Research on Modeling the Super-illuminated Area of Apple Tree Canopies Based on Ray Tracing and Multi-swarm Particle Swarm Optimization

Shuai Xia *

International College, Chongqing University of Posts and Telecommunications, Chongqing, China, 400065

* Corresponding Author Email: sx3351658664@outlook.com

Abstract. In the gully region of the Loess Plateau, the efficient utilization of solar energy within apple tree canopies is crucial for optimizing fruit yield and quality. To address the challenge of optimizing the light use efficiency of apple tree canopies, this paper proposes a cross-scale modeling framework that integrates ray tracing and improved particle swarm optimization. By establishing a dual coordinate system of ground-light cone and a dynamic light cone equation, combined with Monte Carlo random sampling and ray equation solving, efficient determination of points receiving more than eight hours of light is achieved. With the objective of maximizing the canopy over-illuminated area (COA) under surface area constraints, as shown in Equation (17),a multi-swarm cooperative PSO algorithm is used for morphological parameter optimization. The results indicate that the COA of apple tree canopies is influenced by factors such as season, terrain, and canopy shape. The COA is larger in summer and almost zero in winter, and the slope of the terrain also affects the duration of sunlight exposure. Conical canopies have a slightly larger COA than spherical canopies under the same surface area. This paper provides theoretical basis and technical support for the morphological optimization of apple tree canopies.

Keywords: Ray tracing, Multi-swarm particle swarm optimization, Monte Carlo method.

1. Introduction

In recent years, advancements in agricultural technology have driven the refined development of fruit tree cultivation management. Among these, the optimization of apple tree canopy morphology and pruning strategies have become central topics for enhancing orchard efficiency, as they directly impact photosynthetic efficiency, nutrient allocation, and yield. Under dense planting conditions, canopy structure directly influences photosynthetic efficiency and light resource allocation during the fruit enlargement period.

In the research on tree canopy morphology and pruning strategies, the Chaploutskyi A team found through a comparison of Fuji and Honeycrisp apple varieties that although the French spindle-shaped canopy reduced canopy width, it increased specific yield by 17% through light energy redistribution, revealing the light efficiency gain mechanism of compact canopy types. However, the variability among varieties highlights the need for customized pruning strategies[1].The Jonckheere team systematically reviewed direct and indirect measurement methods for the Leaf Area Index (LAI) and proposed that hemispherical photography technology based on high dynamic range digital cameras could reduce indirect measurement errors caused by leaf and branch clustering effects. Nevertheless, the lack of standardized protocols still restricts its field application [2]. The Westling team innovatively combined LiDAR point clouds with solar geometry models to construct an avocado canopy light tracing model, verifying the feasibility of unstructured point clouds in light energy distribution simulation. However, the model did not integrate the dynamic coupling effects of terrain undulations and seasonal solar trajectory shifts [3]. The Qiao Peng team quantified the light interception efficiency of loquat canopies through 3D modeling and radiosity algorithms, discovering that the spreading-type canopy had the highest photosynthetically active radiation interception. Nonetheless, the study was limited to static canopies and did not introduce parameter optimization algorithms [4].

Although the aforementioned achievements have deepened empirical understanding of pruning strategies, existing research still suffers from three major limitations: (1) Light transmission modeling largely relies on empirical parameters and lacks physical mechanism analysis based on ray tracing; (2) Neither LAI measurement nor canopy light efficiency analysis has been coupled with the seasonal variations in terrain slope and solar trajectory, leading to insufficient applicability in complex topographic areas; (3) Morphological optimization depends on expert experience and lacks an efficient search mechanism for high-dimensional parameter spaces, making it difficult to balance global optimal solutions under multi-objective constraints.

This paper innovatively proposes a cross-scale collaborative optimization framework, with core breakthroughs including:

(1) Dynamic light cone modeling: Establishing a dual coordinate system mapping model of the earth and light cone, generating a canopy surface point set through Monte Carlo random sampling. Combining light ray equations with algorithms for finding intersections of enclosed surfaces to rapidly determine over 8 hours of light-receiving points (COP).

(2) Multimodal PSO algorithm: Designing a multi-swarm collaborative optimizer, introducing Lévy flight strategies and dynamic subgroup reorganization mechanisms to enhance convergence speed in high-dimensional parameter space searches.

(3) Topography-season coupling analysis: Establishing dynamic light cone equations to simulate seasonal shifts in solar incidence trajectories and topographic shadow effects.

By integrating physical models of light transmission with intelligent optimization algorithms, this study breaks through the traditional empirical pruning paradigm and provides a decision-making support system that combines physical mechanisms with algorithm-driven approaches for apple canopy design in complex terrain areas. This methodology can also be extended to research on light efficiency optimization for other canopy crops.

