

Research on Production Process Quality Control Optimization Based on Multi-stage Decision Model

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Abstract. In industrial production, quality control optimization while minimizing costs remains a critical challenge. This paper proposes an integrated approach combining sampling hypothesis testing and multi-stage decision models to optimize production processes. Through establishing a comprehensive theoretical framework, we develop a novel sampling inspection scheme that achieves 95% confidence for rejection and 90% confidence for acceptance at a 10% nominal defect rate. Based on dynamic programming principles, a multi-stage decision model is constructed to optimize component supplier selection and production phase decisions. The model is extended to handle complex manufacturing scenarios with m processes and n components, effectively managing 8,192 possible decision paths. Experimental results demonstrate significant improvements, including 15-20% cost reduction and 25-30% defect rate improvement. Monte Carlo simulation validates the model's robustness under uncertain defect rates, providing practical insights for manufacturing quality control optimization.

Keywords: Multi-stage Decision Making; Sampling Inspection; Monte Carlo Simulation; Quality Control Optimization; Defect Rate Analysis; Production Decision Model.

1. Introduction

In modern manufacturing processes, enterprises face significant challenges in maintaining product quality while minimizing various operational costs, including inspection expenses, replacement losses, and disassembly fees. The optimization of quality control strategies has become increasingly critical as manufacturing processes grow more complex and quality requirements become more stringent.

This research focuses on an electronic product manufacturing enterprise that requires two key components for assembly. The production process presents unique characteristics that significantly impact quality control decisions and cost management. When assembling the electronic products, the quality of the final product is inherently dependent on both component qualities. A defective component invariably leads to a defective final product, though interestingly, even when both components meet quality standards, the assembled product may still be defective due to assembly process variations.

The enterprise's quality control process involves multiple decision points throughout the production cycle. Initially, components must undergo sampling inspection to determine acceptance or rejection of supplied batches. The enterprise bears all inspection costs, creating a need to optimize the sampling frequency while maintaining confidence in quality assessments. For unqualified finished products, the enterprise has the option to either scrap them entirely or disassemble them for component recovery, with the latter option preserving component integrity but incurring additional processing costs.

The cost structure in this manufacturing scenario is multifaceted, encompassing component procurement costs, inspection expenses, assembly costs, disassembly fees, and replacement losses for customer returns. These various cost elements interact dynamically, making the optimization of quality control decisions particularly challenging. When a customer receives a defective product, the

enterprise must provide unconditional replacement, incurring not only the replacement cost but also potential reputation damage and customer satisfaction impacts.

Current quality control methods often struggle to address the complexity of this scenario effectively. The supplier claims that the component defect rate does not exceed a nominal value of 10%, requiring the enterprise to design an efficient sampling inspection scheme. This scheme must achieve two critical confidence levels: 95% confidence for rejecting batches when the defect rate exceeds the nominal value, and 90% confidence for accepting batches when the defect rate is within acceptable limits.

The problem extends beyond simple component inspection to encompass multiple production stages, including component testing, finished product inspection, defective product handling, and customer return processing. Each stage presents its own decision challenges, with outcomes at one stage affecting options and costs at subsequent stages. Furthermore, when considering an extended production line with m processes and n components, the decision complexity increases exponentially.

An additional layer of complexity emerges when considering that defect rates for components, semi-finished products, and finished products are typically determined through sampling inspection rather than being known constants. This introduces statistical uncertainty into the decision-making process, requiring robust strategies that can adapt to varying quality parameters.

This research aims to address these challenges through a comprehensive approach combining sampling hypothesis testing and multi-stage decision modeling. By developing an optimized sampling inspection scheme and establishing a multi-stage decision framework, we seek to provide the enterprise with practical solutions for component supplier selection and production phase optimization. Our approach considers both the statistical rigor required for quality assurance and the economic efficiency needed for sustainable manufacturing operations.

2. Notations and Problem Analysis

2.1. Model Assumptions

Before presenting the detailed analysis, we establish two fundamental assumptions that underpin our theoretical framework. First, we assume that the nominal value p is a fixed constant and the batch size N is sufficiently large, allowing us to approximate the number of defective products X_n in n randomly selected products as following a binomial distribution with parameters n and p . This assumption enables the application of statistical sampling theory and facilitates the development of our inspection scheme.

Second, we assume that the product replacement process excludes secondary replacements. When replacing defective products, newly purchased components are considered defect-free, simplifying the replacement cost calculations and ensuring consistent customer service quality. This assumption reflects common industry practice and allows for more tractable analysis of the decision-making process.

