Study on the Variability of the Sex Ratio of Sea Lamprey Based on the Improved Predator Model

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Abstract. The change in sex ratio in response to food availability is a unique ecological trait that distinguishes lampreys from many other aquatic species. This trait not only affects the reproductive dynamics of the lamprey population but also has cascading effects on the broader ecosystem. This paper proposes a functional model that quantitatively describes the relationship between environmental resource abundance and the sex ratio (female-to-male) in sea lampreys. The sex ratio function, characterized by a quadratic growth trend, is integrated into an improved predator - prey model to account for the lamprey's dynamic interactions with food resources, host species, and natural predators such as birds and fish.Subsequently, a system of coupled differential equations is established to simulate the population dynamics of lampreys and other key species within the food web. The model considers parasitic behavior, predator types (human and non-human), and the impact of adaptive sex ratios on ecosystem stability. Numerical simulations compare scenarios with fixed versus resource-driven sex ratios.Results demonstrate that allowing the sex ratio to adapt leads to greater stability in lamprey population size, more regular predator-prey oscillations, and reduced short-term fluctuations in food resources. Although the influence on host populations remains minimal, the overall system exhibits improved resilience and ecological balance.This study provides novel insights into the ecological role of sex ratio plasticity and offers a modeling framework for evaluating species adaptability under varying environmental conditions, contributing to future ecosystem management and conservation planning.

Keywords: Lamprey, Improved Predator Model, Differential Equation, Ecosystem.

1. Introduction

The sea lamprey (Petromyzon marinus), as a jawless vertebrate, exhibits unique biological and ecological characteristics that distinguish it from many other aquatic species. According to existing studies, lampreys display diverse life history strategies: some species are anadromous, migrating between marine and freshwater habitats to spawn, while others remain in freshwater throughout their lives [1]. From a trophic perspective, lampreys can be classified into parasitic species, which attach to and consume the bodily fluids of host fish, and non-parasitic species, which feed primarily on detritus and plankton. Within their respective food webs, lampreys serve as both consumers and prey, with predators including large fish, birds, and, in some regions, humans.

An important biological feature of lampreys is their dioecism and their metamorphic life cycle. More notably, emerging evidence suggests that their sex ratio—particularly the proportion of males to females—is influenced by environmental factors, especially food availability. As food becomes more abundant, the proportion of females increases [2]. This flexible sex ratio, alongside a relatively short post-spawning lifespan, positions lampreys as a sensitive ecological indicator and a potentially powerful regulator of food web dynamics.

Despite increasing attention to lamprey genetics and physiology, their role in ecosystem stability and population dynamics remains underexplored [3]. Most existing ecological models assume static sex ratios, thereby neglecting species like lampreys whose reproductive demographics are resource-dependent. Addressing this limitation, this study introduces a novel modeling approach by incorporating a food-dependent sex ratio function into an extended predator-prey differential equation model. This adjustment enables us to simulate the population dynamics of lampreys, their predators, and food resources under more realistic ecological assumptions.

The core contribution of this paper lies in bridging a theoretical gap in ecological modeling: we quantify the impact of adaptive sex ratios on long-term population stability and ecosystem resilience. Through numerical simulation and comparative analysis, we demonstrate that allowing sex ratios to vary with food availability significantly stabilizes lamprey populations, reduces predator-prey oscillations, and contributes to overall ecosystem equilibrium. These findings hold practical implications for species management, biodiversity conservation, and future ecological modeling frameworks.

2. The Functional Model Between Sex Rate and Food Resources

The sex ratio (females to males) of the lampreys varies by environment and increases with food availability. In environments where food is less available, growth rates will be lower and the proportion of females can reach about 22% of the population. In environments where food is more readily available, the proportion of females was observed to be about 44% of the population. The model was constructed on the following basis: when the amount of environmental resources is relatively scarce, sex ratio increases rapidly with the amount of food resources, and when the amount of resources is more abundant, the sex ratio increases slowly with the amount of food resources in that case, because the amount of resources that can be consumed by a certain population size of lampreys is limited. It can be hypothesized that the change of sex ratio with the amount of food resources is a smooth curve whose slope gradually tends to zero and is always positive. As a result of the study, the paper set the model as a quadratic curve and expressed in terms of differential equations. Its expression is set in the following form.

$$\frac{dR}{dS} = \omega(S_{\max} - S) \tag{1}$$

Where the ω is the scale factor to be determined, $S_{\max}$ is the maximum resources available from ecosystems. R represents the sex ratio of lampreys (female to male), and S represents the resources.