2. Method

To facilitate the paper of the COA of apple tree canopies for more than eight hours in this paper, we take apple trees in Luochuan County, Yan'an City, China as an example, as shown in Figure 1. Luochuan County in Yan'an City is typical of the loess plateau gully landform, with the terrain sloping from northeast to southwest and an average altitude of 1,106 meters. The climate is a warm temperate humid continental monsoon climate, with an annual average temperature of 10.3°C. Luochuan is the core area of the apple optimal production zone on the Weibei Loess Plateau. It features large diurnal temperature differences, deep and loose soil rich in various trace elements such as potassium, calcium, magnesium, zinc, and selenium. With unique natural resource conditions for apple production, it is one of the production areas globally that fully meet all seven meteorological indicators for the most suitable apple production zone. It is unanimously recognized by domestic and foreign fruit tree experts as the world's best apple optimal production area.

Figure 1. Apple trees in Luochuan County, Yan'an City

2.1. Ray tracing method

2.1.1. Establishment of ground coordinate system

In this paper, a ground coordinate system is established with the tree center as the origin, the east direction as the x -axis, the north direction as the y -axis, and the direction perpendicular to the horizontal plane as the z-axis. The solar elevation angle is denoted as α_s, and the azimuth angle as γ_s, as shown in Figure 2.

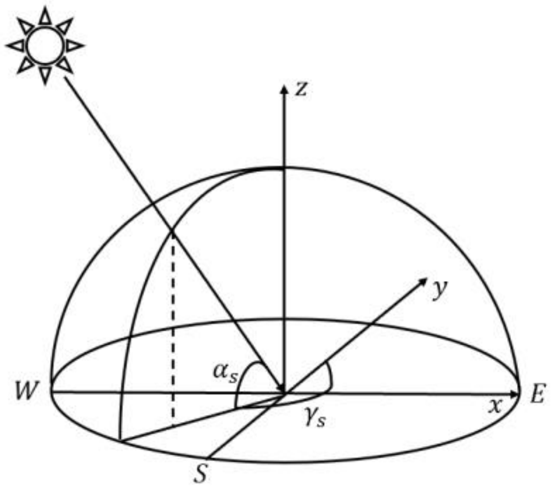

Figure 2. Schematic diagram of ground coordinate system

2.1.2. Direction vector of principal rays of sunlight

With reference to the geodetic coordinate system, sunlight is not parallel rays but a beam of conical light with a certain conical angle. Therefore, it is necessary to first describe a main ray of sunlight using a vector. To describe the direction vector of the incident main ray of sunlight, we first need to know the solar elevation angle and the solar azimuth angle. The solar elevation angle is the angle between the line connecting the sun to the origin of the coordinate system and the horizontal ground. Based on the definition of the solar elevation angle, the formula for the solar elevation angle α_s can be obtained [5]:

$$\begin{cases} \omega = \dfrac{\pi}{12}(ST-12) \\ \sin\delta = \sin\dfrac{2\pi D}{365}\sin(\dfrac{2\pi}{360}23.45) \\ \cos\delta = \sqrt{1-\sin^2\delta} \\ \sin\alpha_s = \cos\delta\cos\varphi\cos\omega + \sin\delta\sin\varphi \\ \alpha_s = \arcsin(\sin\alpha_s) \end{cases} \tag{1}$$

where ω is the solar hour angle, ST is the local time, δ is the solar declination angle [6], D is the day count starting from the vernal equinox as day 0, and φ is the local latitude.

The solar azimuth angle refers to the angle measured clockwise from the starting direction of due north to the terminating direction of the sun's projection point. Based on the definition of the solar azimuth angle, the formula for the solar azimuth angle is [5]:

$$\begin{cases} \cos\gamma_s = \dfrac{\sin\delta - \sin\alpha_s\sin\varphi}{\cos\alpha_s\cos\varphi} \\ \sin\gamma_s = \begin{cases} \sqrt{1-\cos^2\gamma_s}, ST < 12 \\ -\sqrt{1-\cos^2\gamma_s}, ST \geq 12 \end{cases} \end{cases} \tag{2}$$

After obtaining the solar elevation angle and the solar azimuth angle, the direction vector L_s of the incident main ray of sunlight is:

$$L_s = (-\cos\alpha_s \sin\gamma_s, -\cos\alpha_s \cos\gamma_s, -\sin\alpha_s) \tag{3}$$

2.1.3. Establishment of light cone coordinate system

Since sunlight is not parallel rays but a beam of conical light with a certain conical angle, a light cone coordinate system is established with the incident main ray (or reflected main ray) as the central ray of the conical beam, as shown in Figure 3.

