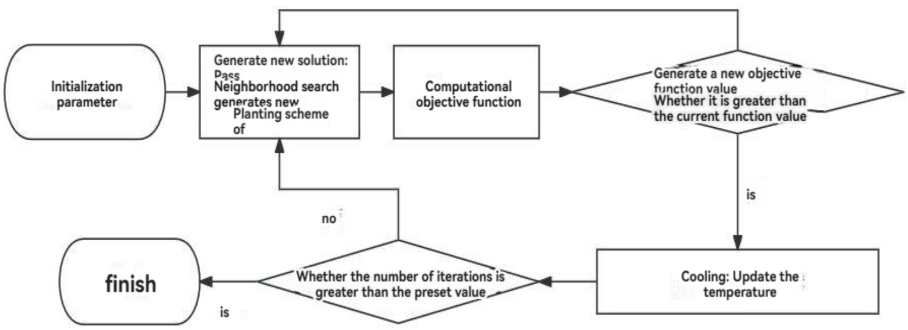

Figure 1. Flowchart of the Simulated Annealing Algorithm

(1) Initial Solution Generation

At the beginning of the solution process, an initial solution is generated based on the suitability conditions of the land and crops. The specific steps are as follows:

1) Each land plot is randomly assigned a planting area for highly suitable crops, ensuring that each plot has a maximum of two crop types.

2) For crops that do not meet the land suitability constraints, their planting area is set to zero.

3) The initial solution must meet all growth conditions and restrictions, including total land area constraints, crop rotation restrictions, bean crop planting requirements, etc.

(2) Parameter Settings

To ensure convergence and computational efficiency in the annealing process, the following key parameters are set:

1) Initial temperature T=1000: Starting with a high initial temperature to ensure strong exploration capability in the early stage.

2) Minimum temperature Tmin=10^{-5}: Set as the stopping condition for the annealing process, ensuring algorithm convergence.

3) Temperature decay rate α=0.99: The temperature decays at a fixed ratio after each iteration, gradually reducing the system's exploration capability.

4) Maximum number of iterations kmax=1000: Controls the total number of iterations to balance computational cost and solution accuracy.

(3) Neighbor Solution Generation

In each iteration, a neighbor solution is generated based on the current solution for local search in the solution space. The specific steps are as follows:

1) Randomly select a land plot and a crop on that plot.

2) Randomly adjust the planting area of the selected crop to form a new planting scheme, while ensuring the new allocation meets the land's total area constraint.

3) Verify the constraints of the new solution to ensure it meets all model requirements (e.g., minimum planting area threshold, crop rotation restriction, etc.).

(4) Objective Function Calculation

For each generated neighbor solution, calculate its objective function value to assess the yield of the scheme in different scenarios. According to the study objectives, the objective functions are as shown in Eq. (8) and (9). In each iteration, the difference between the objective function values of the current solution and the neighbor solution is calculated to decide whether to accept the new solution.

(5) Acceptance Criterion

The Metropolis criterion is used to determine whether to accept the newly generated solution, controlling the acceptance probability of solutions during the annealing process. Specifically: If the objective value Znew of the new solution is better than the current solution Zcurrent, the new solution is accepted directly. If the objective value of the new solution is worse than the current solution, it is accepted with a probability of exp(Znew−Zcurrent)/T, allowing the algorithm to escape from local optima to a certain extent.

This acceptance criterion ensures that the algorithm has high exploratory capability in the early stages and gradually converges to the global optimum as the temperature decreases.

(6) Temperature Decay and Iteration Control

At the end of each iteration, the temperature is updated according to the set decay rate α, proceeding to the next iteration. As the temperature decreases, the algorithm gradually converges, with reduced exploration and a shift toward local fine-tuning. The iterative process continues until either of the following stopping conditions is met: the temperature falls below the minimum temperature Tminor the maximum number of iterations kmax is reached.

2.5. Parameter Settings

Before optimization, to ensure that the model accurately reflects the sales situation of different crops in the market, we set detailed parameters for the sales price Pj and sales ratio Sj of each crop. The setting of sales prices and ratios is based on market demand, planting costs, and crop storage characteristics, aiming to make the model's solution more meaningful in practical applications.

