Dynamic Programming Modeling for Rural Cultivation Strategy

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Abstract. According to the actual situation of rural areas, making full use of limited cultivated land resources, adapting to local conditions and developing organic planting industry are of great practical significance to the sustainable development of rural economy. Selecting appropriate crops and the planting areas is conducive to facilitating field management and reducing the planting risks that may be caused by various uncertain factors. In order to optimize the planting strategy, improve agricultural production efficiency and quality, and increase farmers' income, this article made reasonable assumptions based on the existing data, established a single-objective linear programming model. We took profit maximization as the objective function, set the constraints according to the real-world limitations, conducted iterative calculations, and finally obtained the optimal planting strategy and the expected income. It has practical significance for the planting planning of collective agriculture and has reference value for solving linear programming problems in other fields.

Keywords: Dynamic Programming, Crop Planting, Optimal Strategy, Genetic Algorithm.

1. Introduction

The development of rural areas is the most critical issues in China today, and rural development is often inseparable from agricultural progress. To assist a rural village in the mountainous region of North China in optimizing its planting strategies, this paper establishes a dynamic programming model based on data from the National Bureau of Statistics of China and relevant literature. The aim is to formulate an optimal crop planting plan to enhance production efficiency.

Rural agricultural revitalization is an important pathway to achieving sustainable rural development and has received widespread attention from academia and policymakers in recent years. In studies related to the policies and practices of rural agricultural revitalization, Zhang Tao analyzed the implementation effects of the rural revitalization strategy and highlighted the key role of policies in agricultural development efficiency [1]. Zuhui Huang argued that local agricultural development requires both technological and policy support, such as the construction of ecological greenhouses [2]. Some researchers have used mathematical models to optimize the agricultural industry structure. Li Qiang applied a linear programming model to optimize the types of agricultural crops grown in the North China region [3]. Zhang Hua and Li Jing used a grey prediction model to forecast future crop yields under relevant planting strategies in the region [4][5]. Additionally, some studies have utilized multi-objective optimization models to analyze various factors such as agricultural yield and quality [6][7]. Overall, scientific planning of crop cultivation can improve land use efficiency and farmers' income while reducing land resource waste [8].

Previous studies have mostly approached the topic from a policy perspective, lacking quantitative development research methods and recommendations. Even when dynamic programming models were used, the research areas were often too large, with assumptions and data that were not sufficiently detailed. Our study focuses on the mountainous regions of North China, taking into account the unique local farmland types, such as terraced fields and smart greenhouses. By integrating relevant data and previous computational methods, we provide more targeted and detailed planting strategies to help local residents improve crop yield and quality, optimize the planting structure, and promote rural revitalization.

2. Model Formulation

2.1. Basic Information

This region is an agricultural area located in the Taihang Mountain area of North China. It is characterized by a cold winter, a hot summer, and distinct seasons. The arable land in this region can be categorized into five types: flat dryland, terraced fields, hillside land, irrigated land, and greenhouses. Among these, greenhouses can be further divided into conventional greenhouses and smart greenhouses, which provide a stable growth environment and enable seasonal crop cultivation. The region supports a total of 41 crop types across five categories: cereals, leguminous cereals, vegetables, leguminous vegetables, and edible fungi. Cereals are cultivated in a single season, while vegetables and edible fungi are grown seasonally. The majority of the crops cultivated in the current season are sold in the same season, with minimal storage.

2.2. Data Collection and Organization

Based on the information obtained from the Department of Agricultural Information in China, we have organized the following data:

All crops and plots of land have been assigned unique identification numbers. Additionally, we have collated the planting area, yield per mu (a unit of area), planting costs, and selling prices for each crop in different plots within the region for the year 2023.

Considering the losses of crops caused by various uncontrollable factors, we have conducted a literature review and determined an 8% loss rate for the final agricultural products.

2.3. Model Objective

We finally decided to employ a linear programming model for this study, with the objective of maximizing the profit from crop cultivation. The model will integrate various constraints, including crop growth patterns, market dynamics in agriculture, and climate change, to identify the optimal cropping strategies for the region over the period of 2024 to 2030.