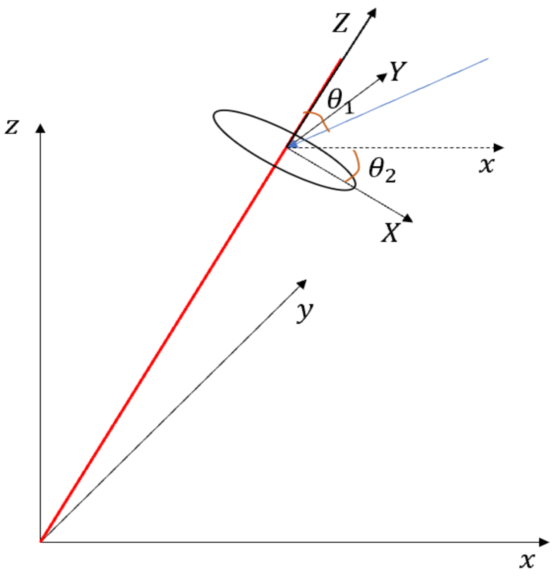

Figure 3. Schematic diagram of light cone coordinate system

In the light cone coordinate system [5], assuming an arbitrary ray is $L = (m, n, p)$, its unit vector in the z-axis direction is $s = (0,0,1)$, its unit vector in the X-axis direction is $s_1 = (1,0,0)$, and its projection vector on the plane of the light cone coordinate system is $L_1 = (m, n, 0)$. Thus, we obtain its elevation angle θ_1 and azimuth angle θ_2 in the light cone coordinate system.

$$\theta_1 = \frac{\pi}{2} - \arccos\theta \frac{L \cdot s}{|L| \cdot |s|}, \theta_2 = \begin{cases} \arccos \dfrac{L_1 \cdot s_1}{|L_1| \cdot |s_1|}, n <= 0, \\ -\arccos \dfrac{L_1 \cdot s_1}{|L_1| \cdot |s_1|}, n > 0. \end{cases} \tag{4}$$

In summary, in the light cone coordinate system, the unit direction vector of any ray on the side of the central ray of the cone is described as:

$$f = (\sin\theta_1 \cos\theta_2, \sin\theta_1 \sin\theta_2, \cos\theta_1) \tag{5}$$

where $\theta_1 \in [0, 0.00465], \theta_2 \in [0, 2\pi)$. When $\theta_1 = 0$, f is the direction vector of the incident main ray in the light cone coordinate system.

2.1.4. Coordinate system transformation

To calculate the intersection point of any ray on the side of the central ray of the cone hitting the tree canopy, we need to perform a coordinate system transformation to unify the coordinate systems and facilitate the simultaneous solution of equations. Given that the azimuth angle of the main ray (i.e., the parallel sunlight) in the geodetic coordinate system is γ_s and the elevation angle is α_s, the coordinate transformation matrix from the light cone coordinate system to the geodetic coordinate system is [5].

$$T(\gamma_s, \alpha_s) = B \cdot A \tag{6}$$

$$\text{where } A=\begin{bmatrix}\cos(\frac{\pi}{2}-\alpha_s) & 0 & \sin(\frac{\pi}{2}-\alpha_s)\\ 0 & 1 & 0\\ -\sin(\frac{\pi}{2}-\alpha_s) & 0 & \cos(\frac{\pi}{2}-\alpha_s)\end{bmatrix}, B=\begin{bmatrix}\cos\gamma_s & -\sin\gamma_s & 0\\ -\sin\gamma_s & \cos\gamma_s & 0\\ 0 & 0 & 1\end{bmatrix}$$

The direction vector of any ray on the side of the central ray of the cone in the geodetic coordinate system is:

$$f_1 = f \cdot T(\gamma_s, \alpha_s) = (m_1, n_1, p_1) \tag{7}$$

2.1.5. Ray tracing

Given the direction vector $f = (\sin\theta_1 \cos\theta_2, \sin\theta_1 \sin\theta_2, \cos\theta_1)$ of the solar conical light in the light cone coordinate system, the direction vector $f_1 = f \cdot T(\beta_1, \beta_2) = (m_1, n_1, p_1)$ of the conical light (i.e., the incident light beam) in the geodetic coordinate system can be obtained through the transformation matrix. Knowing a point (x_0, y_0, z_0) on the surface of the apple tree canopy in the geodetic coordinate system, combined with the direction vector of the incident light beam, the light ray equation can be derived [7].

$$\frac{x-x_0}{m_1} = \frac{y-y_0}{n_1} = \frac{z-z_0}{p_1} = t \tag{8}$$

2.2. Establishment of COA model for different crown shapes

2.2.1. Establishment of daylight time

Since the tree canopy is only illuminated by the sun during the daytime, it is necessary to determine the daylight hours of a day. Daytime refers to the period from when the sun rises above the horizon to when it sets below the horizon. According to the definition, the sunrise time is when the solar elevation angle is 0° for the first time, and the sunset time is when the solar elevation angle is 0° for the last time.

$$\begin{cases} sunrise = t_1(\alpha_s == 0) \\ sunset = t_2(\alpha_s == 0) \end{cases} \tag{9}$$

where $sunrise$、$sunset$ represent the sunrise and sunset times, respectively.