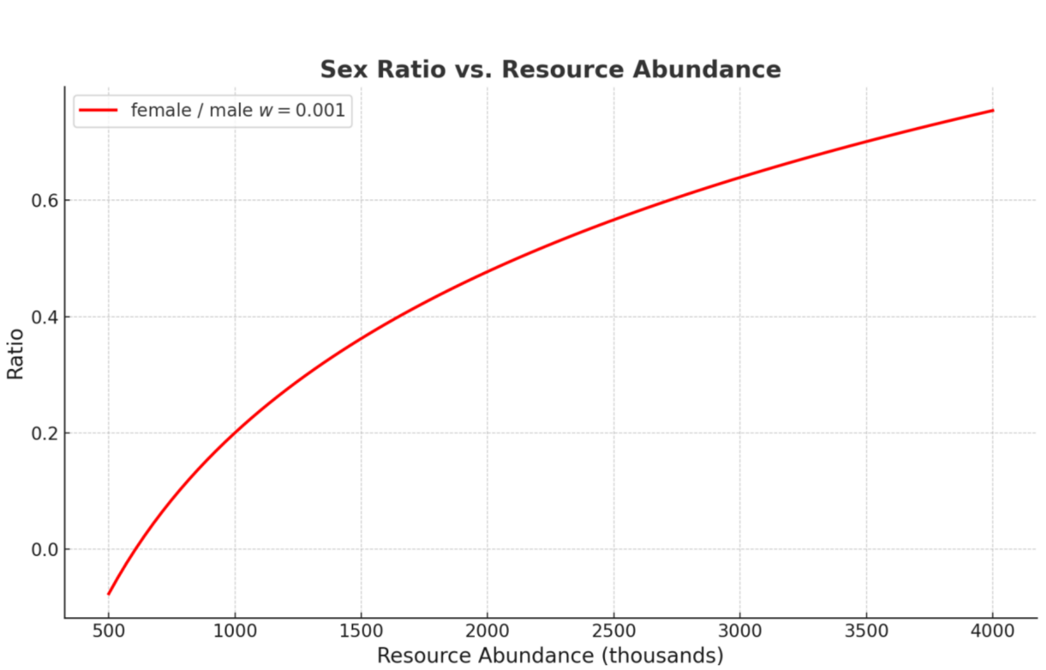

Figure 1 The relationship curve between gender ratio and resource quantity

The relationship between gender ratio and resource changes in lampreys: R=f (S) is indeed a curve with increasing S and decreasing growth rate as shown in figure 1. When resources are very abundant, the male to female ratio approaches 1, which in turn verifies the correctness of this model.

3. Improved Predator Model

3.1. The Growth Model Base on Logistic Growth Model

The Logistic Growth Model [4] is a population size growth model in which only environmental resistance is taken into account, and the curve of the population size over time shows an S-shape, and the differential equations of the model are expressed in the following form:

$$\frac{dN(t)}{dt} = r_1 N(t)\left(1-\frac{N(t)}{N_{\max}}\right) \tag{2}$$

Where the r_1 is the natural growth rate of the lamprey's population, the $N_{\max}$ is the environmental capacity. Considering the impact of the sex ratio on the birth rate, we obtain: $r_1 = k_1 R$,Substituting into equation (2), we obtain the growth model that takes the sex ratio into account

$$\frac{dN(t)}{dt} = k_1 R N(t)\left(1-\frac{N(t)}{N_{\max}}\right) \tag{3}$$

The lampreys has a complex food chain in the ecosystem in which it is found, and within the food chain, the paper focus on species that form predatory or competitive relationships with the lampreys population. We model the differential equations for the growth of the lampreys and other species in the food chain based on the predator model.

3.2. Introducing more complex ecological roles

Before introducing the predator of lampreys into the model, we divided the predator into two parts: human predator and non-human predator (birds, large fish, etc.). Assuming that humans are independent of marine ecosystems and their numbers do not change within the time range under investigation. According to the reality , the intensity of human predation of lampreys is directly proportional to the population size of lampreys.

Next, The paper will use a predator model[5]to introduce predator factors into the population growth model of lampreys. human factors and non-human factors (birds, large fish) will be introduced separately.

