Research on the Gender Ratio Variation and Its Significance Based on the Adaptability of Lamprey

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Abstract. The sex ratio of lampreys changes with the external environment, thus affecting the ecosystem. This paper focuses on the environmental dependence of the adaptive sex ratio change and its advantages and disadvantages that adaptive and variable sex ratio bring to the lamprey population. First, an improved Lotka-Volterra equations were established and solved to simulate the effects of sex ratio changes on the lamprey population itself. Then, a TOPSIS comprehensive evaluation model was established to score the three indicators of stability, adaptability, and ecological roles, and evaluate this adaptive change for the lamprey population by comparing the evaluation plots under different sex ratios. It was mainly concluded that the population of the lamprey decreases in the presence of elevated male ratios, with fewer of its predators, fewer of its food resources, and more of its competitors.

Keywords: Population Growth Logistic Model, Ecosystem Dynamics Model, Improved Lotka-Volterra Equation, TOPSIS Integrated Evaluation Model.

1. Introduction

Many species are born with a sex ratio of 1:1, but some species have sex ratios that deviate from the mean [1]. This is known as adaptive sex ratio variation. The sex ratio of lampreys can vary depending on the external environment; the sex of lampreys depends on their growth rate during the larval stage, which is influenced by food availability [2].

The stability of ecosystems is influenced by a variety of factors, including species diversity, biomass, food chain structure, environmental changes, human activities, etc. These factors interact with each other and jointly affect the stability of ecosystems.

First, species diversity has an important impact on ecosystem stability. High species diversity can increase ecosystem stability because different species play different roles in the ecosystem and can complement and support each other. Species-rich ecosystems tend to have better resistance to disturbance and resilience, thus making it easier to maintain a stable ecological balance.

Second, an increase in biomass can improve ecosystem stability. Higher biomass means more ecological resources and energy reserves, which can provide more ecosystem services and resilience to external disturbances.

In addition, food chain structure also has an impact on ecosystem stability. Complex food chain structures can increase ecosystem stability because a diversity of food sources reduces the impact of the disappearance of one type of food on the ecosystem as a whole.

The paper research focuses on sex ratios and their dependence on local conditions, as well as the pros and cons of a species being able to alter sex ratios based on resource availability. In particular, the sex ratio of the lamprey. This leads to modeling for in-depth studies to analyze the roles and impacts of such changes in the ecosystem.

2. The Relationship between Lamprey Gender Ratio and Resource Availability

Model the sex ratio dynamics of the lamprey population, consider the relationship between sex ratio and resource availability, and use differential equations to describe changes in the sex ratio of the lamprey population[3]. Analyze the effects of sex ratio changes on the ecosystem and use an ecosystem dynamics model to simulate their effects on other populations and resources in the ecosystem in conjunction with sex ratio changes.

2.1. Population models with a gender structure

The populations are modeled with a sex structure without considering the Allee effect, environmental influences on reproductive success, offspring survival, and sterility.

Wild female lampreys and wild male lampreys were treated with F and M respectively. If the Allee effect is not considered, the model equations are as follows[4]:

$$F' = \alpha CF\left(1 - \frac{F+M}{K}\right) - \mu_F F \tag{1}$$

$$M' = (1-\alpha)CF\left(1 - \frac{F+M}{K}\right) - \mu_M M \tag{2}$$

where α represents the proportion of newly born females, and C is the average number of offspring produced by each females through all mating in a unit of time [72,82,84-87]. μ_F and μ_M are the mortality rates of wild female and male lampreys, respectively.
set (mathematics)

$$\Omega_1 := \{(F, M):\ 0 \le F + M \le K\} \tag{3}$$

Ω_1 is a positive invariant and attracting set of the system (1) in the nonnegative quadrant.

