Dynamic Wear Simulation of Steps Based on Archard and Numerical Difference Algorithm

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Abstract. In this paper, a dynamic wear multifactor coupling model is proposed, focusing on model construction, typical walking pattern analysis and cross-scale validation. Firstly, based on the theory of wear and elastic mechanics, a single-step wear depth calculation model is established, and the time discretization is carried out by using the explicit difference format, the spatial discretization by using the center difference format, and the two-dimensional discrete numerical simulation is carried out by combining with the adaptive time-step strategy. Secondly, the travel patterns are defined, and the “pattern matrix” framework is constructed by linear superposition, which transforms the pattern distribution into a matrix operation to quantify the incremental wear weights. Finally, the single-step model is extended to the full staircase 3D dataset, and the human flow distribution is inverted using the least squares method and maximum likelihood estimation. The model integrates multi-physical field coupling, and realizes wear prediction with the help of numerical discretization algorithm and matrix superposition, which can accurately reflect the impacts of different walking modes on the structure, and provides a cross-scale quantitative analysis method for the durability assessment of staircases.

Keywords: Archard; numerical difference algorithm; pattern matrix; Piecewise spatial modeling method.

1. Introduction

In this paper the wear evolution of staircase structures under dynamic loading is emphasized, with the aim of constructing a multi-factor coupled dynamic wear model and quantifying the influence of typical walking modes on the wear mechanism [1]. Firstly, based on Archard's wear theory and elastic mechanics, a single-step wear depth calculation model is established, and the model system with multi-physical field coupling is improved by introducing attenuation function, correction factor and diffusion effect term; the numerical difference algorithm is used to carry out the spatial discretization, and combined with the adaptive time-step strategy, the dynamic numerical simulation of the two-dimensional discrete wear model is realized [2-4]. Secondly, multiple modes are defined for the behavioral characteristics of the crowd, and the “mode matrix” framework is constructed through linear superposition, so that the wear distribution of different modes is transformed into matrix operations, and the quantitative analysis of the weight of the wear increment of each mode is realized. Finally, the single-step model is extended to the three-dimensional space-time scale of the whole staircase, and the least squares method with maximum likelihood estimation is utilized to invert the distribution and time pattern of foot traffic [5] [6]. The experimental results show that the constructed model can effectively simulate the actual wear and tear distribution, which provides theoretical support and methodological reference for the durability assessment of the staircase structure, the development of maintenance strategies and the monitoring of pedestrian flow.

2. Multi-Factor Coupled Wear Modeling and Numerical Methods

2.1. Single Step Wear Model Based on Archard's Theory

2.1.1. Dynamic friction and wear evolution model establishment

On the microscopic scale, first consider the surface loss of the material due to a single tread. According to Archard's wear theory, the amount of single wear is proportional to the normal force and sliding distance, and inversely proportional to the material hardness. Let the depth of single wear be Δh_1 , then there are:

$$\Delta h_l = k\frac{F_n \cdot s}{H \cdot A} \tag{1}$$

Where k is the wear coefficient, F_n is the normal force, s is the sliding distance, H is the material hardness and A is the contact area.

2.1.2. Modeling of dynamic pressure wear evolution

Considering the actual role of stairs, small deformations under prolonged treading cannot be ignored. To derive the small deformation of the slab under a vertically downward force, the basic concepts are used in the mechanics of materials, especially Hooke's law. Hooke's law states that the stress of a material is proportional to the strain, with the coefficient of proportionality being the elastic modulus of the material.