2.4. Modeling process

2.4.1 Objective Function:

Planting Cost:

$$\boldsymbol{W_{i,t} = \sum_j x_{i,j,t} \cdot S_{i,j,t} \cdot b_{i,j}^t,\ i = 1\sim41\ ,\ j = 1\sim82\ ,\ t = 2023\sim2028} \tag{1}$$

where $x_{i,j,t}$ is a binary variable indicating whether crop i is planted on plot j in year t, with "1" for planting and "0" for not planting; $S_{i,j,t}$ is the area of crop i planted on plot j in year t; $b_{i,j}^t$ is the unit planting cost of crop i on plot j in year t.

Revenue from Normal Sales:

$$\boldsymbol{H_{i,t}^{(1)} = \sum_j min(P_{i,j,t}, e_{i,j}) \cdot c_{i,j}} \tag{2}$$

$$\boldsymbol{e_{i,\mathrm{j}} = P_{i,\mathrm{j},2023} \times 92\%, \forall i, \forall \mathrm{j}} \tag{3}$$

where $H_{i,t}^{(1)}$ is the revenue obtained from the normal sale of crop i in year t, $c_{i,j}$ is the unit selling price of crop i on plot j, and $e_{i,j}$ is the expected sales volume, which is 92% of the actual yield of crop i on plot j.

Expected Selling Price:

When $P_{i,j,t} < e_{i,j}$, the production is less than the expected sales volume, there is no excess part, so the additional income is 0:

$$\boldsymbol{H_{i,t}^{(2)} = 0} \tag{4}$$

When $P_{i,j,t} \geq e_{i,j}$, the production is greater than or equal to the expected sales volume, it is necessary to further discuss whether to "hold back" or "sell at a 50% discount" for the excess part:

$$\boldsymbol{H_{i,t}^{(2)}} = \begin{cases} \mathbf{0} \\ \sum_{j} (\boldsymbol{P_{i,j,t}} - \boldsymbol{e_{i,j}}) \cdot \mathbf{0.5}\boldsymbol{c_{i,j}} \end{cases}, \boldsymbol{P_{i,j,t}} \geq \boldsymbol{e_{i,j}} \tag{5}$$

Final Profit Expression:

$$\boldsymbol{max\ Q} = \sum_{t=2024}^{2030} \sum_{i} \left(\boldsymbol{H_{i,t}^{(1)}} - \boldsymbol{W_{i,t}} + \boldsymbol{H_{i,t}^{(2)}}\right) \tag{6}$$

2.4.2 Constraints:

(1) Crop Planting Area Restriction

The planting area of crops on each plot cannot exceed the area of the plot itself:

$$\sum_{i=1}^{41} \boldsymbol{x_{i,j,t}} \cdot \boldsymbol{S_{i,j,t}} \leq \boldsymbol{d_j}, \forall \boldsymbol{t} \in [\mathbf{2024}, \mathbf{2030}], \forall \boldsymbol{j} \in [\mathbf{1}, \mathbf{82}] \tag{7}$$

In addition, according to expert statistics, the planting area of each crop on a single plot (including greenhouses) should not be less than 30%-50% of the plot's area, otherwise it may affect the normal growth of the crop and reduce production efficiency:

$$\boldsymbol{S_{i,j,t}} \geq \boldsymbol{\alpha} \cdot \boldsymbol{d_j}, \forall \boldsymbol{i}, \forall \mathbf{j}, \forall \boldsymbol{t}, \boldsymbol{\alpha} \in [\mathbf{30}\%, \mathbf{50}\%] \tag{8}$$

(2) Crop Planting Condition Restrictions

- Flat dryland, terraced fields, and hillside land (j:1-26) can only be planted with one season of grain crops (excluding rice, i:1-15) each year.
- When choosing to plant rice (i:16), irrigated land can only be planted with a single season each year (j:27-34).
- The first season of conventional greenhouses (j:35-50) and the upper and lower seasons of smart greenhouses (j:51-54,79-82) can only be planted with vegetables excluding Chinese cabbage, white radish, and red radish (i:17-34).
- The second season of conventional greenhouses (j:63-78) can only be planted with edible fungi (i:38-41).
- Chinese cabbage, white radish, and red radish (i:35-37) are only planted in the second season of irrigated land (j:55-62) and cannot be mixed.

