Analysis and Optimization of Bench Dragon Model: A Joint Application of Traversal optimization method and Differential Method

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Abstract. Bench dragon is a series of benches that go end to end and move along an equidistant spiral. Through mathematical modeling, the movement process of the bench dragon and the shortest turning distance under specific circumstances are discussed. Firstly, with the help of Runge-Kutta 4th Order Method and the Microelement Method, the model of the relationship between the position of the dragon head and the time, and the model of the relationship between the position of each bench and the time were constructed. Then, to determine the minimum pitch so that the benches would not collide with each other, A spiral model with variable pitch is established, as well as the relationship between the pitch and the position of the front handle of the dragon head at the end time. Then, a small enough step size is taken to traverse within a certain range to obtain the minimum pitch that satisfies the condition. Lastly, to determine the shortest turning path and the movement of the bench dragon when coiling out, a possible turning path was put forward. The shortest possible turning path and the movement of each bench was thus calculated and solved by using Microelement Method and Backward Differential Method.

Keywords: Bench Dragon, Runge-Kutta 4th Order Method, Traversal Algorithms, Backward Differential Method, Limits and Microelements.

1. Introduction

The bench dragon is a traditional folk activity with a long history. During the Lantern Festival, hundreds of benches are drilled and connected to each other with handles to form a spectacular bench dragon [1]. In Fujian, welcoming the gods and praying for blessings is the importance of the bench dragon. During this event, people express their devotion and prayers to the gods by dancing this giant bench dragon. This custom shows their yearning and pursuit of a better life [2]. In the case that the dragon dancer can dance smoothly, the smaller the area occupied by the bench dragon coil and the faster the dragon travels, the more ornamental the bench dragon is. In the process of dancing the bench dragon, people cooperate with each other and cooperate tacitly, which not only exercises the body, but also cultivates team spirit. At the same time, this activity also passed on the knowledge of traditional culture and folklore to the younger generation, so that they can better understand and pass on their roots.

Suppose that a bench-dragon consists of 223 benches, where the first section is the dragon head, the following 221 sections are the dragon body, and the last one section is the tail. The length of the dragon head is 341 cm, the length of the dragon body and tail is 220 cm, and the width of all benches is 30 cm. There are two holes on each bench, the aperture (diameter of the hole) is 5.5 cm, and the center of the hole is 27.5 cm from the nearest plate head, as is shown in Figure 1 and 2. Two adjacent benches are connected by a handle.

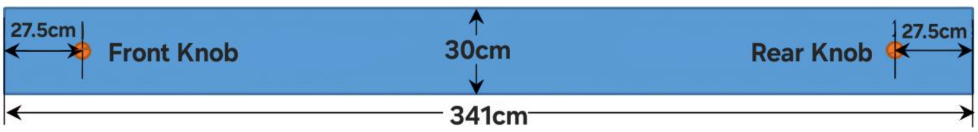

Figure 1. Top view of the dragon head

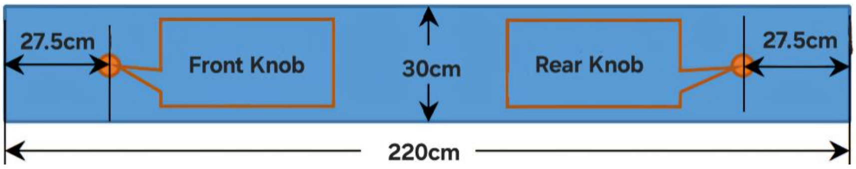

Figure 2. Top view of the dragon body and tail

2. The basic methods used in solving the bench dragon problem

2.1. Runge-Kutta 4th Order Method

Runge-Kutta methods are an important class of implicit or explicit iterative methods for solving nonlinear ordinary differential equations. These techniques were invented by mathematicians Carl Runge and Martin Wilheim Kutta around 1900 [3]. Among which the fourth-order Runge-Kutta method is a high-precision classical single-step method for solving ordinary differential equations, as is shown in equation (1) [4].