Firstly, the sales price Pj is adjusted according to crop type and land type. Considering the differences in planting costs and market demand for different land types, we divide the land into conventional greenhouses, intelligent greenhouses, and other land types (e.g., terraces, slopes), with different price ranges for each. For example, for the same crop type, the sales price is higher in intelligent greenhouses due to lower planting costs and higher market demand; conversely, the sales price is lower in conventional greenhouses due to relatively higher planting costs. Specifically, the sales price for each crop can be determined by the following formula:

$$P = \begin{cases} b, Conventional\ Greenhouse \\ \frac{c+d}{2},\ Other\ Land\ Types \\ \frac{e+f}{2}, Intelligent\ Greenhouse \end{cases} \tag{11}$$

This tiered price setting ensures that the model can accurately reflect market conditions under different land types, optimizing the planting strategy for each crop. Fig. 2 and Fig. 3 show the sales prices of bean and grain crops.

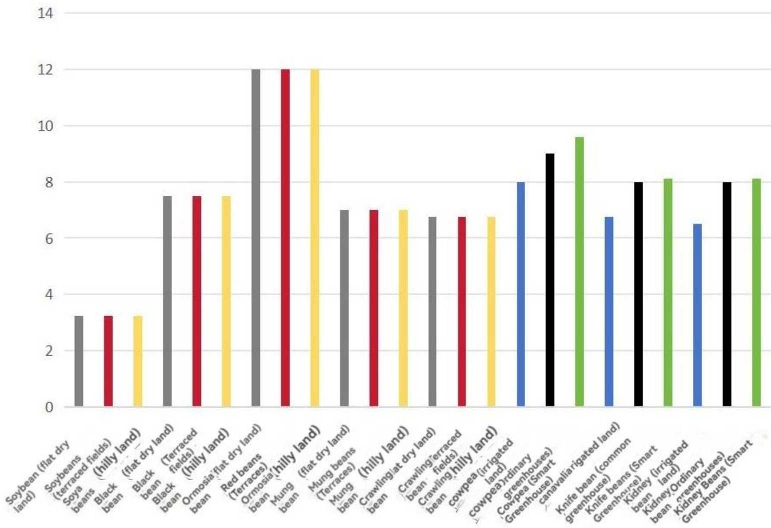

Figure 2. Sales Prices of Bean Crops

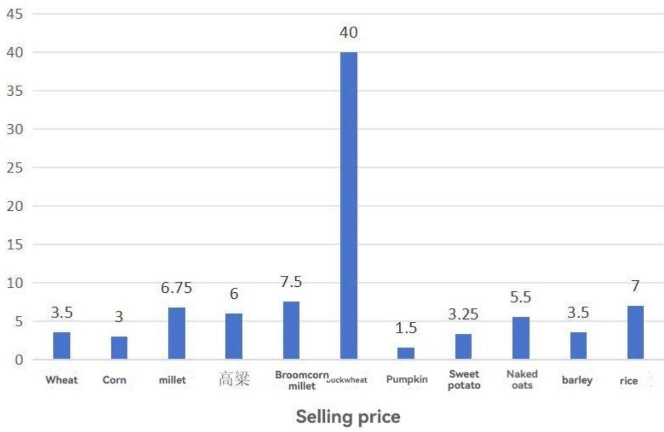

Figure 3. Sales Prices of Grain Crops

Secondly, the sales ratio Sj is based on market demand and crop storage characteristics. The sales ratio represents the proportion of total production that can be successfully sold, directly influencing the accuracy of revenue calculations. Depending on market demand stability and crop shelf life, the sales ratio varies. For example, grain crops (such as wheat and corn) have a high sales ratio due to stable demand and suitability for long-term storage, whereas mushroom crops have a relatively low sales ratio due to short shelf life and fluctuating market demand. The specific sales ratio settings are as follows:

$$S_1 = 0.80, S_2 = 0.85, S_3 = 0.70, S_4 = 0.60 \tag{12}$$

This setting ensures that the objective function in the solution process more accurately reflects the economic value of different crops in the market.