Let us assume that the slab is a homogeneous, isotropic elastomer with a thickness of d, an area of A and a normal force of F_n. The material of the slab has a modulus of elasticity E. From the definition of stress σ. According to Hooke's law, the stress σ is proportional to the strain ϵ and the proportionality factor is the modulus of elasticity E:

$$\sigma = E\epsilon = \frac{F_n}{A} \tag{2}$$

Since the strain ϵ is the ratio of the shape variable Δh_2 to the original size d:

$$\frac{F_n}{A} = \epsilon E = E\frac{\Delta h_2}{d} \tag{3}$$

From this the single wear depth Δh can be got:

$$\Delta h = \Delta h_1 + \Delta h_2 = k\frac{F_n \cdot s}{H \cdot A} + \frac{F_n d}{AE} \tag{4}$$

Considering the dynamic nature of human motion, the normal force F_n can be expressed as a function of time:

$$F_n(t) = mg(1 + \alpha \sin(\omega t)) \tag{5}$$

Where m the body is mass, g is the gravitational acceleration, α is the dynamic coefficient, ω and is the step frequency. And ωt is satisfied: $\omega t \in [0, \pi]$

On the macro scale, need to consider cumulative effects. Let the total wear depth in time t be $h(t)$, then there are:

$$h(t) = \int_0^t \frac{k}{H \cdot A}\int_0^\tau F_n\left(t'\right) \cdot v\left(t'\right) \partial t' \partial \tau + \frac{d}{AE}\int_0^t F_n\left(t'\right) \partial t' \tag{6}$$

Where $v(t)$ is the sliding speed?

2.1.3. Model optimization

To deal with the degradation of material properties over time, the material property decay function is introduced:

$$H(t)=\left(H_o\right)^{-\beta t} \tag{7}$$

Where H_0 is the initial hardness and β is the attenuation factor. The influence of environmental factors can be expressed by a correction factor:

$$k(T,\eta)=k_\theta\left(1+\gamma_T\left(T-T_\theta\right)+\gamma_\eta\left(\eta-\eta_\theta\right)\right) \tag{8}$$

Where T is the temperature, η is the humidity, and γ_T and γ_η are the influence coefficients of temperature and humidity, respectively. Finally, obtain the complete wear evolution equation:

$$\frac{\partial h}{\partial t}=\frac{k(T,\eta)}{H(t)\cdot A}F_n(t)\cdot v(t)+\frac{dF_n(t)}{AE}+D\nabla^2 h \tag{9}$$

Where the last part represents the diffusion effect of wear and D is the diffusion coefficient.

2.1.4. Numerical simulation based on numerical difference algorithm

After modeling the dynamic friction wear evolution and dynamic pressure wear evolution, a 2D discrete wear model for a single step is developed.

At the time discretization level, use an explicit difference format

$$h^{n+1}=h^n+\frac{k(T,\eta)}{H\left(t_n\right)\cdot A}F_n\left(t_n\right)\cdot v\left(t_n\right)+\frac{dF_n\left(t_n\right)}{AE}+\Delta t\cdot D\nabla^2 h^n \tag{10}$$

At the level of spatial discretization, use the central difference format:

$$\nabla^2 h_{i,j}\approx\frac{h_{i+l,j}+h_{i-1,j}+h_{i,j+l}+h_{i,j-l}-4h_{i,j}}{\Delta x^2} \tag{11}$$

Meanwhile, to improve the computational efficiency and ensure the numerical stability, the adaptive time-step strategy is adopted:

$$\Delta t=\min\left(\frac{C\Delta x^2}{2D},\frac{H(t)\cdot A}{k(T,\eta)\left|F_n(t)\cdot v(t)\right|},\frac{AE}{dF_n(t)}\right) \tag{12}$$

Where C is the Courant number, usually taken as 0.5.

At each time step, need to perform a convergence test:

$$\max_{i,j}\left|h_{i,j}^{(n+D)}-h_{i,j}^{(n)}\right|<\varepsilon \tag{13}$$

Where ε is the preset convergence threshold? A no-flow boundary condition is used at the edges of the staircase: $\left.\frac{\partial h}{\partial n}\right|_{\partial\Omega}=0$

A continuity condition is applied at the step joints:

$$h^-=h^+,\left.\frac{\partial h}{\partial n}\right|^-=\left.\frac{\partial h}{\partial n}\right|^+ \tag{14}$$

Accordingly, the observed final wear depth $\Delta h_{i,j}$ can be obtained by summing up all the tread events, and include some random error terms, $\varepsilon_{i,j}$, to better fit the actual situation:

$$\Delta h_{i,j} = \sum_{n=1}^{N} h_{i,j}^{(n)} + \varepsilon_{i,j} \tag{15}$$

Where $\varepsilon_{i,j}$ is used to indicate deviations due to measurement noise, environmental errors or inhomogeneities in the material itself, N indicates the total time required to cause the final wear depth $\Delta h_{i,j}$.