$$\begin{cases} \sum_{j=1}^{26} \sum_{i=16}^{41} \boldsymbol{x_{i,j,t}} = \mathbf{0} \\ \sum_{j=27}^{54} \left(\sum_{i=1}^{15} \boldsymbol{x_{i,j,t}} + \sum_{i=35}^{41} \boldsymbol{x_{i,j,t}}\right) = \mathbf{0} \\ \sum_{j=55}^{62} \left(\sum_{i=1}^{34} \boldsymbol{x_{i,j,t}} + \sum_{i=38}^{41} \boldsymbol{x_{i,j,t}}\right) = \mathbf{0} \\ \sum_{j=63}^{78} \sum_{i=1}^{37} \boldsymbol{x_{i,j,t}} = \mathbf{0} \\ \sum_{j=79}^{82} \left(\sum_{i=1}^{16} \boldsymbol{x_{i,j,t}} + \sum_{i=35}^{41} \boldsymbol{x_{i,j,t}}\right) = \mathbf{0} \\ \boldsymbol{x_{16,j,t}} + \boldsymbol{x_{16,j+28,t}} \leq \mathbf{1}, \boldsymbol{j} \in [\mathbf{27}, \mathbf{34}] \end{cases} \tag{9}$$

(3) Crop Rotation Restriction

To avoid reduced yields, the same crop cannot be planted on the same plot for two consecutive years:

$$\boldsymbol{x_{i,j,t}} + \boldsymbol{x_{i,j,t+1}} \leq \mathbf{1}, \forall \boldsymbol{i}, \forall \boldsymbol{j}, \forall \boldsymbol{t} \tag{10}$$

(4) Three-Year Legume Planting Restriction

To optimize soil quality and promote the growth of other crops, each plot (including greenhouses) should be planted with legume crops at least once within a three-year period:

$$\sum_{t=t_0}^{t_0+2} \sum_{i \in \mathbf{legumes}} \boldsymbol{x_{i,j,t}} \geq \mathbf{1}, \forall \boldsymbol{j}, \boldsymbol{i} \in [\mathbf{1}, \mathbf{5}] \cup [\mathbf{17}, \mathbf{19}], \boldsymbol{t_0} \in [\mathbf{2023}, \mathbf{2028}] \tag{11}$$

Here, "legumes" refer to the set of legume crop types, and the specific crop numbers are given as $i \in [1,5] \cup [17,19]$. The time period t_0 ranges from 2023 to 2028, ensuring that each plot has at least one season of legume crops within any three-year span.

3. Results

We constructed a single-objective linear programming model with the goal of maximizing profit as the objective function. Multiple constraints were set based on actual conditions, and iterative calculations were performed through programming. Using data from 2023 and assuming that the expected sales volumes, planting costs, yields per unit area, and selling prices of various crops remain stable compared to 2023, we provided optimal planting plans for various crops from 2024 to 2030 under two scenarios: choosing to let surplus produce go to waste or selling it at a 50% discount when supply exceeds demand. The following are some of the results of the planting scheme under the situation where the excess part is left unsold and wasted, as shown in Table.1-Table.3.

Table.1. The planting strategies for three crops on plots D1 - D9 in the second quarter of 2024

	D1	D2	D3	D4	D5	D6	D7	D8
Cabbage	0	0	8	0	0	0	22	0
Mooli	0	0	6	4	0	12	0	0
Carrot	0	0	0	2	10	0	0	0

Table.2. The planting strategies for three crops on plots D1 - D9 in the second quarter of 2025

	D1	D2	D3	D4	D5	D6	D7	D8
Cabbage	0	10	0	0	0	0	0	20
Mooli	3	0	0	0	0	0	22	0
Carrot	12	0	0	0	0	0	0	0

Table.3. The planting strategies for three crops on plots D1 - D9 in the second quarter of 2026

	D1	D2	D3	D4	D5	D6	D7	D8
Cabbage	0	0	14	0	4	12	0	0
Mooli	0	0	0	6	0	0	12	0
Carrot	0	0	0	0	6	0	10	0

The following are some of the results of the planting scheme under the condition that the excess part is sold at a 50% discount, as shown in Table.4-Table.6.