Based on the growth model of the lamprey population in the previous section, obtain the self growth rate of the lamprey population and input it into the predator model[5-9]:

$$\frac{dN(t)}{dt} = N(t)\left(k_1 S\left(1-\frac{N(t)}{N_{\max}}\right) - \lambda_1 y\right) \tag{4}$$

Introducing human predators and non-human predators, we obtain:

$$\frac{dN(t)}{dt} = N(t)\left(k_1 R\left(1-\frac{N(t)}{N_{\max}}\right) - k_2 N(t) - \lambda_1 Z(t)\right) \tag{5}$$

And

$$\frac{dZ(t)}{dt} = Z(t)\left(-r_2 + \lambda_2 N(t)\right) \tag{6}$$

Equation (5) represents the lamprey population dynamics equation after incorporating multiple factors, while Equation (6) represents the differential equation for the change in the number of natural predators.

Then Leading the Prey and the Parasites into Growth Model. Assuming that α is the percentage of lampreys living as parasitic organisms in the population of lampreys is the total population. The population growth model of lampreys for the introduction of parasitized and preyed:

$$\frac{dN(t)}{dt} = N(t)\left[k_1 R\left(\left(1-\frac{N(t)}{N_{\max}}\right)-k_2 N(t)-\lambda_1 Z(t)\right)+\lambda_3\alpha X+\lambda_4(1-\alpha)s\right] \tag{7}$$

Where the λ_3 is the feeding ability of parasitized individuals to the population of lampreys, the λ_4 is the feeding capacity of prey on the population of lampreys, the X is the quantity of the parasitized, the S is the quantity of prey. This is the final growth rate model of the lamprey population under the influence of the environment and other species. corresponding population growth models for both parasitized and preyed individuals can be obtained:

$$\frac{dX}{dt} = X\left[r_3\left(1-\frac{X}{X_{\max}}\right)-\gamma\alpha\lambda_5 N\right] \tag{8}$$

$$\frac{dS}{dt} = S\left[r_4\left(1-\frac{S}{S_{\max}}\right)-(1-\alpha)\lambda_6 N\right] \tag{9}$$

Where the γ is the mortality rate of parasitized individuals. The r_3 is the natural growth rate of parasitized individuals, the r_4 is the natural growth rate of prey. The λ_5 is the predation intensity of lampreys on parasitized individuals, the λ_6 is the predation intensity of lampreys on prey. The paper also studied the variation of the sex ratio over time,It satisfies the following differential equation:

$$\frac{dR}{dt} = \frac{dR}{dS}\frac{dS}{dt} = \omega(S_{\max}-S)R\left[r_4\left(1-\frac{S}{S_{\max}}\right)-(1-\alpha)\lambda_6 N\right] \tag{10}$$

Based on the differential equations for the population dynamics of the various species, as well as the equation for the variation in the sex ratio of the sea lamprey, we have constructed an ecosystem closely related to the sex ratio of the sea lamprey.

4. Results Analysis

The paper simulates the changes in gender ratio over time and the changes in population numbers of various species in the ecosystem where the lamprey is located under constant gender ratio (1:1). By comparing the differences between the two situations, this study aims to illustrate the impact of gender ratios in the population changes of lampreys on species and ecosystems. In the simulation process, we control whether the gender ratio changes over time by controlling $\omega > 0$ or $\omega = 0$.

The paper will take the average of the statistical values of the population numbers of various species in the lamprey food chain from several existing literature as the initial data. The initial data is as follows Table1:

Table1: Partial parameter values

Population	N	Z	S	X	R	Nmax	Smax	Xmax	a
Quantity	54	19.6	500	50	0.2821	200	5000	200	0.914

Where a is the S/S+X. (The raw data comes from NOAA, WOCE, EMODnet, IODE.)

4.1. The Impact of Changing Sex Rate on the Larger Ecological System

The paper discussed the impact of changes in the sex ratio of the lamprey population on the population itself, predators, food resources, parasitized individuals, and the overall ecosystem. We also analyzed the phase trajectory between the number of lamprey populations and predators, as well as the sex ratio, to complete a comprehensive analysis of the impact of lamprey sex ratio on a larger ecosystem. We simulated a model of the gender ratio changes in the lamprey population and the population size of various species in the ecosystem over time under constant conditions.