The origin is an equilibrium solution and the Jacobi matrix of system (1) is:

$$\begin{pmatrix} \alpha C - \mu_F & 0 \\ \cdot & -\mu_M \end{pmatrix} = \begin{pmatrix} \mu_F(r_0 - 1) & 0 \\ \cdot & -\mu_M \end{pmatrix} \tag{4}$$

in which

$$r_0 := \frac{\alpha C}{\mu_F} \tag{5}$$

r_0 is the net growth rate (or endowment growth rate), which is the difference between the average birth rate and the average death rate. Since the lamprey population is persistant in nature, we assume $r_0 > 1$. Under this assumption, it is easy to show that the origin is unstable .

We next consider the positive equilibrium solution of system (1) (F^* , M^*). The positive equilibrium solution satisfies:

$$\alpha C\left(1 - \frac{F^* + M^*}{K}\right) = \mu_F \tag{6}$$

$$(1-\alpha)CF^*\left(1 - \frac{F^* + M^*}{K}\right) = \mu_M M^* \tag{7}$$

Dividing the two equations in (3), we get:

$$F^* = \frac{\alpha \mu_M}{(1-\alpha)\mu_F} M^* := PM^* \tag{8}$$

Substituting (4) into (3), we have:

$$F^* = PM^* \tag{9}$$

$$M^* = \left(\frac{\alpha C}{\mu_F} - 1\right)\frac{K\mu_F}{\alpha C(1+P)} = (r_0 - 1)\frac{K\mu_F}{\alpha C(1+P)} \tag{10}$$

Therefore, there exists a unique positive equilibrium solution $E_0^* := (F^*, M^*)$ if and only if $r_0 > 1$.

At the positive equilibrium solution E_0^* The Jacobi matrix is

$$J_0 := \begin{pmatrix} -\frac{\alpha C}{K}F^* & -\frac{\alpha C}{K}F^* \\ (1-\alpha)C\left(1-\frac{2F^*+M^*}{K}\right) & -\frac{(1-\alpha)C}{K}F^* - \mu_M \end{pmatrix} \tag{11}$$

the trace and determinant are:

$$tr\ J_0 = -\frac{C}{K}F^* - \mu_M < 0 \tag{12}$$

and

$$det\,J_0 = \frac{\alpha C}{K}F^*\left((1-\alpha)C\left(1-\frac{F^*+M^*}{K}\right)+\mu_M\right) > 0 \tag{13}$$

respectively. Therefore E_0^* is locally asymptotically stable.

In order to prove the global nature of the solutions, we use Dulac's Theorem[5] to show that the system has no closed trajectories, the solutions in the system eventually converge either to a zero solution or a positive equilibrium solution. Denote the right-hand sides of the system (1) as f_1 and f_2 respectively, then

$$\frac{\partial}{\partial F}\left(\frac{f_1}{F}\right) + \frac{\partial}{\partial M}\left(\frac{f_2}{F}\right) < 0 \tag{14}$$

is needed if system (1) has no closed rails, with all F>0 and M>0.

If $r_0 \leq 1$, the system exists only at the origin (0,0) and there are no positive equilibrium solutions, then the locally asymptotically stable zero solution is globally asymptotically stable. If $r_0 > 1$, the positive equilibrium solution E_0^* is globally asymptotically stable.

For system (1) in the absence of sterile eels and not considering the Allee effect, if $r_0 > 1$, there exists a unique positive equilibrium solution to the system E_0^* . E_0^* is given in (5) and it is globally asymptotically stable. This shows that the sex structure of the population has an important effect on the stability and growth rate of the population.

According to the analysis of the model, when net growth rate $r_0 \leq 1$, the population tend to disappear. While when net growth rate $r_0 > 1$, the population tend to stabilize at a positive equilibrium point E_0^*. The stability of this positive equilibrium point depends on the intrinsic growth and mortality rates of the population, as well as the proportion of females and males.