$$\begin{cases} k_1 = f(t_{n-1}, \varphi_{n-1}) \\ k_2 = f(t_{n-1} + \frac{h}{2}, \varphi_{n-1} + \frac{h}{2}k_1) \\ k_3 = f(t_{n-1} + \frac{h}{2}, \varphi_{n-1} + \frac{h}{2}k_2) \\ k_4 = f(t_{n-1} + h, \varphi_{n-1} + hk_3) \\ \varphi_n = \varphi_{n-1} + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{cases} \tag{1}$$

Where h is the step size, set to 0.01, use the MATLAB program to traverse, and find k_1, k_2, k_3, k_4 in turn. Then find the next discrete value φ_n.

2.2. The coordinate system transformation method

In order to facilitate the description of each point on the bench, it is chosen to establish the following bench coordinate system: for any bench, the center of the former handle is the coordinate origin, the width of the parallel plate is outside the spiral as the X-axis forward direction, and the length of the parallel plate is away from the forward direction of the bench as the Y-axis forward direction, as shown in Figure 3. Take any two different benches to establish the coordinate system called the I system and the J system respectively [5]. Where θ_i is the Angle between the Y-axis of the bench coordinate system and the Y-axis of the earth coordinate system, and the coordinates of the origins O of the bench systems in the earth coordinate system are(x_i, y_i), (x_j, y_j), respectively.

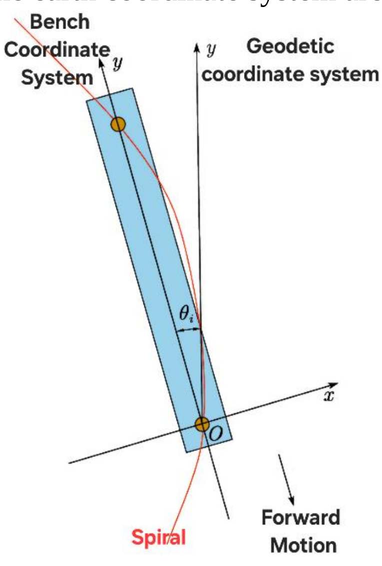

Figure 3. Schematic diagram of the bench and geodetic coordinate system

In addition, it is necessary to establish the transformation relationship between the I-system and the J-system in order to judge whether the benches i and j collide [6]. For any point on bench i in frame I, the coordinate is (x, y), the coordinate of the point transformed to the geodetic coordinate system is (x', y'), and then transformed to the coordinate system of bench j is (x'', y''), according to the coordinate transformation formula, shown in equation (2) and (3) [7]:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} cos\theta_i & -sin\theta_i \\ sin\theta_i & cos\theta_i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x_i \\ y_i \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} cos\theta_j & sin\theta_j \\ -sin\theta_j & cos\theta_j \end{pmatrix} \left(\begin{pmatrix} x' \\ y' \end{pmatrix} - \begin{pmatrix} x_j \\ y_j \end{pmatrix} \right) \tag{3}$$

Where x and x'' are in the range [-15,15], while y and y'' are in the range [-27.5,192.5].

In order to improve the running efficiency of the program, the judgment condition of bench collision can be simplified. As shown in Figure 4, when $i < j$, the position of bench i must be inside bench j. If i meets j, it must be that the edge of i located outside the spiral meets j. And the top of the bench is furthest from the handle. So, it is only needed to traverse j and transform the coordinates of the two vertices located outside the spiral in bench i to each J coordinate system for collision comparison with the position of j bench. At the same time, because the dragon head is the longest bench, if there is a collision, it must be the dragon head and the dragon body collide, so it is only needed to judge the situation of the dragon head and the dragon body collide.