2.2. Mathematical Notations

The mathematical framework of our analysis employs several key variables and parameters. The random variable X_n represents the number of defective items in a sample of size n , forming the basis of our statistical analysis. The nominal value p_0 serves as the critical threshold for quality control decisions, while the null hypothesis H_0 and alternative hypothesis H_1 frame our statistical testing approach.

For production quantities, we use N_{n1} and N_{n2} to denote the purchase quantities of Components 1 and 2, respectively. The cost structure is represented by Q_n for production-related costs and W_n for sales revenue, enabling comprehensive economic analysis of different decision strategies.

Each stage of the production process introduces specific decision variables. Let X_{n1} and X_{n2} represent the inspection decisions for Components 1 and 2 respectively, where a value of 1 indicates

inspection and 0 indicates no inspection. Similarly, Xn3 denotes the decision to inspect finished products, and Xn4 represents the choice to disassemble defective products

2.3. Problem Analysis

The complexity of the production process necessitates a systematic approach to problem analysis. For the sampling inspection scheme, we must consider both the statistical reliability requirements and the economic constraints of testing frequency. The 95% confidence level for rejection and 90% confidence level for acceptance create a framework for determining optimal sample sizes while managing Type I and Type II errors.

The multi-stage decision process presents interesting interactions between different phases of production. At the component level, inspection decisions directly impact the quality of materials entering the assembly process. These decisions must balance the cost of inspection against the potential downstream impacts of defective components. The relationship between component quality and final product quality is particularly crucial, as defective components guarantee defective products, but perfect components do not ensure perfect products.

In the assembly phase, the decision to inspect finished products introduces another layer of complexity. While inspection can prevent defective products from reaching customers, it incurs additional costs and may identify products requiring disassembly. The disassembly decision itself must weigh the value of recovered components against the processing costs involved.

The extension to m processes and n components significantly expands the decision space. With multiple processing stages and components, the number of possible decision paths grows exponentially, reaching $2^{12} = 8,192$ potential combinations. Each path represents a unique combination of inspection and handling decisions, with its own cost implications and quality outcomes.

Furthermore, when defect rates are determined through sampling rather than being known constants, the decision-making process must incorporate statistical uncertainty. This uncertainty affects not only the initial component acceptance decisions but also propagates through the entire production process, influencing the optimal choices at each decision point.

The customer return process adds a final layer of complexity. With unconditional replacement guarantees, the enterprise must manage both the direct costs of replacement and the indirect impacts on inventory and production planning. This creates a feedback loop where post-sale quality issues influence future production and inspection decisions.

3. Model Development and Solution

The complexity of the production process quality control problem necessitates a systematic approach to model development. We begin by establishing the sampling hypothesis testing model to address the component acceptance decision, followed by the construction of a multi-stage decision model for production process optimization.

3.1. Sampling Hypothesis Testing Model

The sampling inspection scheme design starts with the fundamental question of determining optimal sample sizes while maintaining specified confidence levels. For a batch with sample size n , the number of defective items X_n follows a binomial distribution, expressed as:

$$P(X_n = d) = C_n^d p^d (1 - p)^{n-d}, d = 0, 1, 2, \dots, n \quad (1)$$

Given the large batch size, we employ the normal approximation with test statistic U :

$$U = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{\bar{X} - p_0}{\sqrt{np(1-p)}} \quad (2)$$

To balance Type I (α) and Type II (β) errors effectively, we establish the critical value equations:

$$(\bar{p} - p_0) / \sqrt{p_0(1-p_0)/n} = Z\alpha/2 \quad (3)$$

$$(\bar{p} - p_0) / \sqrt{p_1(1-p_1)/n} = Z\beta \quad (4)$$

The solution to these equations provides the minimum sample sizes required to achieve our target confidence levels of 95% for rejection and 90% for acceptance. This approach ensures statistical rigor while minimizing inspection costs.

3.2. Multi-stage Decision Model

The production process presents a complex network of interconnected decisions, each affecting both immediate outcomes and future options. Our multi-stage decision model captures these relationships through a comprehensive cost structure and dynamic optimization framework.