2.2. Parameterized Pedaling Patterns

Take the three most common patterns: people single-footed in the middle of the step, two people walking side by side, and people single-footed on the edge of the step, each of which is denoted as $P_1, P_2, \cdots, P_n$. For each mode, its unit tread on the grid (i,j) is estimated experimentally or theoretically beforehand as a function of the distribution $\delta_{i,j}^{(n)}$, $n = \{1, 2, 3\}$ and each time the n mode occurs, the wear increment for (i,j) can be written as follows:

$$h_{i,j}^{(n)} = \delta_{i,j}^{(n)} \tag{16}$$

2.2.1. Mode 1: person with one foot in the middle of the step

When a single person uses the stairs, either going up or down, assume that the distribution of footsteps is concentrated in the middle or slightly to the side of the step.

$$\delta_{i,j}^{(n)} = A_l e^{\left(-\frac{(i-x_j)^2}{2\sigma_x^2} - \frac{(j-y_0 \pm \Delta y)^2}{2\sigma_y^2}\right)} \tag{17}$$

Where: x_c indicates the center position of the step; y_c indicates the middle position of the step tread; Δy indicates the offset, $\Delta y > 0$ indicates the upper floor offset leading edge, $\Delta y < 0$ indicates the lower floor offset trailing edge; σ_x , σ_y indicates the range of footstep distribution.

2.2.2. Mode 2: pair walking side by side

Two people walking side-by-side, both up and down the stairs, have their feet concentrated on the left and right sides of the steps in a symmetrical distribution.

$$\delta_{i,j}^{(2)} = A_2 \left(e^{\left(-\frac{(i-x_j)^2}{2\sigma_x^2} - \frac{(j-y_y \pm \Delta y)^2}{2\sigma_y^2}\right)} + e^{\left(-\frac{(i-x_j)^2}{2\sigma_x^2} - \frac{(j-y_z \pm \Delta y)^2}{2\sigma_y^2}\right)} \right) \tag{18}$$

Where: $x_l < x_c$, $x_r > x_c$ indicates the center position of the left and right feet and x_l , x_r are symmetrically distributed; y_c indicates the position of the middle of the step; σ_x , σ_y indicates the distribution range of the footsteps.

2.2.3. Mode 3: person with one foot on the edge of the step

When a person steps on the edge of a step with one foot, the foot is centered on the edge of the step.

$$\delta_{i,j}^{(3)} = A_3 e^{\left(-\frac{(i-x_x)^2}{2\sigma_x^2}-\frac{j^2}{2\sigma_y^2}\right)} \tag{19}$$

2.2.4. Superposition construction

In summary, the wear distribution of a given step of a staircase consists of a linear superposition of these three patterns.

If write the distribution function of all modes as a "mode matrix", denoted $\vec{\delta}^{(n)}$; arrange $\{\Delta h_{i,j}\}$ as a vector $\vec{h}$; arrange $\varepsilon_{i,j}$ as a vector $\vec{\varepsilon}$, and make $\vec{c}=(c_1,\cdots c_n)^T$; then there are:

$$\vec{h}=\sum_{n=1}^{N} c_n \vec{\delta}^{(n)}+\vec{\varepsilon}=\left[\vec{\delta}^{(1)}\vec{\delta}^{(2)}\cdots\vec{\delta}^{(N)}\right]\vec{c}+\vec{\varepsilon} \tag{20}$$

This results in a linear modeling framework. By measuring $\vec{h}$, first understand a priori (or fit) the pattern matrix, and thus inversely perform the tread weights for each pattern c_n

2.3. Solving the Model

It is possible to derive the amount of wear and tear on stone and wood for a single person in a single use based on the modeling above $h_{i,j}$. The results are shown in the Fig. 1.