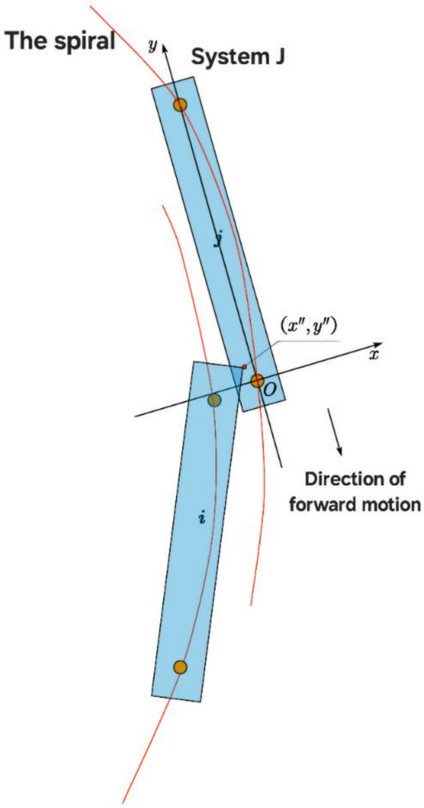

Figure 4. Schematic diagram of benches colliding

The coordinates of the two vertices of the dragon head are $(15, -27.5)$ and $(15, 313.5)$ in the coordinate system of the bench of dragon head, which are transformed into the coordinate system of the dragon body and bench j. When it is within the boundary of bench j, it can be judged that the dragon head and the dragon body collide, as shown in equation (4), that is, it satisfies:

$$\begin{cases} x'' \in [-15, 15] \\ y'' \in [-27.5, 192.5] \end{cases} \tag{4}$$

To traverse each bench, the recurrence relation between θ_i and the bench should be found. In the geodetic coordinate system, we have (5):

$$tan\theta_i = \frac{x_{i+1}-x_i}{y_{i+1}-y_i} \tag{5}$$

Due to the periodic nature of the tangent function, a classification discussion is needed to determine the value of θ_i[8].

2.3. Geometric optimization design model of bench dragon turning path

The turning path is an S-shaped curve formed by the tangent connection of two circular arcs. The radius of the former arc is twice that of the latter one, and it is tangent to the coil inlet and coil outlet spirals. The length of the turning path is (6), and the schematic diagram is given by Figure 5.

$$l = \theta_1 \times R + \theta_2 \times r \tag{6}$$

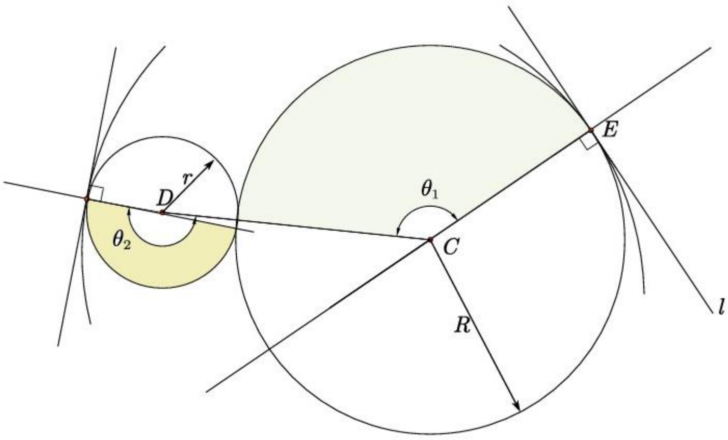

Figure 5. Schematic diagram of the turning path

To determine the inscribed circle of the spiral, the tangent lines of the spiral at each point need determining first. According to the algebraic meaning of tangent line, the equation is deduced (7) [9]:

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \tag{7}$$

The diameter of the circle is perpendicular to the tangent line. Because the arcs are tangent to each other, the distance between the arcs' centers C and D should be the sum of R and r (8). The geometric relationship is shown in Figure 6 [10].