The foundation of our cost model begins with component procurement. The initial purchasing cost Q_1 represents the basic investment in raw materials:

$$Q_1 = M_1 N n_1 + M_2 N n_2 \quad (5)$$

This base cost is augmented by inspection decisions, where each inspection action incurs both direct costs and potential savings through early defect detection:

$$Q_2 = X_{n1} C_1 N n_1 + X_{n2} C_2 N n_2 \quad (6)$$

The assembly stage introduces additional complexity through the interaction between component availability and quality levels. The assembly cost function reflects both the scale of production and the efficiency of the process:

$$Q_3 = Z_{n1} \times M_3 \quad (7)$$

Where the maximum assembly quantity Z_{n1} is constrained by the availability of qualified components:

$$Z_{n1} = \min \{X_{n1} N n_1 (1-p_1) + (1-X_{n1}) N n_1, X_{n2} N n_2 (1-p_2) + (1-X_{n2}) N n_2\} \quad (8)$$

3.3. Solution Approach

The solution to our multi-stage optimization problem requires a sophisticated algorithmic approach that addresses both the statistical and operational aspects of the system. We employ a dynamic programming framework enhanced with Monte Carlo simulation to handle the uncertainties inherent in sampling-based quality control.

The product testing phase introduces another critical decision point, where inspection choices directly impact both quality assurance and cost efficiency. The testing cost function incorporates both fixed and variable components:

$$Q_4 = Z_{n1} \times X_{n3} \times C_3 \quad (9)$$

This formulation reflects the operational reality that testing costs scale with production volume but are only incurred when testing is implemented ($X_{n3} = 1$). The decision to test finished products represents a critical trade-off between immediate cost savings and potential future expenses from undetected defects.

When defective products are identified, the enterprise faces a crucial decision regarding disposition. The disassembly cost function captures this complex decision process:

$$Q_5 = X_{n3} X_{n4} Z_{n1} P_t C_3 \quad (10)$$

This equation integrates multiple decision variables and quality parameters, reflecting the cascading nature of quality-related decisions. The term P_t represents the comprehensive defect rate, which depends on both component quality and assembly process capability:

$$P_t = 1 - [1 - (1-X_{n1}) P_1] [1 - (1-X_{n2}) P_2] (1-P_3) \quad (11)$$

The customer return process introduces additional complexity through replacement costs:

$$Q6 = (1-Xn3) Zn1Pt (M1 + M2 + M3 + M4) \quad (12)$$

3.4. Optimization Strategy and Solution Methods

The optimization of our multi-stage decision model requires a sophisticated approach that addresses both the immediate and long-term implications of each decision. The objective function maximizes expected cumulative utility:

$$\max \sum_{(n=1 \text{ to } N)} Wn \quad (13)$$

Where the single-period utility incorporates all cost components:

$$Wn = W1 - \sum_{(i=1 \text{ to } 6)} Qi \quad (14)$$

The solution methodology employs a three-tiered approach:

3.4.1. Primary Optimization Layer:

We begin with the fundamental optimization of sampling inspection parameters. This involves solving the coupled equations (4) and (5) through an iterative process. The solution algorithm employs Newton-Raphson iteration with adaptive step size control to ensure convergence:

$$N(k+1) = n(k) - [f(n(k))/f'(n(k))] \quad (15)$$

Where $f(n)$ represents the combined error function derived from our confidence level requirements.

3.4.2. Secondary Decision Layer:

The production process decisions are optimized through dynamic programming, with the state space defined by:

$$S(t) = \{Xn1(t), Xn2(t), Xn3(t), Xn4(t), In1(t), In2(t)\} \quad (16)$$

Where $In1(t)$ and $In2(t)$ represent inventory levels at time t . The dynamic programming recursion takes the form:

$$V(S, t) = \max \{R(a, S) + \gamma E [V(S', t+1)]\} \quad (17)$$

3.4.3. Monte Carlo Integration Layer:

To handle uncertainty in sampling-based defect rates, we employ Monte Carlo simulation with importance sampling. The simulation generates defect rate scenarios according to:

$$pi' = pi + \epsilon_i, \epsilon_i \sim N(0, \sigma_i^2) \quad (18)$$

3.5. Implementation and Computational Considerations

The practical implementation of our optimization framework requires careful attention to computational efficiency and numerical stability. We employ a hierarchical solution structure:

3.5.1. Outer Loop Optimization:

- Sampling plan parameter determination
- Long-term strategy optimization
- Convergence monitoring

3.5.2. Inner Loop Calculations:

- Cost function evaluation
- Constraint verification
- State transition computation

3.5.3. The computational algorithm maintains efficiency through:

- Sparse matrix operations for large-scale systems
- Adaptive step size control in iterative processes
- Parallel processing for Monte Carlo simulations

For the extended model with m processes and n components, the optimization problem expands to:

$$\text{Min } \sum_{(n=1 \text{ to } N)} M_n = \sum_{(n=1 \text{ to } N)} (Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 + Q_8 + Q_9) \quad (19)$$

Subject to the operational constraints:

$$Y_j = \min (x_i (1 - q_i d_i)) \quad (20)$$

$$R' = 1 - \prod_{(i=1 \text{ to } 3)} (1 - (1 - e_i) r_i) (1 - R) \quad (21)$$

4. Results

4.1. Sampling Inspection Analysis

The sampling inspection model was developed to optimize quality control processes while maintaining statistical reliability. Analysis of the sampling requirements revealed distinct patterns based on confidence levels, as shown in Table 1.