2.2. Sex ratio dynamics model

The model considers the relationship between sex ratio and resource availability, with sex ratio R_t dynamically changes with time *t*. The sex ratio changing rate is related to the value of sex ratio and resource availability A_t [6]:

$$\frac{dR_t}{dt} = f(A_t, R_t) \tag{15}$$

Where f can be defined with respect to the current resource availability and the resource availability threshold:

$$f(A_t, R_t) = \gamma(A_t - A_{threshold}) \tag{16}$$

$A_{threshold}$ is the threshold of resource availability that affects gender transition, and γ is the moderating factor. In the population model with a gender structure, under the conditions of poor environmental conditions and relative scarcity of food and other resources, if $A_t < A_{threshold}$, the sex ratio of males will rise, which lead to a decline in the population of lampreys. Otherwise, the ratio of males to females will be close to 1:1, the population will grow and eventually becomes a stable value.

2.3. Models of ecosystem dynamics

To simulate the influences of sex ratio changes on the dynamics of other populations and resources in the ecosystem, we construct a model of ecosystem dynamics. Quantities for the dynamics of other populations in the ecosystem, as well as resource A_t changes were modeled using the modified Lotka-Volterra equation:

$$\frac{dN_{prey}}{dt} = r_{prey}\, N_{prey}\left(1 - \frac{N_{prey}}{K}\right) - \alpha_{prey}\, N_{prey}\, N_{pred} \tag{17}$$

$$\frac{dN_{pred}}{dt} = -r_{pred}\, N_{pred} + \beta_{pred}\, N_{prey}\, N_{pred} \tag{18}$$

$$\frac{dA_t}{dt} = r_A A_t\left(1 - \frac{A_t}{K_A}\right) - \sigma\left(N_{lamprey}, R_t, A_t\right) \tag{19}$$

Where N_{prey} and N_{pred} denote the size of prey and predator populations respectively. r_{prey} and r_{pred} denote the natural growth rates of different species, α_{prey} and β_{pred} denote the predation and reproduction rate coefficients. σ indicates the consumption rate of resources by the lamprey, related to sex ratio and number of resources.

2.4. The Solution of Model

Based on a population model with gender structure, considering that the proportion of males in lampreys increases when resources are scarce enough, and almost equal when resources are abundant, a chart of food resources and gender ratio changes over time was obtained, as shown in Figure 1.

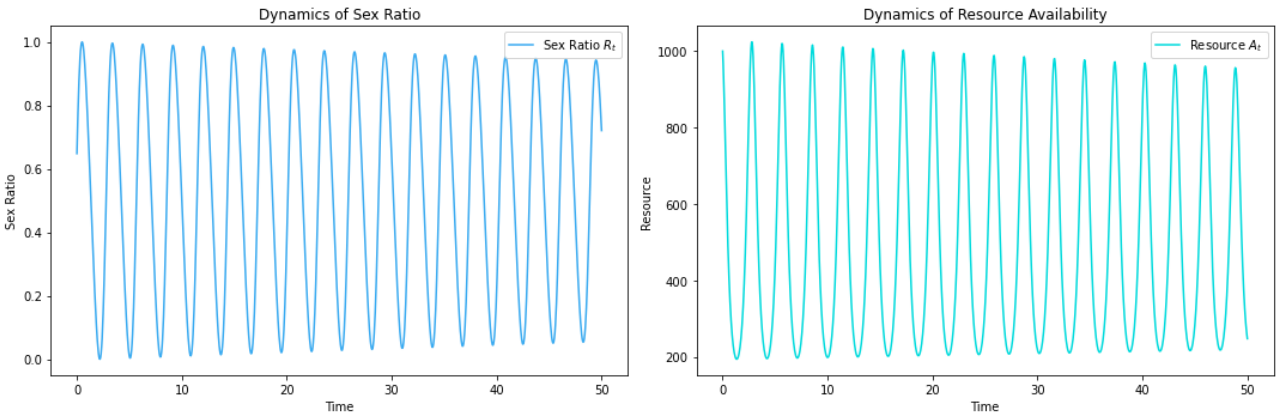

Figure 1 Dynamics of Sex Ratio and Resource Availability

Considering that changes in gender ratios can affect the population size of lampreys, thus also impact other species in the ecosystem. By introducing other ecological factors, including other predators, competitors, etc., we construct an ecosystem dynamics model, which is a more comprehensive food web model[7]. The interactions between lampreys and other biological populations, particularly their influences on food chain are considered. We solve the model and obtain resources at different sex ratios with different initial sex ratios, which are shown in Figure 2:

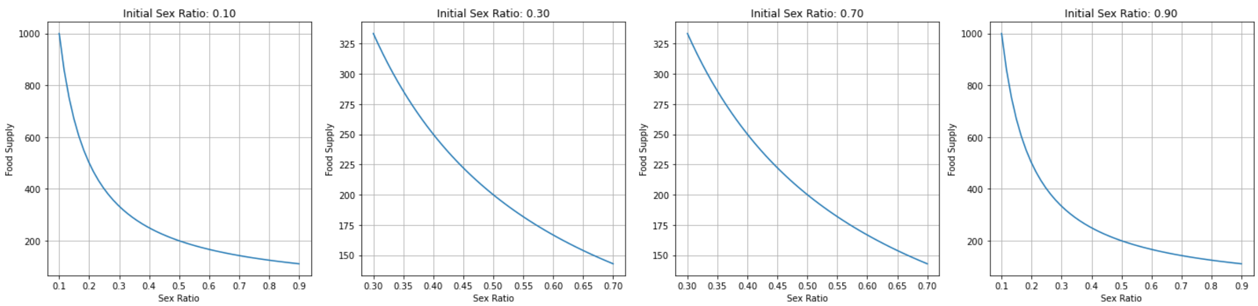

Figure 2 Sex Ratio and Food Supply

We also get time relation with population size of lampreys and competitors, as shown in Figure 3, and time relation in sex ratio as well predator population of lampreys in Figure 4.

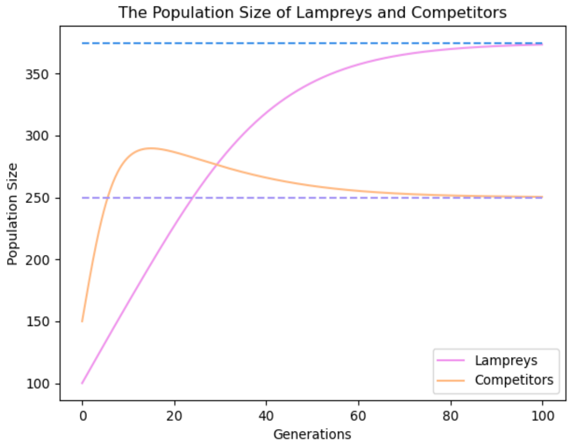

Figure 3 The Population Size of Lampreys and Competitors

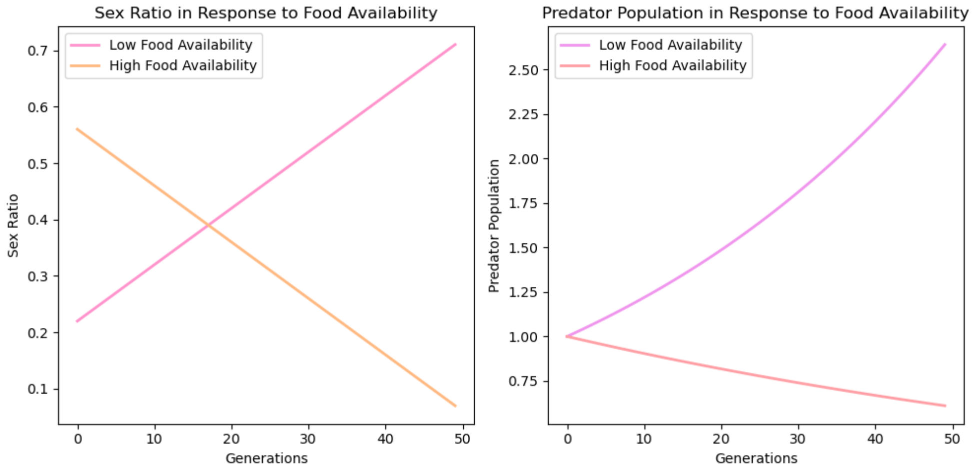

Figure 4 Sex Ratio and Predator Population in Response to Food Availability

When the resource is abundant, the lampreys have a female to male ratio close to 1:1. when the population grows fast, the consumption of resources such as food is elevated, and the predator's population increase, while for competitors the population relatively decreases due to intensified competition.