$$\sqrt{(x_C - x_D)^2 + (y_C - y_D)^2} = R + r \tag{8}$$

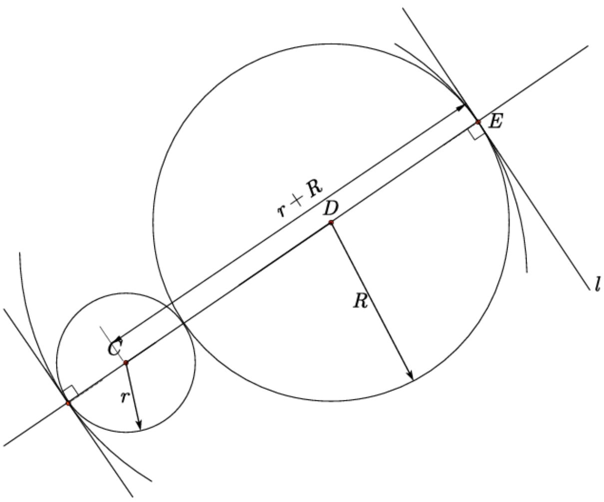

Figure 6. Schematic diagram of the geometric relationship

The distance between C and D is calculated according to the distance formula between two points. If it is not satisfied, the value of the circle radius r is adjusted, and the circle radius satisfying the condition is found by traversing the method. Then the length of the turning path is calculated according to the geometric relationship.

3. Results

3.1. The establishment of coiling-in model

Firstly, assume that the spiral is an equidistant spiral with a pitch of 55cm. According to the knowledge of advanced mathematics, the parametric equation of the spiral can be written as equation (9) [11]:

$$\begin{cases} x = \frac{55}{2\pi}\varphi cos\varphi \\ y = \frac{55}{2\pi}\varphi sin\varphi \end{cases} \tag{9}$$

Where φ is the polar angle of any point on the spiral, the range is $[0, 32\pi]$, φ is 32π at the starting point, and x, y is the value of the horizontal and vertical coordinates of the point.

φ decreases with time because the bench dragon coils in. Now, assuming that the linear velocity of the dragon head is constant at $1m/s$, the differential equation of the polar angle φ changing with time t can be obtained as equation (10) [12].

$$\varphi'(t) = -\frac{100}{\frac{55}{2\pi}\sqrt{1+\varphi^2(t)}} \tag{10}$$

The fourth-order Runge-Kutta method is used to obtain the motion results of the dragon head in the first 300 seconds. The trajectory of the front knob of the dragon head is given by Figure 7, while the change in dragon head-to center-angle by Figure 8.