2.2.2. Establishment of research time period model

Since Luochuan County in Yan'an City is typical loess plateau gully landform with the terrain sloping from northeast to southwest, sunlight cannot illuminate the apple tree canopy at sunrise during the daytime. The parallel sunlight can only reach the apple tree canopy when the solar elevation angle is greater than the inclination angle of the terrain. Therefore, we need to separately identify the study period within the daytime. (Since the terrain slopes from northeast to southwest, the sunset time is the last moment when the sunlight illuminates the apple trees.) The formula for the study period is as follows:

$$\begin{cases} t_1 = when(\alpha_s \ge \zeta) \\ t_2 = sunset \end{cases} \tag{10}$$

where t_1 is the moment when the sunlight first illuminates the apple tree, t_2 is the moment when the sunlight last illuminates the apple tree, and ζ is the inclination angle of the terrain. The sunlight blocked by the terrain is shown in Figure 4.

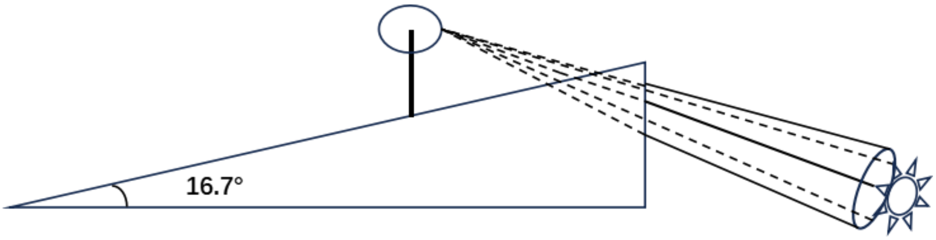

Figure 4. Schematic diagram of terrain blocking sunlight

2.2.3. COA model of different crown shapes

A point that is continuously illuminated for 8 hours can be called a COP n_1, and the area of the apple tree canopy that is continuously illuminated by sunlight for 8 hours is called the COA. The Monte Carlo method [8] is used to generate a total of N surface points. The steps to establish a COA model for tree canopies of different shapes are as follows:

(1) Given the surface equations of tree canopies of different shapes, use the Monte Carlo method to generate points on the surface and the direction vectors of three light rays in the light cone coordinate system at time t.

(2)Determination of COP n_1

For a closed shape, if a point on the surface is illuminated by sunlight at time t and is also illuminated at time t+8 during the study period, then this point will be continuously illuminated by sunlight for 8 hours. Similarly, if a point (x_0, y_0, z_0) on the surface is illuminated by sunlight at time t but is shaded at time t+8, then this point will not be continuously illuminated by sunlight for 8 hours and will not be a COP.

Combining the ray tracing method [7], the generated points that are shaded are shown in Figure 5:

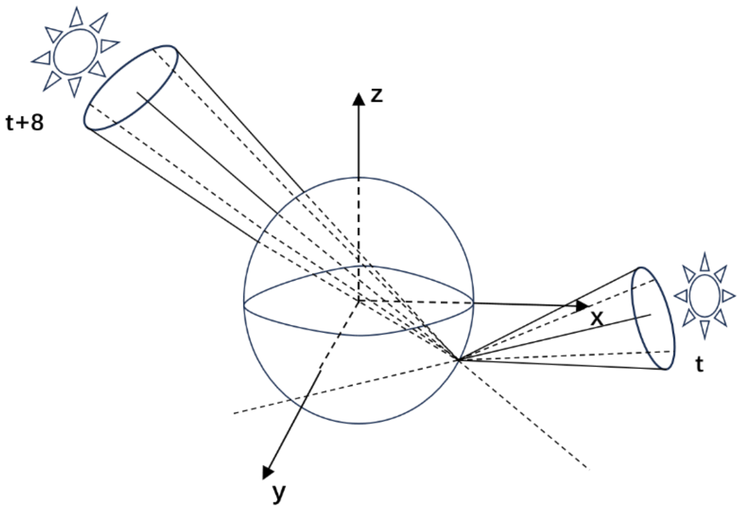

Figure 5. Schematic diagram of occluded COP

Determine the light ray vector $f = (m_1, n_1, z_1)$ at time t, and derive the light ray equation at time t. Then, solve it simultaneously with the equation of the closed shape.

$$\begin{cases} \dfrac{x - x_0}{m_1} = \dfrac{y - y_0}{n_1} = \dfrac{z - z_0}{p_1} \\ z = f(x, y) \end{cases} \tag{11}$$

We can obtain the intersection points z_n, where $n \in (1, 2, 3...)$. If all intersection points are less than or equal to the generated points on the surface, i.e., $z_n \le z_0$, then this generated point is illuminated at time t.