(1) The impact of changes in sex ratio on lampreys

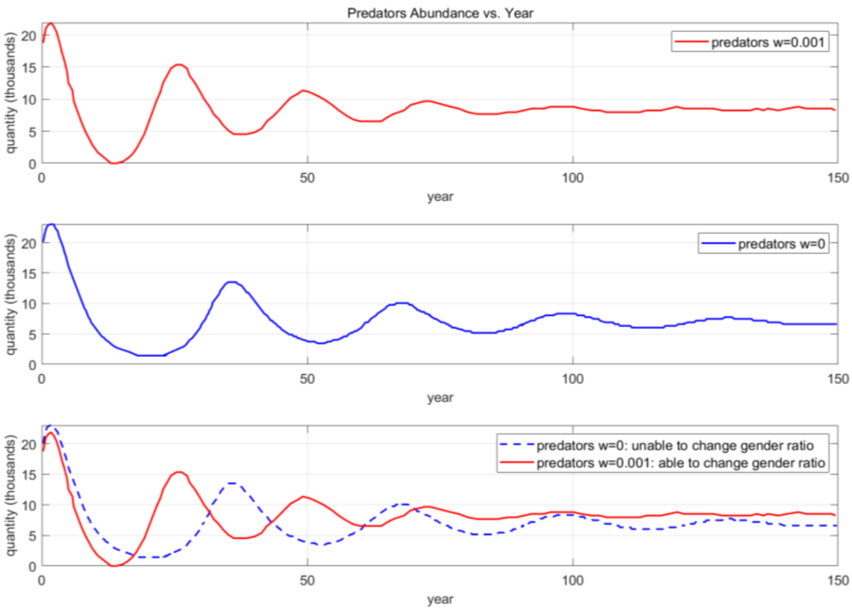

Figure 2 Comparison Chart of Lampreys Population

As shown in figure 2, in the long run, when the ecosystem tends to balance, the population of lampreys is basically the same; From a period of time, the change in gender ratio has made the fluctuation of the population of lampreys more stable, which will greatly help its environmental stability, adaptability, and anti-interference ability (we will use more specific indicators to demonstrate in detail in the future) [10-13]. At the same time, this stability also helps to maintain the stability of the population changes of other species.

(2) The impact of changes in sex ratio on predators

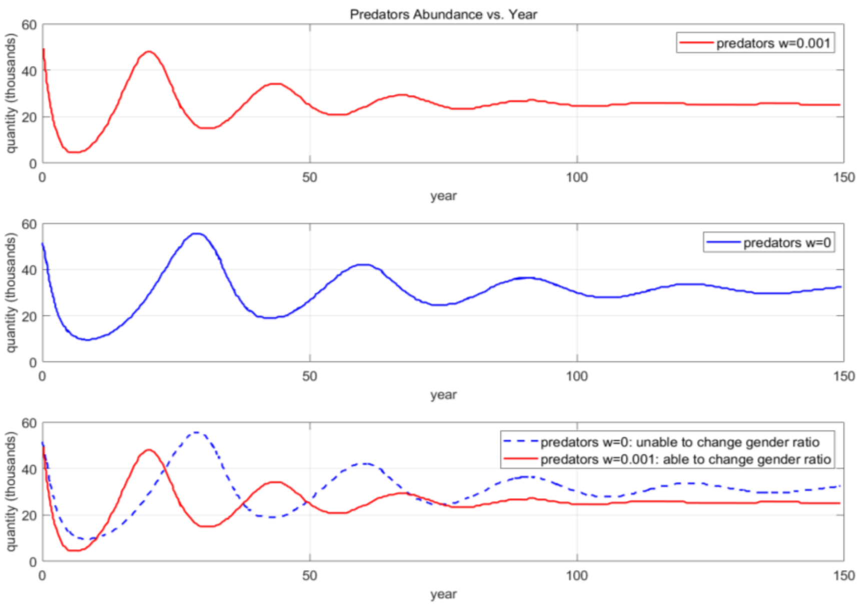

Figure 3 Comparison of Predators Population

As shown in figure 3, the population of predators in a stable state increase, and the fluctuations in the number of predators are more stable.

Cause analysis: The number of predators is limited by the population of lampreys, and the stability of lampreys due to their ability to change gender ratios leads to fluctuations in the number of predators becoming more stable.