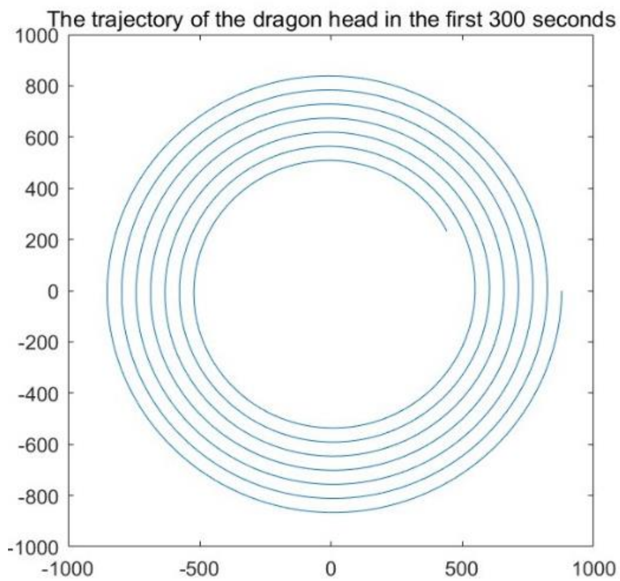

Figure 7. The trajectory of the dragon head in the first 300 seconds

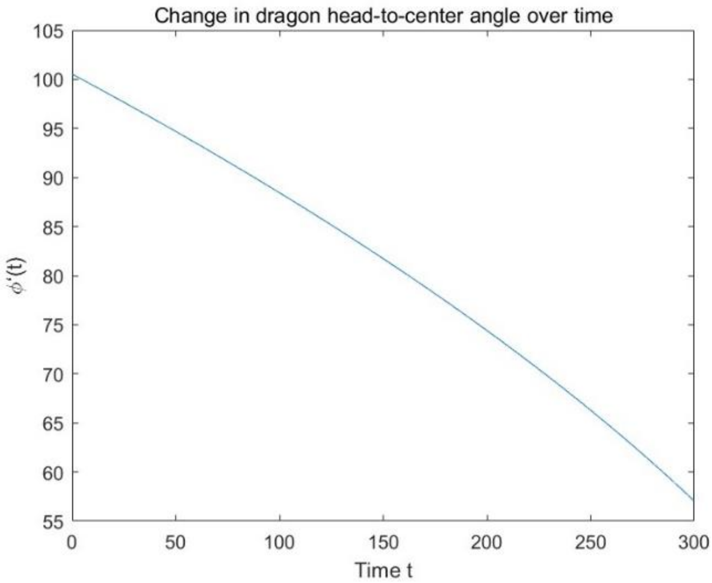

Figure 8. Change in dragon head-to center-angle in the first 300 seconds

The motion speed of the front knobs of some segments of the dragon body at specific times is given by Table 1.

Table 1. Motion speed of the front knobs (m/s)

	0s	120s	180s	240s	300s
The 1st section	0.882884	1.509208	0.681470	1.051889	0.599483
The 101st section	1.033734	1.389970	0.868177	0.563850	0.563850
The 201st section	1.048662	1.607009	1.021213	0.484260	0.500408

3.2. Solution of the minimum pitch

Since Bench-dragon needs to make a turning, we define a turning space, which is a circular region with a diameter of 9m. Before turning around, the bench dragon does not collide with each other, as is shown in Figure 9.

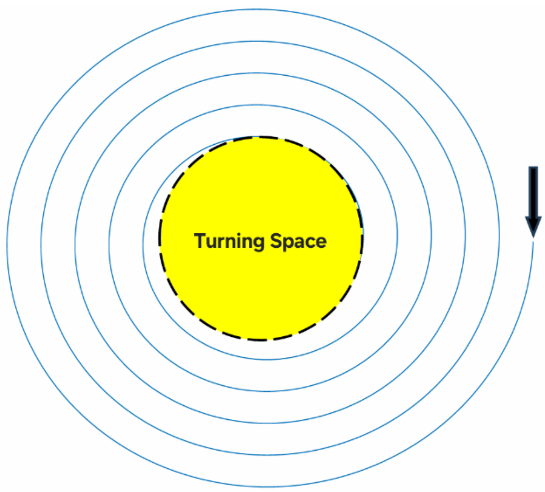

Figure 8. Change in dragon head-to center-angle in the first 300 seconds

Referring to the model of the relationship between the position and time of the head of the bench dragon, the pitch in the model was modified. Then, using the method of judging whether the benches collide, the minimum pitch can be obtained to make the benches not collide before turning round.

Using the traversal method, the minimum pitch is calculated as equation (11).

$$d_{min} = 45.2389cm \tag{11}$$

3.3. Solution of the turning path

The turning curve consists of two arcs tangent to each other, and the radius of the latter arc is half of the former one. The model is solved by using the traversal method and the idea of micro-element method. The length of the turning path is 972.3223cm. The trajectory of the dragon head is shown in Figure 9.