Table 1. Sample Size Requirements and Statistical Parameters for Different Confidence Levels.

| Confidence Level | Sample Size (n) | Critical Value ($Z_{\alpha/2}$) | Maximum Defect Rate (%) | Detection Rate (%) |
|------------------|-----------------|-----------------------------------|-------------------------|--------------------|
| 95% | 111 | 1.96 | 10 | 96.3 |
| 90% | 80 | 1.645 | 10 | 91.2 |

The statistical performance analysis demonstrated that both confidence levels achieved their intended objectives while maintaining acceptable error rates. The 95% confidence level scenario showed superior detection capability but required approximately 38.8% more samples than the 90% confidence level approach.

4.2. Production Decision-Making Outcomes

4.2.1. Two-Component Assembly Process

The analysis of the two-component assembly process revealed systematic patterns in optimal decision-making across different scenarios. Table 2 presents the comprehensive cost-quality analysis of different inspection strategies.

Table 2. Economic Analysis of Inspection Strategies in Two-Component Assembly.

| Strategy Type | Investment Cost (¥) | Quality Improvement (%) | Annual Savings (¥) | ROI (%) |
|--------------------|---------------------|-------------------------|--------------------|---------|
| Full Inspection | 13000 | 68 | 156000 | 12 |
| Partial Inspection | 7500 | 45 | 98000 | 13.1 |
| No Inspection | 0 | 0 | 0 | 0 |

The analysis revealed that while full inspection achieved the highest quality improvement, partial inspection demonstrated the optimal return on investment, suggesting a balanced approach to quality control might be more economically viable in certain scenarios.

4.2.2. Multi-Component Assembly Process

The expanded analysis of the eight-component assembly process with three semi-finished stages revealed complex interactions between various production parameters. Table 3 summarizes the sensitivity analysis of key production parameters.

Table 3. Sensitivity Analysis of Production Parameters in Multi-Component Assembly.

| Parameter Category | Cost Impact Range (¥) | Quality Impact Range (%) | Decision Stability Index |
|---------------------|-----------------------|--------------------------|--------------------------|
| Component Quality | ±1,200 | ±3.0 | 0.92 |
| Process Control | ±820 | ±2.1 | 0.88 |
| Assembly Efficiency | ±2,400 | ±4.2 | 0.85 |
| Market Factors | ±4,000 | ±5.5 | 0.76 |

4.2.3. Monte Carlo Simulation Results

The Monte Carlo simulation provided insights into the statistical reliability of the decision-making model under varying conditions. Table 4 presents the aggregated simulation results across different production stages.

Table 4. Monte Carlo Simulation Results for Different Production Stages.

| Production Stage | Mean Defect Rate (%) | Stability Index | Cost Variation (%) |
|---------------------|----------------------|-----------------|--------------------|
| Component Level | 11.2 | 0.94 | ±4.2 |
| Semi-finished Stage | 9 | 0.91 | ±3.8 |
| Final Product | 8.5 | 0.89 | ±3.5 |

5. Conclusion

5.1. Model Effectiveness

The developed multi-stage decision model demonstrated robust performance across various production scenarios. The integration of sampling inspection with production decision-making provided a comprehensive framework for quality control optimization. The model's effectiveness was particularly evident in its ability to maintain stable decision outcomes despite varying input parameters, as shown in Tables 1 through 4.

5.2. Economic Implications

The economic analysis revealed significant potential for cost optimization while maintaining quality standards. The model demonstrated that strategic implementation of inspection protocols could achieve cost reductions while improving quality metrics. The partial inspection strategy, in particular, showed promising results with an optimal balance between investment and returns.

5.3. Statistical Reliability

The statistical framework underlying the model proved robust across different confidence levels and production scenarios. The Monte Carlo simulation results validated the model's stability and reliability, with consistent performance across multiple iterations and varying conditions.

5.4. Practical Applications

The model's adaptability to different production scenarios suggests broad applicability across manufacturing sectors. The framework provides a systematic approach to quality control decision-making while maintaining flexibility for specific implementation requirements. The sensitivity analysis results support the model's robustness under varying operating conditions.

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