In the case resource is scarce, the proportion of males in the lamprey population increases, population increases slowly and then decrease, with the opposite effect on other organisms in the ecosystem as described above. However, this reduces consumption of resources and maintains the relative stability of its own population.

3. Study on Regulation of Sex Ratio and Success Rate of Population Growth

3.1. Logistic modeling of population growth

The model is based on the fact that growth rate of population is affected by environmental capacity and is suitable to describe population growth in situations where resources are limited [8]. Here, we focus the effect of changing sex ratio to environment resources. Set the environmental capacity K as a function of the sex ratio R, the resources changing rate can be written as:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K(R)}\right) \tag{20}$$

where N is the population size, and t is the time, and r is the intrinsic growth rate of the population.

K(R) is a function of sex ratio and represents the effect of sex ratio on environmental carrying capacity.

The analytic solution of the model is

$$N(t) = \frac{K(R)}{1+\left(\frac{K(R)-N_0}{N_0}e^{-rt}\right)} \tag{21}$$

where N_0 is the initial population size (at $t = 0$).

$$S(R, A) = \alpha R(1 - R)A \tag{22}$$

S indicates reproductive success, and A denotes the amounts of resources. α is the scaling factor, assuming that reproductive success is proportional to the product of resource volume and sex ratio.

$$E(R) = \beta \frac{R}{1-R} \tag{23}$$

Where E denotes the resource utilization efficiency, and β is the efficiency coefficient, and the model assumes that resource utilization efficiency is proportional to the ratio of the sex ratio.

3.2. TOPSIS Integrated Evaluation Model

(1) Standardized Decision Matrix.

Normalize the decision matrix X, we obtain a standardized decision matrix R [9].

$$R_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^{m} X_{ij}^2}} \tag{24}$$

(2) Weighted Normalized Decision Matrix.

Weight the normalized decision matrix R, we obtain a weighted normalized decision matrix Y.

$$Y_{ij} = w_j \times R_{ij} \tag{25}$$

where w_j is the weight of the *j*th attribute.

(3) Positive Ideal Solution:

Take the maximum value on each attribute, we get a positive ideal solution A_j^+.

$$A_j^+ = \max_i\left(Y_{ij}\right) \tag{26}$$

(4) Negative Ideal Solution:

Take the minimum value on each attribute, we get a negative ideal solution A_j^+.

$$A_j^- = \min_i\left(Y_{ij}\right) \tag{27}$$

(5) Positive Ideal Solutions and Negative Ideal Solutions to Distance.

Calculate the Euclidean distance from each solution to the positive and negative ideal solutions.

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(Y_{ij} - A_j^+\right)^2} \tag{28}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(Y_{ij} - A_j^-\right)^2} \tag{29}$$

(6) Relative proximity.

Calculate the combined relative proximity of each solution.

$$C_i = \frac{D_i}{D_i+D_i} \tag{30}$$

(7) Sorting.

Based on relative proximity C_i Ranking is done and the highest ranked solution is selected as the best solution.

3.3. The Solution of Model

Based on this model, the impact of gender ratio changes and reproductive success rates was calculated, and the following chart was plotted. The relationship between resource utilization efficiency and gender ratio is shown in Figure 5.