Table.4. The planting strategies for three crops on plots D1 - D9 in the second quarter of 2024

	D1	D2	D3	D4	D5	D6	D7	D8
Cabbage	0	0	0	2	0	0	22	0
Mooli	0	0	0	0.2	0	0	0	20
Carrot	0	0	0	3.8	10	0	0	0

Table.5. The planting strategies for three crops on plots D1 - D9 in the second quarter of 2025

	D1	D2	D3	D4	D5	D6	D7	D8
Cabbage	0	4.4	0	0	0	12	0	0
Mooli	6.2	0	14	0	0	0	0	0
Carrot	8.8	5.6	0	0	0	0	0	0

Table.6. The planting strategies for three crops on plots D1 - D9 in the second quarter of 2026

	D1	D2	D3	D4	D5	D6	D7	D8
Cabbage	0	0	0	0	2	0	22	0
Mooli	0	0	0	0	0	0.2	0	20
Carrot	0	0	0	6	8	0.4	0	0

According to the planting strategy, the expected profit Q is estimated under the following two scenarios:

When the expected production is less than or equal to the expected sales volume.:

When $P_{i,j,t} \le e_{i,j}$: $\max Q = 4.3225e+07$

When the expected production is more than or equal to the expected sales volume.:

When $P_{i,j,t} > e_{i,j}$: $\max Q = 4.7172e+07$

This analysis demonstrates that when projected yield exceeds expected sales volume, greater profits can be achieved. This occurs because even when the excess produce is sold at a 50% discount, crops with high profit ratios (selling price/cost) remain profitable. Therefore, it is advisable to cultivate more high-profit-ratio crops within appropriate planting ranges. Conversely, scenarios where actual yield falls short of or equals projected sales correspond to conditions where excess production remains unsold and wasted. In such cases, the portion exceeding expected sales volume will incur additional costs and diminish overall profitability.

4. Conclusions

Agriculture is the foundation of a country's economy and an important component of the national economy. Rural areas, as significant regions for agricultural production, have made substantial contributions to the social and economic development of rural communities. Supporting agricultural development not only helps maintain national food security but also promotes the prosperity and development of rural communities. Our model can assist farmers in effectively utilizing limited arable land resources in actual agricultural production based on the model's results, and in adopting location-specific strategies to promote sustainable economic development. Moreover, by planting crops that are more suitable for the local environment and improving planting strategies, farmers can more easily manage their farmland, reduce planting risks, and enhance agricultural productivity and quality, thereby increasing their income.

The model adopts planting area as the decision variable and introduces binary auxiliary variables to construct constraint relationships, which moderately increases computational complexity but significantly enhances the convenience of constraint expression. In the solution process, the pulp library is systematically integrated to address linear programming problems, while the Monte Carlo method and genetic algorithms are synergistically employed for iterative optimization, achieving a balance between computational efficiency and solution accuracy. Innovatively, adjacency matrices are utilized to analyze management constraints for adjacent plots, complemented by visualized spatial layouts to strengthen the model's practicality. However, limitations persist: First, parameters such as yield per unit area were simplified as static assumptions in the initial analysis, whereas real-world scenarios would require modeling them as white noise sequences. Second, the rigid threshold setting of a minimum 30% planting area ratio for each plot lacks flexibility, and incorporating reasonable fluctuation ranges would better align with practical conditions. Future enhancements could focus on sensitivity analyses of genetic algorithm parameters, comparative multi-scenario evaluations, and dynamic visualizations to improve model adaptability. Although these extensions remain unexplored in this study, they provide critical technical insights for subsequent research endeavors.

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