(3) The impact of changes in sex ratio on food resources

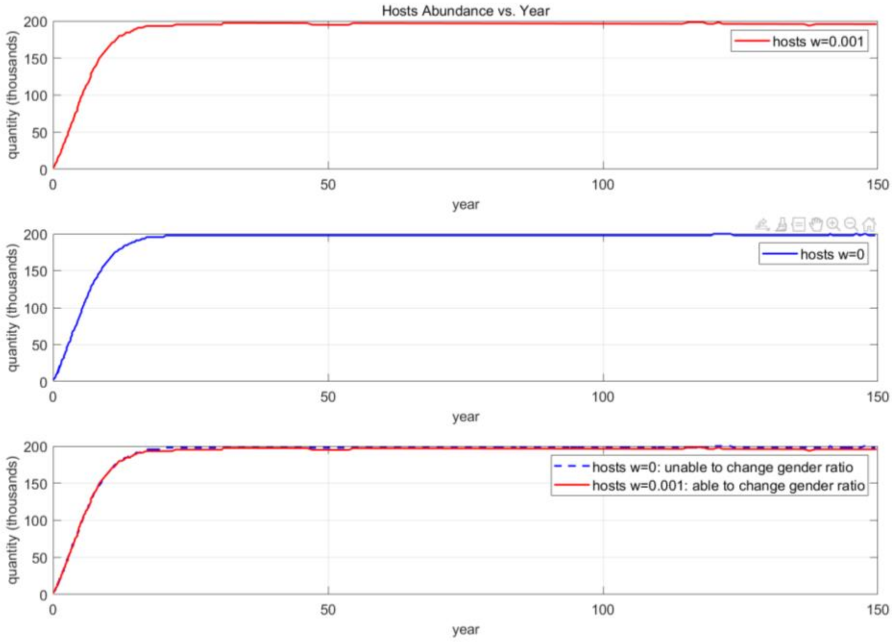

Figure 4 Comparison of Resources Population

As shown in figure 4, in the long run, it has little impact on the total amount of resources, but it makes the short-term fluctuations of resources smaller and the quantity more stable.

(4) The impact of changes in sex ratio on the hosts

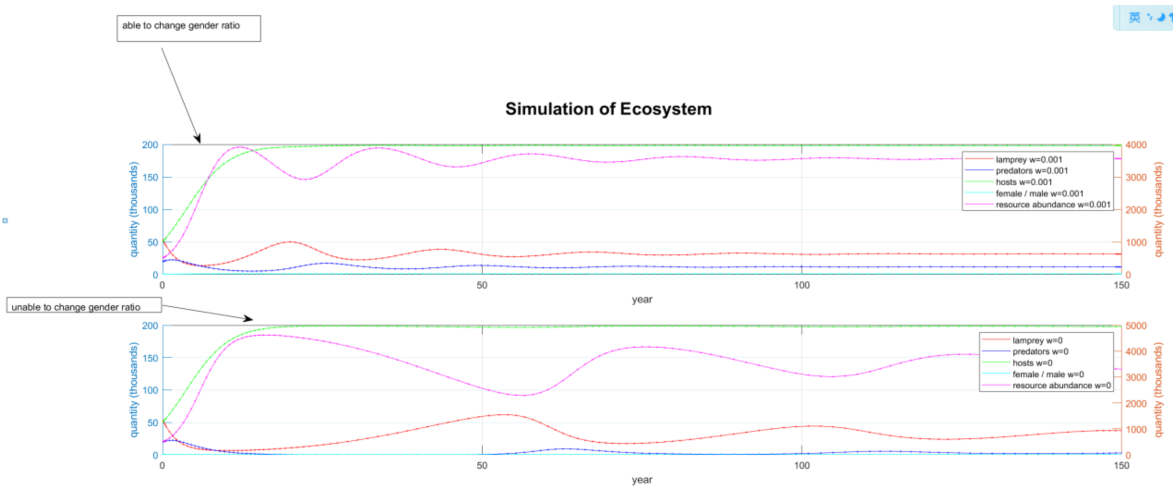

Figure 5 Comparison of the hosts Population

As shown in figure 5, from the graph, it can be seen that gender changes in lampreys have almost no effect on the number of hosts.

Cause analysis: The parasitic behavior of lampreys is not related to gender, but rather to the total population size of lampreys. As shown in Figure 1, it can be concluded that lampreys exhibit parasitic behavior at $\omega = 0$ and $\omega > 0$, there is not much difference in population size, but more importantly, it affects the stability of population size. Therefore, there is not much difference in the impact on the host between the two situations.