Similarly, let the light ray vector of the conical light at time t+8 be $f=(m_2,n_2,z_2)$, derive the light ray equation, and solve it simultaneously with the closed shape:

$$\begin{cases}\dfrac{x-x_0}{m_2}=\dfrac{y-y_0}{n_2}=\dfrac{z-z_0}{p_2}\\ z=f(x,y)\end{cases}\tag{12}$$

We can obtain the intersection points z_m, where $m\in(1,2,3...)$. If all intersection points are less than or equal to the generated points on the surface, i.e., $z_m\le z_0$, then this generated point is illuminated by sunlight at time t+8. If this point on the surface is illuminated by sunlight at both time t and time t+8, i.e.,

$$(z_n\le z_0)\cap(z_m\le z_0)\tag{13}$$

Then this point is COP.

(3)Calculate the surface area of different surfaces

If the surface equation is known, the surface area can be calculated [9]:

$$\begin{cases}z=f(x,y)\\ S=\iint\sqrt{1+(\dfrac{\partial z}{\partial x})^2+(\dfrac{\partial y}{\partial x})^2}\,dxdy\end{cases}\tag{14}$$

(4) Solution of COA for different crown shapes

Iterate through all generated points, count the number of COPs, and then determine the COA S_1 for tree canopies of different shapes.

$$S_1=\frac{n_1}{N}S\tag{15}$$

where S is the surface area of tree canopies of different shapes, n_1 is the number of COPs, and N is the total number of generated points using the Monte Carlo method.

Often, the COA of an apple tree canopy in a single day is random, so this paper focuses on the annual average COA of tree canopies of different shapes, which is represented as:

$$S_1=\frac{\sum_{t=1}^{365}\frac{n_1}{N}S}{365}\tag{16}$$

2.2.4. COA model of apple tree crown based on multi population particle swarm optimization

The paper of apple tree canopy COA is often conducted for the purpose of canopy pruning. Through reasonable pruning, not only can the appearance of the apple tree be changed, but it can also regulate its growth and development, enhancing ecological function and ornamental value. Furthermore, even within the same shape and canopy surface area, there can be different canopy postures. Therefore, to investigate the maximum COA of a canopy within a given shape, this paper employs a multi-swarm particle swarm optimization algorithm to optimize the apple tree canopy COA model [10].

The objective function is to maximize the apple tree canopy COA for the same shape and canopy surface area, with the decision variable being the parameter ∂ that determines the surface area of the canopy of this shape. Thus, the objective function is:

$$\max F=\frac{\sum_{t=1}^{365}\frac{n_1(\partial)}{N}S(\partial)}{365}\tag{17}$$

Since this paper adopts the method of controlling variables to constrain the surface area of tree crowns of the same shape, the constraint condition is defined as follows.

$$S(\partial_1)=S(\partial_2)\tag{18}$$

In the optimization model established in this paper, the objective function is a nonlinear function, which is solved using a multi-swarm particle swarm optimization algorithm. This paper adopts an intelligent algorithm strategy that divides the entire population into multiple swarms for parallel processing. The key to its optimization lies in establishing an information exchange mechanism among sub-swarms and utilizing strategies such as Lévy flight and hybrid particles to enhance execution capability. Additionally, a dynamic random reorganization method, rather than a fixed partition method, is chosen to improve information exchange capability.

3. Results

For the convenience of result analysis, this paper selects apple trees with apple-shaped canopies and conical canopies to solve the COA models for different shapes. And for the ease of result comparison, this paper stipulates that both the apple trees with spherical canopies and those with conical canopies have a surface area of 36π square meters.