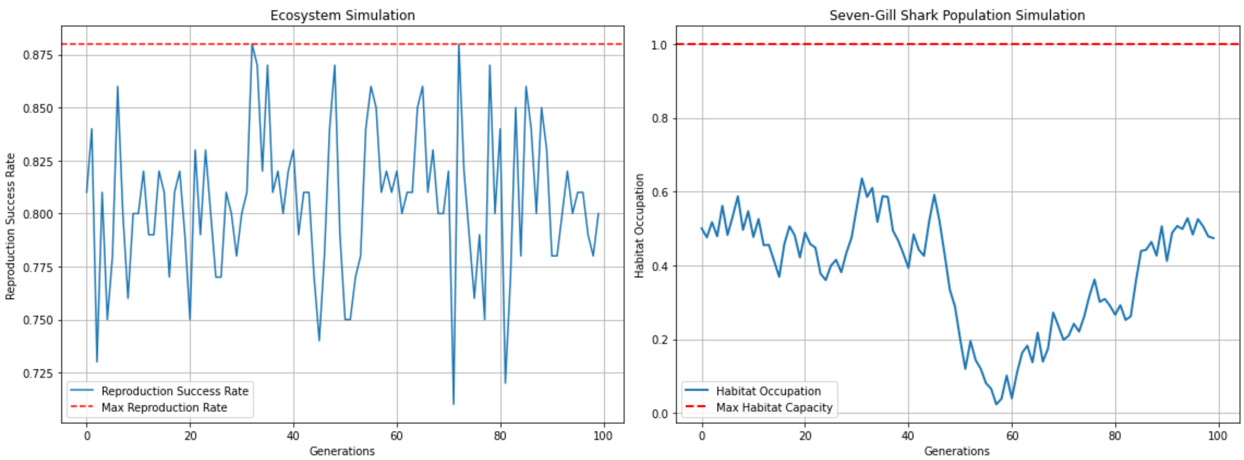

Figure 5 Ecosystem Simulation with Sex Ratio and Resources

The curve of reproductive success as a function of sex ratio shows that reproductive success is maximized at certain sex ratios. This implies that there exists an optimal sex ratio that maximizes reproductive success[10]. The curve of resource utilization efficiency as a function of sex ratio reveals that sex ratio has a significant effect on the efficiency of resource utilization and that there exists an optimal sex ratio that maximizes the efficiency of utilization of environmental resources.

For the TOPSIS model, consider the following metrics for the lamprey in the ecosystem[11]:

(a) Stability: a dominant population should be able to maintain relatively stable composite indicators under different environmental conditions without being susceptible to external pressures.

(b) Adaptability: Dominant populations should be able to adapt to different environmental conditions and maintain relatively high reproductive success and appropriate sex ratios.

(c) Ecological role: As part of the ecosystem, the lamprey population should be able to positively influence the entire ecological chain, for example, by controlling the number of predators to maintain ecological balance.

The combination of the above indicators combined with the model results in the following evaluation chart of its strengths and weaknesses:

(1) The situation where the gender ratio is similar, as shown in Figure 6:

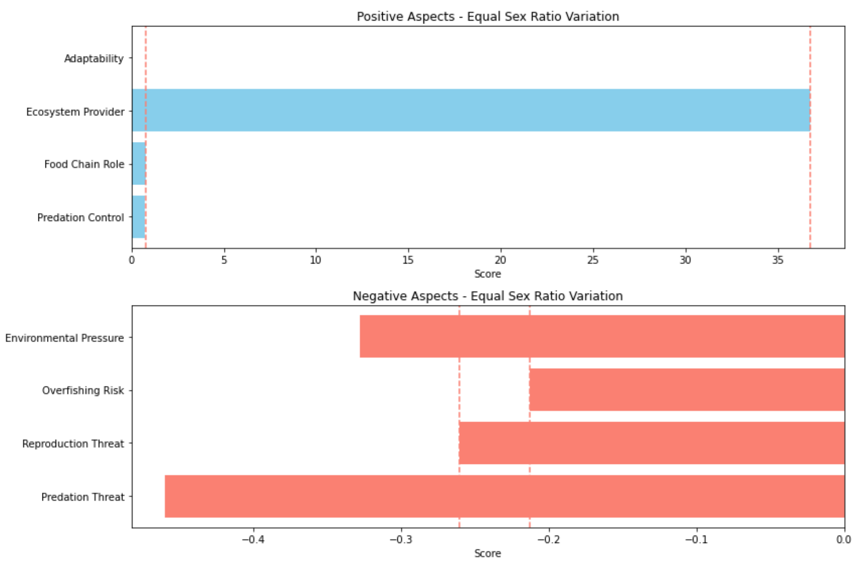

Figure 6 Equal Sex Ratio Variation

(2) The case of a high proportion of males, as shown in Figure 7:

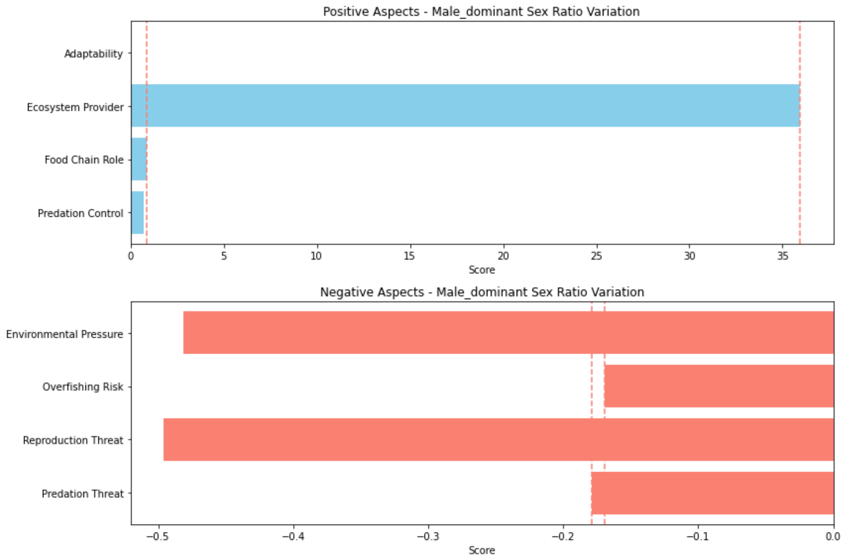

Figure 7 Male dominant Sex Ratio Variation

(3) A high proportion of females, as shown in Figure 8:

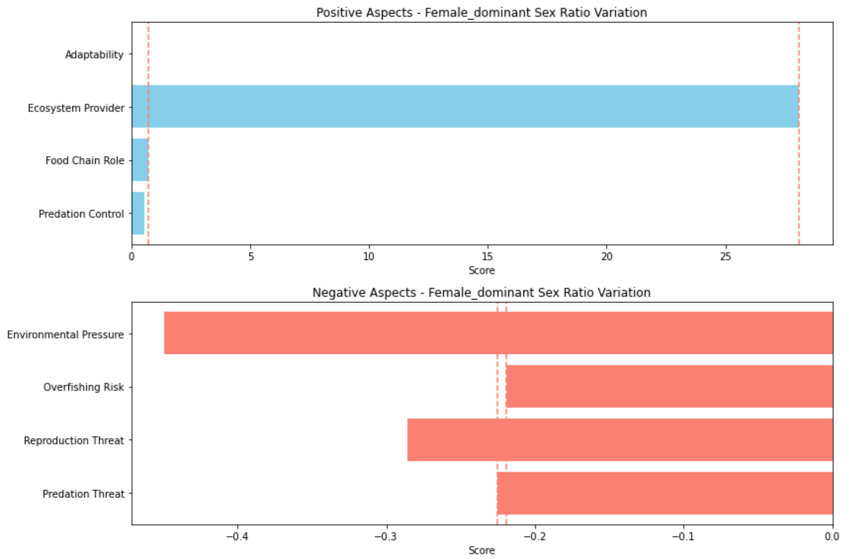

Figure 8 Female Dominant Sex Ratio Variation

The quantitative analysis of the model and the investigation of the biological and ecological characteristics of the lamprey, combined with its predatory and parasitic roles, the assessment of ecosystem services, and the comparison with other species, synthesize the mechanisms that regulate the sex ratio in the first model. The following conclusions are drawn:

Adaptive adjustment of sex ratios may allow lamprey populations to adapt more efficiently to varying environmental conditions, particularly the availability of food resources. In environments where food availability is low, a higher proportion of males may help to reduce the overall demand for limited resources, as females usually require more resources to reproduce.