(5) The impact of changes in sex ratio on ecosystem

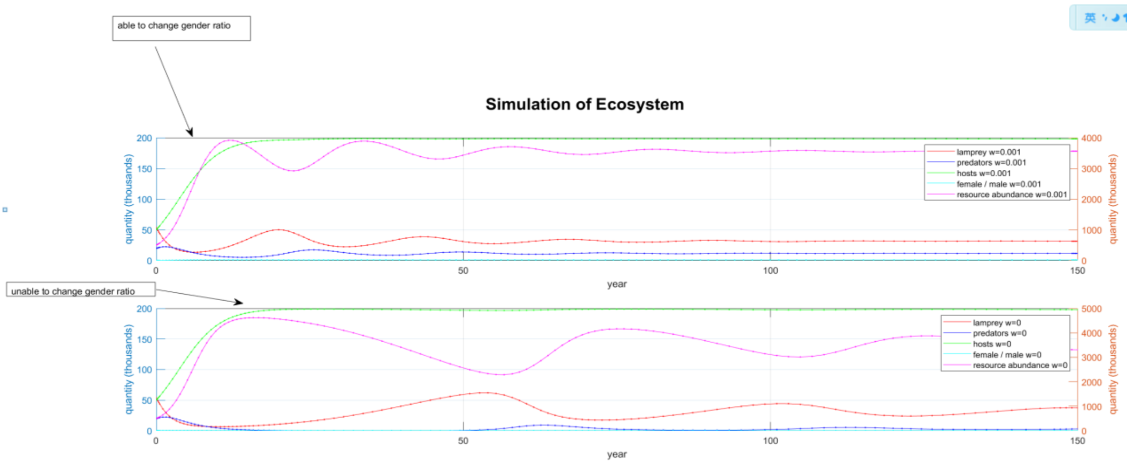

Figure 6 Comparison Chart of the Ecosystem

As shown in figure 6, from the perspective of the overall ecosystem, when the gender ratio can change, the changes in the number of various species in the system become more stable, indicating that the lamprey plays a role in serving the ecosystem and maintaining ecosystem balance.

4.1.1 Analysis of Phase Trajectory Diagram

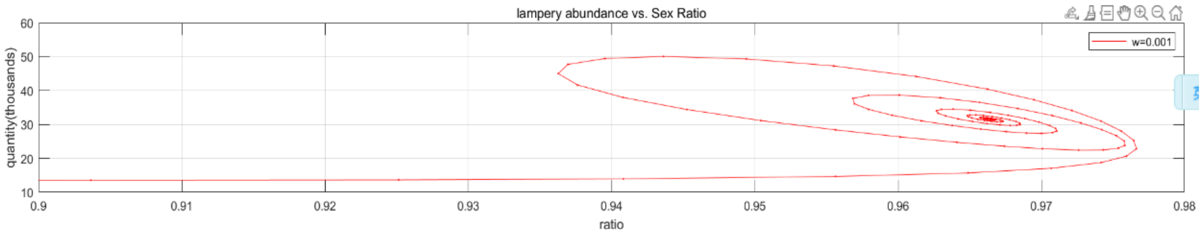

Figure 7 Comparison Chart of Trajectory chart of the number of lampreys and predators

Trajectory diagrams [14-15] help in analyzing and predicting the behavior of systems by clearly representing their movements and changes over time or space. As shown in figure 7, the phase trajectory diagram of the changes in predator numbers and lampreys reflects the relationship between the two, with slight differences between genders that can be changed and genders that cannot be changed. The graph of gender ratio changes is more stable.

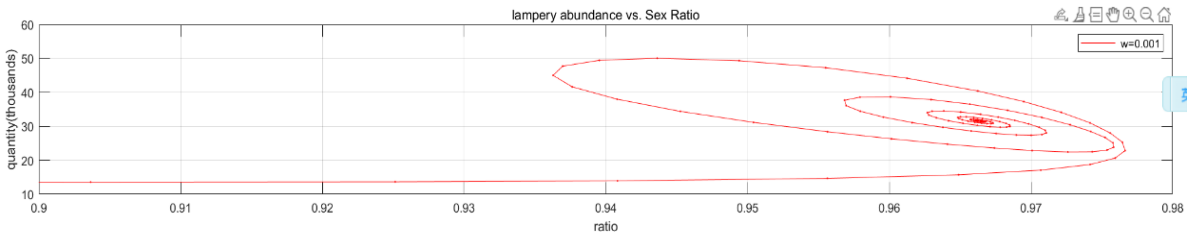

Figure 8 Comparison Chart of Trajectory chart of the number of lampreys and predators

As shown in figure 8, the dynamic relationship between the quantity and sex ratio changes of lampreys reflects the regulation of sex ratio changes on the population quantity of lampreys when resources are relatively abundant.