3.1. Daytime time in Luochuan County, Yan'an City

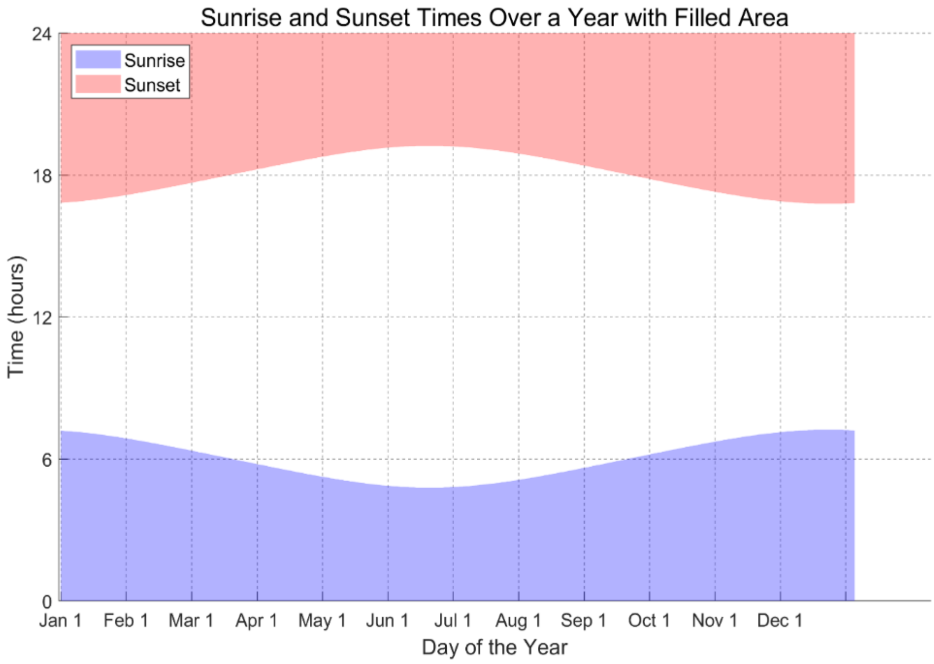

Figure 6. Daily sunrise and sunset periods

Through solving the model, as shown in Figure 6, the white portion represents daytime hours, indicating that the total duration of daylight first increases and then decreases throughout the year, with longer daylight hours in summer and shorter ones in winter. The purple curve represents sunrise time, and the pink curve represents sunset time, illustrating that as we get closer to summer, sunrise occurs earlier and sunset later, which generally aligns with real-world patterns.

3.2. Study period of Luochuan County, Yan'an City

Taking the terrain inclination angle of 16.7 ° as an example, the research time period of Luochuan County, Yan'an City is shown in Figure 7.

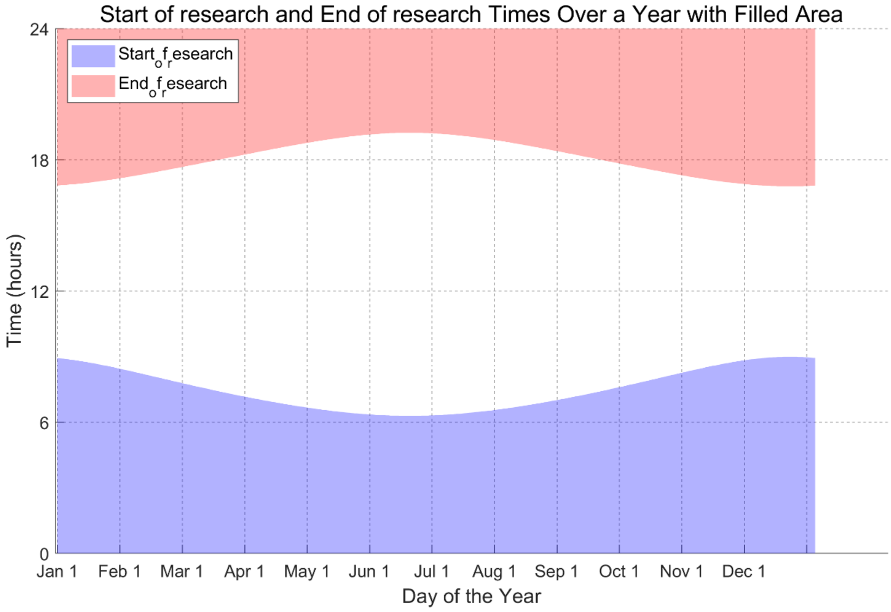

Figure 7. Study period of Luochuan County, Yan'an City

Through solving the problem, it is found that due to the presence of topography, the study period is generally shorter than the daytime period, but the end of the study period still coincides with sunset time, which is determined by the topography sloping from northeast to southwest.

3.3. Spherical crown apple tree COA

3.3.1. Crown COA of spherical apple trees with a radius of 3M

By solving the COA for a spherical apple tree canopy with a radius of 3m, the annual average COA for the spherical apple tree canopy is found to be 54.254 m^2 , and the daily COA for the spherical apple tree canopy is shown in Figure 8.

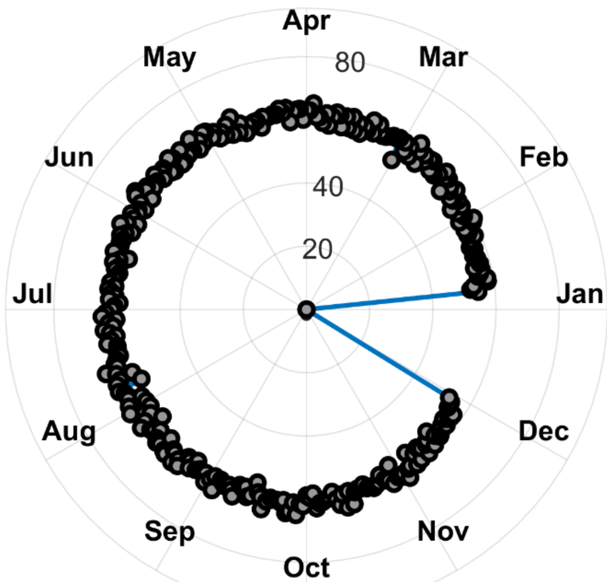

Figure 8. Daily COA of spherical crown

From Figure 8, it can be seen that for a spherical canopy with a radius of 3 meters, the daily COA remains basically around 60 square meters, accounting for about half of the surface area of the apple tree canopy. This indicates that for most of the year, a relatively stable portion of the apple tree canopy receives continuous sunlight for more than eight hours each day. This lighting condition may affect physiological processes such as temperature distribution, photosynthesis, and transpiration within the canopy.