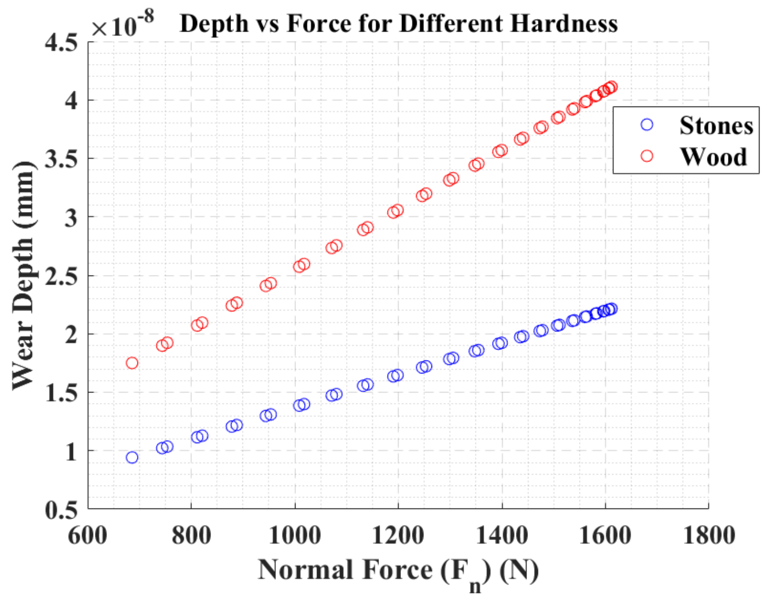

Fig 1. Scatterplot of normal force versus wear depth for different material backgrounds.

Assume the same number of people pass each time, and for total wear and tear $\Delta h_{i,j}$, there are:

$$P=\frac{\Delta h_{i,j}}{h_{i,j}\cdot n} \tag{21}$$

Where P represents frequency and n represents number.

3. Three-Dimensional Segmental Modeling: Full Staircase Wear and Temporal Analysis

The wear of staircase surfaces is not only related to the magnitude of forces but also to many factors, so a dynamic wear model with multi-factor coupling is needed.

3.1. Integrated Time-Repair-Materials-Number-of-People Model for Multi-Steps

In practical problems, it is necessary to analyze not only a single step, but also the whole staircase, different time periods. The special model for single steps is extended to a general model for the whole section of steps, the core of which lies in the introduction of spatial modeling with more constraints in blocks or segments.

3.1.1. Spatial extension of multi-steps

Let there be a total of s steps in the entire staircase (numbered $s = 1, 2, \cdots, S$). For each step, a 2D discrete data collection at $\{\Delta h_{i,j}^{(s)}\}$ can still be done in the same way as for the "single step". In this way, there are a three-dimensional number set $\Delta h_{i,j,s}$, and $s = 1, 2, \cdots, S$

The stepping pattern can also be defined as a spatial pattern $\delta_{i,j,s}^{(n)}$ that corresponds to the typical stepping pattern of a person on the first s step, respectively. The number of uses of $c_{n,s}$ could perhaps vary with floor position; thus $c_{n,s}$ could be made to represent the number or weight of occurrences of the "pattern n " on the s step;

Ultimately, if the material is the same or less different at each step, the following can be standardized: k for the wear factor, H for the hardness of the material, and E for the modulus of elasticity; if the material is different at each step (e.g., for a new step material added or repaired at a different time), the following can be noted: k' for the wear factor, H' for the hardness of the material, and E' for the modulus of elasticity to differentiate between the two.