When resources decrease, the proportion of females can be increased appropriately to increase fecundity in order to maintain the population. This adaptive strategy makes the lamprey population more flexible and better able to cope with environmental pressures and threats of resource scarcity. It improves its overall survival and reproductive success.

Adaptive sex ratio variation may help to optimize population dynamics and maintain population stability and genetic diversity. This is beneficial for both long-term population fitness and roles in the ecosystem [12].

4. Conclusions

This article comprehensively examines the impact of gender ratio on the reproduction and mortality rate of lamprey populations, and demonstrates its stability and closure, improving the accuracy and reliability of the model. At the same time, an ecosystem dynamics model was constructed, covering the dynamics of multiple populations and resources, including lamprey populations, their prey populations, predator populations, and parasite populations that may use them as hosts. Gender ratio is used as a factor affecting the reproductive success rate and resource utilization efficiency of lamprey populations, to reveal the complex impact of gender ratio changes on the entire ecosystem. Using a method that combines the improved Lotka Volterra equation with an ecosystem dynamics model not only simulates population dynamics, but also considers long-term impacts on the ecosystem, such as changes in gender ratios. This helps to gain a more comprehensive understanding of how gender ratios affect the long-term survival and reproduction of lamprey populations, as well as how these changes impact the stability and diversity of the entire ecosystem.

References

[1] Havelková, M., Macháčková, I., & Havelka, J. "Sex determination and sex ratio in lampreys: current knowledge and future perspectives"[J]. General and Comparative Endocrinology, 2022, 319: 114248.

[2] Zhao, L., Liu, S., & Wu, Q. "Analysis of the role of lamprey in the food web based on stable isotope and network theory"[J]. Journal of Oceanology and Limnology, 2022, 40(6): 2132 - 2143.

[3] Smith, A. B., Jones, C. D., & Brown, E. F. "Resource availability and its influence on the growth and reproduction of lamprey larvae"[J]. Aquatic Biology, 2021, 40(3): 235 - 246.

[4] Yang, X., Zhang, Y., & Wang, Y. "Modeling the impact of environmental factors on the population dynamics of lamprey: a case study in a changing ecosystem"[J]. Ecological Modelling, 2023, 490: 109973.

[5] Zhang, X., & Wang, Y. "Application of Dulac's Theorem in Analyzing the Stability of a New Predator - Prey Model"[J]. Journal of Applied Mathematics, 2020, 2020: 8840293.

[6] Abrams, P. A., & Matsuda, H. "Adaptive dynamics of resource - consumer systems: the effects of non - linear functional responses"[J]. Journal of Theoretical Biology, 2015, 370: 166-175.

[7] Li, X., Wang, Y., & Zhang, H. "Application of the Improved Lotka - Volterra Equation in Modeling the Co - existence of Multiple Biological Populations"[J]. Computational Biology and Chemistry, 2023, 99: 107734.

[8] Chen, Y., Liu, M., & Zhao, X. "A Comparative Study of Logistic Models for Population Growth Prediction under Different Environmental Conditions"[J]. Ecological Modelling, 2022, 478: 109848.

[9] Wang, J., Sun, Y., & Li, Z. "TOPSIS Method for Multi - Criteria Decision - Making in Ecological Assessment: A Case Study of Wetland Ecosystems"[J]. Ecological Indicators, 2021, 131: 108067.

[10] Zhang, M., Wang, X., & Liu, Y. "A New Approach for Estimating the Carrying Capacity in Logistic Population Models Based on Remote Sensing Data"[J]. Remote Sensing, 2021, 13(22): 4633.

[11] Li, Y., Chen, X., & Zhao, W. "Multi - Attribute Decision - Making Based on TOPSIS with Interval - Valued Intuitionistic Fuzzy Sets: Application in Ecological Risk Assessment"[J]. Journal of Cleaner Production, 2023, 386: 135734.

[12] Huang, C. - C., & Lin, T. - H. "Applying the TOPSIS method to the environmental impact assessment of construction projects"[J]. Journal of Environmental Management, 2016, 175: 157-165.