Particularly noteworthy is that the COA is zero from December to January. This is because during this period, the duration of the study period is usually only about eight hours, or even less, so there is no COA. This phenomenon suggests that under extremely short daylight conditions in winter, no part of the apple tree canopy can receive continuous sunlight for more than eight hours, which may have an impact on the tree's dormancy and metabolic processes.

3.3.2. Crown COR of spherical apple trees with different radii

The Continuously Over-illuminated Ratio (COR) is defined as the ratio of COA to the surface area of the apple tree canopy. By calculating the COR for spherical apple tree canopies of different radii, the annual average COA for spherical canopies of various radii is obtained as shown in Figure 9.

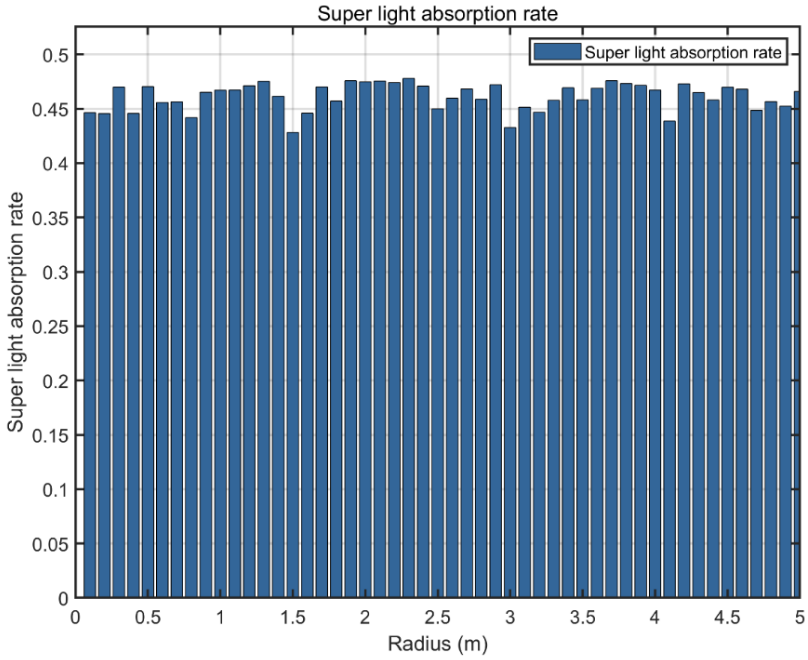

Figure 9. Annual average COR of spherical crown with different radii

From the Figure 9, it can be seen that as the radius increases, the COR of the spherical canopy exhibits certain fluctuations overall, but the overall variation is not significant. For most canopy radii, the annual average COR is concentrated between 0.3 and 0.5. This indicates that the annual average COR is relatively similar for both small and slightly larger spherical canopies. This suggests that the canopy radius has a relatively small impact on the annual average COA of spherical canopies. This means that when managing canopies, there is no need to excessively pursue the size of spherical canopies; instead, reasonable pruning and management should be carried out by comprehensively considering factors such as lighting, ventilation, and pest and disease control.

3.4. Conic apple tree crown COA

3.4.1. Solution of COA of conic apple tree crown under 36π square meters surface area

By applying a multi-swarm particle swarm optimization algorithm to solve for the conical apple tree canopy, we found that the optimal radius for the conical canopy with a surface area of 36π square meters is $0.16\,m$, and the COA is $52.17m^2$. In comparison, the COA for a spherical canopy with the same surface area is $54.254\,m^2$. Additionally, the COA for conical apple tree canopies with different radii under the surface area of $36\pi m^2$ square meters is shown in Figure 10.