Thus, the overall wear model of a multistage step can be written as:

$$\Delta h_{i,j,s} = \sum_{n=1}^{N} \left[c_{n,s} \left(\delta_{i,j,s}^{(n)} + \delta_{i,j,s}^{'(n)} \right) \right] + \varepsilon_{i,j,s} \tag{22}$$

The main parameters to be determined here include 1, the material coefficients of each step 2, the weights of each mode on each step $c_{n,s}$; 3, the distribution characteristics of the random error $\varepsilon_{i,j,s}$ (Gaussian or other can be assumed and defined according to the actual situation).

3.1.2. From wear and tear to usage currents - time extrapolation

When there are completed the measurements for $\{\Delta h_{i,j,s}\}$ and want to inversely infer "how many people pass this staircase per day" or "whether the number of users is relatively stable over time or concentrated in certain periods", the following inversions can be performed or fitted under the modeling architecture described above:

1. Start by assuming a segmented time model (L segments) or by guessing the initial footfall distribution λ_l and the pattern distribution $\{p_{s,l}\}$ from historical data;

2. Bring in the material factor and the repair factor $\{p_{s,l}\}$ to get the theoretical prediction $\Delta h_{i,j,s}$;

3. Least Squares or Maximum Likelihood Estimation (MLE) fit to the actual measurements, adjusting the parameters to minimize the difference between the observed wear and the model predictions;

4. Based on the optimal solution $\hat{\lambda}_l$, etc., determine the average number of users per year and whether there is a concentrated period with a high frequency of foot traffic. If $\hat{\lambda}_l$ is significantly larger than other periods, it can be assumed that the flow of people is particularly intensive during that period.

3.2. Analysis of Results

Based on the data, see the actual friction loss distribution for the five randomized steps by plotting the heat map. The results are shown in the Fig. 2:

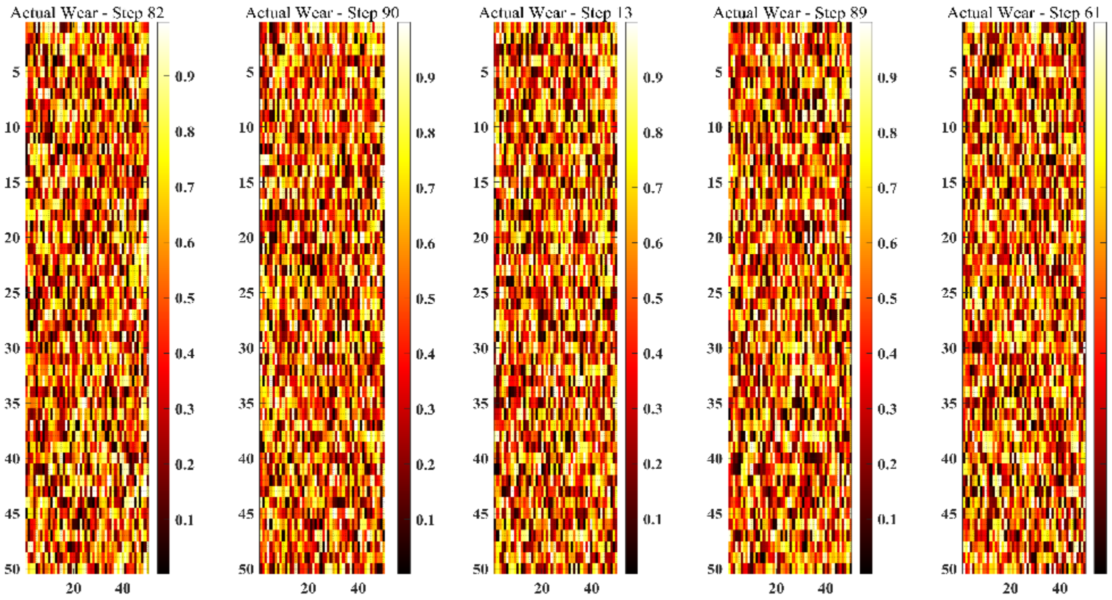

Fig 2. Distribution of real data for randomized 5-steps.