4.2. The Effects of Changing Sex Rate on the Lampreys

The paper simulated the growth models of the population of lampreys with and without changes in gender ratio, as shown in figure 9. And then used species evaluation models to calculate the evaluation values for both scenarios, studying the impact of gender ratio changes on the lampreys themselves.

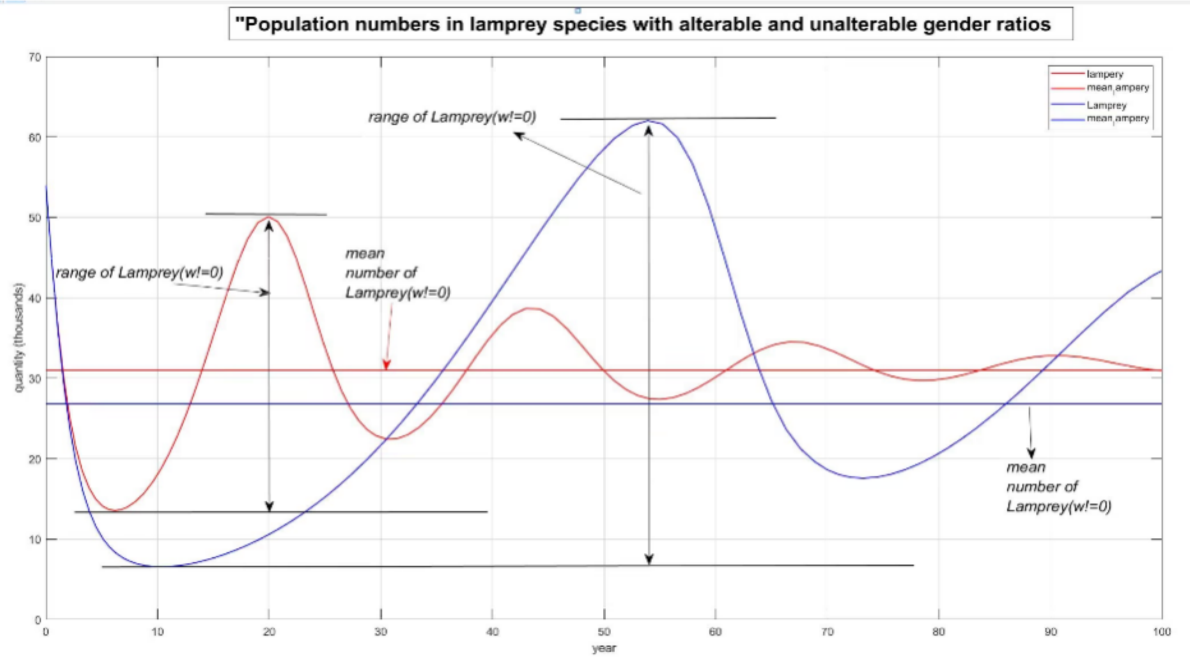

Figure 9 Comparison chart of the population of lampreys (ω is 0.001and 0)

5. Conclusion

This study presents a novel analysis of how variability in the sex ratio, driven by food availability, affects lamprey population dynamics and broader ecosystem stability. Compared with models where the sex ratio is fixed, our improved model shows that allowing the sex ratio to adapt significantly enhances the stability of the lamprey population, reduces fluctuations in predator abundance, and minimizes short-term variations in resource availability. For example, in Figure 2 and Figure 3, we observe that adaptive sex ratio models yield smoother population curves and higher predator population stability in the long term. Moreover, our model introduces a new quadratic relationship between food resources and gender ratio, a contribution not addressed in prior ecological modeling literature. This functional linkage allows for a more realistic representation of biological behavior and improves predictive capability of ecosystem evolution. Therefore, our findings not only provide theoretical insights into the ecological role of sea lampreys but also offer a modeling framework that can inform species management and conservation under varying environmental conditions.

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