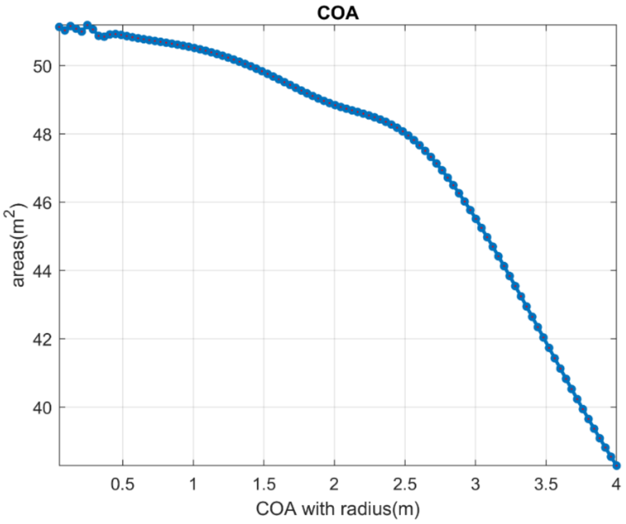

Figure 10. COA of conic apple tree crown with different radii under $36\pi m^2$ surface area

As shown in Figure 10, as the canopy radius decreases, the COA exhibits a certain growth trend, but it does not grow indefinitely. When the canopy radius reaches a certain specific value, the growth rate of COA begins to slow down or even decline.

When the canopy radius is 0.16, the COA reaches its maximum value. This means that at this radius, the conical apple tree canopy has the largest area on average each day throughout the year that is continuously illuminated by sunlight for more than eight hours. In practical operation, we can adjust the canopy shape and size to make the COA approach or reach this optimal value, thereby improving the growth efficiency and fruit quality of apple trees.

Furthermore, under the same surface area conditions, the COA of a spherical canopy is slightly smaller than that of a conical canopy. However, in practical applications, a spherical canopy may have better structural stability and air circulation, which helps to reduce pests and diseases and improve fruit quality. Therefore, when choosing the canopy shape, we need to weigh the pros and cons and make reasonable decisions based on the specific conditions of the orchard and planting goals.

3.4.2. Cone shaped apple tree crown COR under different surface areas

By solving for the over-illumination rate of conical apple tree canopies with different surface areas and proportionally increasing conical canopy radii, we can obtain Figure 11.

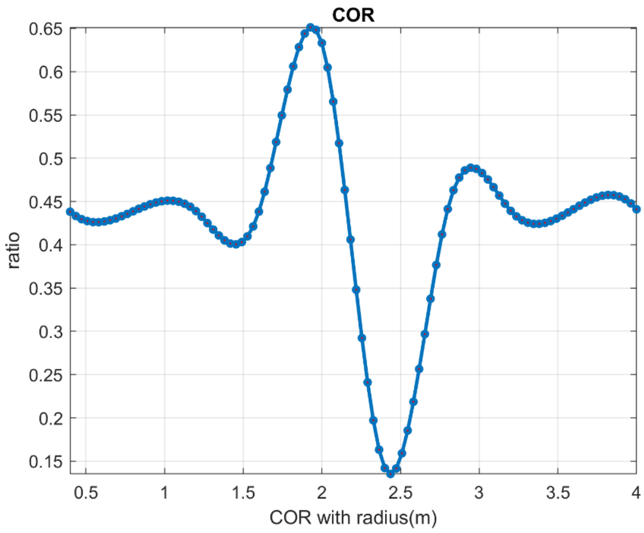

Figure 11. Solution of cone-shaped apple tree crown COR with different surface areas

From the figure, it can be observed that the COR values fluctuate with increasing radius. Specifically, the COR values reach a peak and a trough at radii of approximately 1.75 meters and 2.75 meters, respectively. The peak COR value is close to 0.65, while the trough value is below 0.2.

This implies that there are significant differences in the ratio of COA to surface area of the canopy at these two specific radii.

The fluctuation pattern of COR values suggests that lighting conditions need to be fully considered when designing the canopy. For example, in the stage when the canopy radius is small, COA can be increased by increasing the width and height of the canopy; whereas in the stage when the canopy radius is large, attention should be paid to the compactness and density of the canopy to avoid excessive shading of sunlight.

4. Conclusions

This paper established a dynamic quantitative model for the COA of apple tree canopies by integrating ray tracing with a multi-swarm particle swarm optimization algorithm, revealing the intrinsic correlation mechanism between canopy morphology and light energy utilization efficiency. Taking apple trees in Luochuan County, Yan'an City, China as an example, this paper successfully established both a ground coordinate system and a light cone coordinate system, and used the Monte Carlo method to generate canopy surface points. Combined with ray tracing technology, the COA of canopies of different shapes throughout the year was accurately calculated. The research results showed that the COA of apple tree canopies is significantly influenced by multiple factors such as season, terrain, and canopy morphology. Especially in summer, due to the long sunshine duration and high solar elevation angle, the COA of the canopy reaches its maximum value; conversely, it is almost zero in winter. By setting a clear objective function and optimizing the key parameters that determine canopy morphology, this paper achieved results where the COA of conical canopies is slightly better than that of spherical canopies under the same surface area conditions.

In summary, this paper provides a novel perspective and method for modeling and optimizing the COA of apple tree canopies, not only deeply revealing the variation patterns and influencing factors of apple tree canopy COA but also providing strong technical support for the refinement and scientification of fruit tree cultivation management.

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