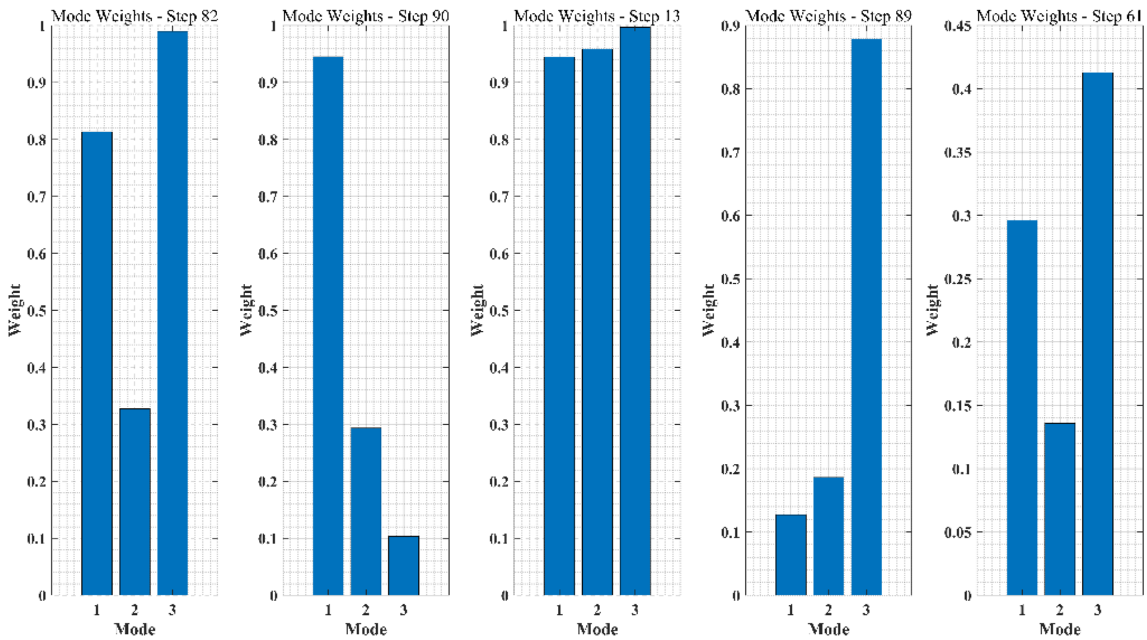

Fig 3. Distribution of weights for different modes under randomized 5 steps.

From the Fig. 3, it can be concluded that the fitted data and the actual data are close to each other, and both are characterized by the obvious difference between the middle and the edge, which indicates that the model we built can better simulate the actual situation. Analyzing the images of the fitted data and the actual data, we can learn that the model we built can well show the effects of different walking patterns on the steps.

4. Conclusion

In this paper, a multifactor coupled dynamic wear model incorporating Archard's wear theory is proposed, which combines the explicit difference-centered difference algorithm with the pattern matrix framework to realize the cross-scale quantitative analysis of staircase wear. First, the model builds a single-step wear calculation framework based on Archard's theory and Hooke's law, and analyzes the wear mechanism from a micromechanical perspective; second, the numerical difference algorithm and the adaptive time-step strategy are used to dynamically wear the staircase, and accurately depict the coupling between the time-varying load and the spatial wear gradient; and then the principle of linear superposition of pattern matrices is utilized to quantify the contributions of different modes to the wear, such as the center of the staircase with a high proportion of centralized modes. The average annual increment of wear depth in the step area with a high proportion of centralized modes is significantly higher than that in the edge area; finally, the model fitting accuracy is verified through the 3D modeling of the whole staircase and parameter backpropagation, which provides data support for staircase maintenance. Future research can further explore the applicability of the model in multi-material combination staircases and extreme environments and introduce machine learning to optimize the pattern matrix updating mechanism.

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