3. Resulet

Through simulated annealing-based optimization, the model has derived the optimal multi-variable planting strategies under different sales scenarios. We analyzed planting data for various land categories, covering terrains such as plains, terraces, and slopes, and planning strategies for the years 2024, 2026, 2027, 2029, and 2030. The results show that high-demand crops exhibit differentiated distribution trends across different terrains and years. For instance, certain drought-resistant crops like soybeans and seed crops demonstrate high adaptability and planting value on plains, whereas moisture-tolerant root crops are better suited to terraces and slopes. In specific years, such as 2024, 2026, and 2027, economically viable crops like beans and vegetables reach peak planting revenue in some plots, aligning closely with major consumer demands. Different land types exhibit specific adaptability and revenue characteristics over the coming years, providing a dynamic basis for optimal planting strategies.

Additionally, by analyzing sales and demand data from 2025 to 2028, the model further validates the adaptability of the planting strategy. The results reveal that demand fluctuations for certain crop types show distinct regional characteristics across different years and plots. For example, certain cold-resistant varieties on slopes demonstrate strong demand stability, making them suitable for expansion in areas with harsher climate conditions. In the market forecast for 2030, heat-resistant crops with high demand elasticity, such as vegetables, show greater growth potential, contributing to optimized land planting decisions and enhancing the system's adaptability in a fluctuating market.

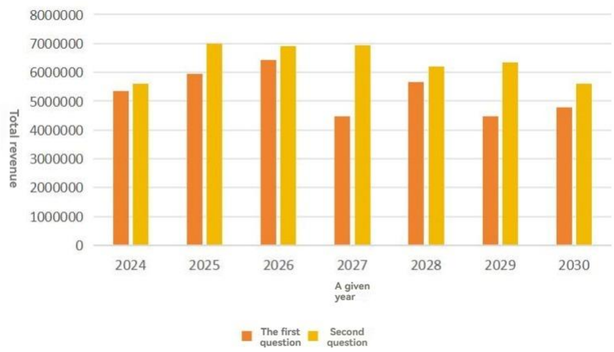

Figure 4. Comparison of Total Revenue from 2024 to 2030

As shown in Figure 4, the maximum revenue for 2024-2030 yields the following conclusions:

In a premium sales scenario, most plots are planted with beans (e.g., black beans, red beans) and grain crops (e.g., sorghum, millet), which achieve high returns during peak demand periods. However, under premium sales conditions, the total revenue of some crops remains capped due to price limits.

When the sales price is reduced to 50% of the 2023 level, crop planting revenue decreases, although some crops still maintain relatively high profit rates. The analysis indicates that adaptable varieties, such as beans and grains, can still generate certain revenue even with a significant price reduction; however, the scenario of reduced prices shows a marked downward trend in revenue, highlighting the need for strategy stability.

In comparison, the premium sales scenario allows for revenue enhancement, while the reduced-price scenario results in greater uncertainty in revenue. Therefore, the system requires robust optimization through reasonable multi-plot, multi-crop allocation. These strategic recommendations provide valuable guidance for future planting planning, and the simulated annealing method demonstrates strong adaptability in optimizing under different scenarios.

4. Conclusion

This study developed a multi-objective optimization model based on simulated annealing to address the challenges of crop strategy optimization under varying demand and sales conditions. By integrating constraint-based analysis with sensitivity evaluation, the model effectively identified optimal planting strategies across different types of land and market scenarios. The results demonstrate that simulated annealing, with its robust global search capabilities, can successfully balance multiple objectives, ensuring maximum profitability while maintaining resilience under uncertainty.

In particular, the model revealed critical insights into the adaptability of different crop categories under premium and discounted sales conditions, highlighting the importance of diversified planting strategies. Through a combination of dynamic response analysis and multi-variable optimization, the model provides a flexible framework for decision-making in complex market environments. Future applications of this framework could extend to other high-dimensional optimization problems, proving its value in adaptive strategy development and robust decision support.

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