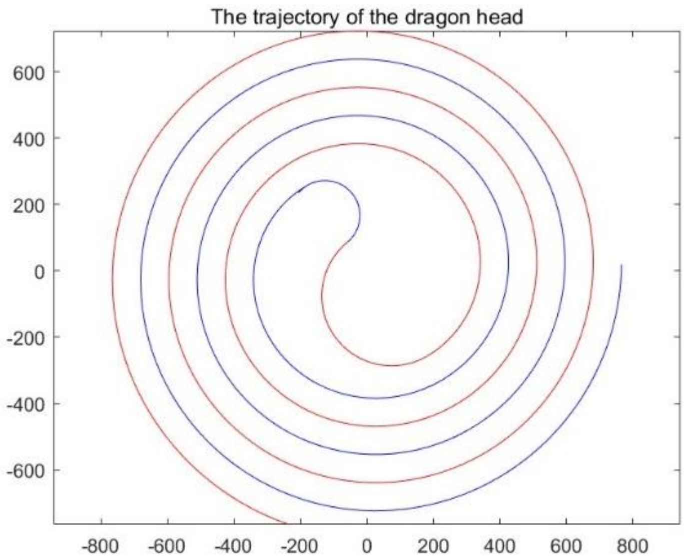

Figure 9. The trajectory of the dragon head

The relationship between the dragon head's pole angle and time is given in Figure 10.

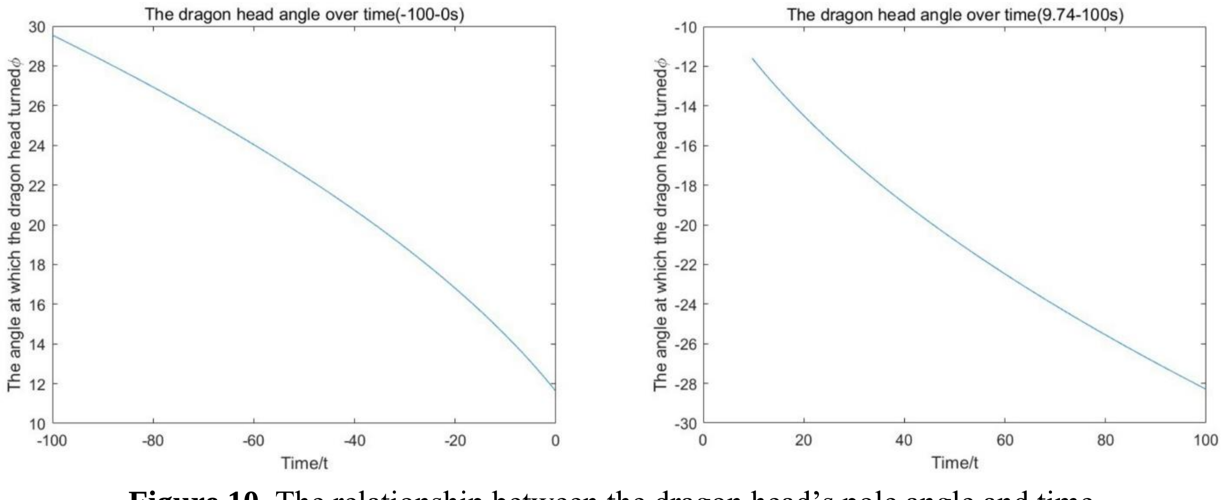

Figure 10. The relationship between the dragon head's pole angle and time

4. Conclusion

In this paper, the movement of bench dragon in a specific situation is studied, and the micro-element method and the traversal method are used in a comprehensive way. By constructing a variable pitch spiral model, the minimum pitch of the bench is calculated to be 45.2389 cm. Through the micro-element method and the traversal method, the length of the turning path is calculated to be 972.3223cm. Since our algorithm does not involve random sampling, only the error resulting from discretizing continuous values, the model should be reliable. The traversal step size changing, the results are found to be always within 5% of each other. Therefore, the algorithm in this paper is reliable.

As a tool to study and express the behavior of dynamic systems, the mathematical model of bench dragon can be extended to many fields. In the field of artificial intelligence, mathematical models of bench dragon can be used to develop intelligent control systems that enable robots or automated devices to imitate the movement of bench dragon. Such applications are not limited to performance, but can be extended to other domains that require complex motor control. The trajectory and mechanical properties of bench dragon can be analyzed in detail by mathematical models, which has potential applications in mechanical design and bionics. For example, studying the movement patterns of bench dragons could help engineers design more efficient locomotion devices or robots.

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